Caltech

Machine Learning & Data Mining

CS/CNS/EE 155

Lecture 8:

Structural SVMs Part 2 &

General Structured Prediction

Announcements

- Homework 2 due next Tuesday
  - 2pm via Moodle
- Homework 3 out next week
  - Due 2 weeks later
  - (Easier than HW2)
- Kaggle Mini-Project out next week
  - Due ~3 weeks later

Kaggle Mini-Project

- Training set of ~5K labeled data points
  - ~50 features
- Test set of unlabeled data points
  - Submit predictions on test set
- You choose the methods, loss functions, feature manipulations, etc.
  - Expected to do cross validation & model selection
  - Written report
    - Clearly & concisely documenting your process
    - Template will be provided
  - Groups of up to 3
- Due after ~3 weeks

Today

- Structural SVMs
  - Recap of Previous Lecture
  - Training
- General Structured Prediction
  - Brief Overview

Recap: 1st Order Sequential Model

- Input: $x = (x^1,\ldots,x^M)$
- Predict: $y = (y^1,\ldots,y^M)$
  - Each y^i one of L labels.

POS Tags:
Det, Noun, Verb, Adj, Adv, Prep

L=6

- Linear Model w.r.t. pairwise features $\phi^j(a,b \mid x)$:

$$F(y,x) \equiv \sum_{j=1}^{M} \left[w^T \varphi^j(y^j, y^{j-1} \mid x) \right]$$

- Prediction via maximizing F:

$$h(x) = \operatorname*{argmax}_y F(y,x)$$

Recap: Simple Example

$$F(y,x) \equiv \sum_{j=1}^{M} \left[w^T \varphi^j(y^j, y^{j-1} \mid x) \right]$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad \varphi^j(a,b \mid x) = \begin{bmatrix} \varphi_1^j(a \mid x) \\ \varphi_2(a,b) \end{bmatrix}$$

- "Unary Features"
- "Pairwise Transition Features"

$$\varphi_1^j(a \mid x) = \begin{bmatrix} 1_{[(a=Noun) \wedge (x^j = 'Fish')]} \\ 1_{[(a=Noun) \wedge (x^j = 'Sleep')]} \\ 1_{[(a=Verb) \wedge (x^j = 'Fish')]} \\ 1_{[(a=Verb) \wedge (x^j = 'Sleep')]} \end{bmatrix}$$

$$\varphi_2(a,b) = \begin{bmatrix} 1_{[(a=Noun) \wedge (b=Start)]} \\ 1_{[(a=Noun) \wedge (b=Noun)]} \\ 1_{[(a=Noun) \wedge (b=Verb)]} \\ 1_{[(a=Verb) \wedge (b=Start)]} \\ 1_{[(a=Verb) \wedge (b=Noun)]} \\ 1_{[(a=Verb) \wedge (b=Verb)]} \end{bmatrix}$$

x = “Fish Sleep” y = (N,V)

$$w_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \varphi_1^j(a \mid x) = \begin{bmatrix} 1_{[(a=Noun)\wedge(x^j='Fish')]} \\ 1_{[(a=Noun)\wedge(x^j='Sleep')]} \\ 1_{[(a=Verb)\wedge(x^j='Fish')]} \\ 1_{[(a=Verb)\wedge(x^j='Sleep')]} \end{bmatrix}$$

$$w_2 = \begin{bmatrix} 1 \\ -2 \\ 2 \\ -1 \\ 1 \\ -2 \end{bmatrix} \quad \varphi_2^j(a,b) = \begin{bmatrix} 1_{[(a=Noun)\wedge(b=Start)]} \\ 1_{[(a=Noun)\wedge(b=Noun)]} \\ 1_{[(a=Noun)\wedge(b=Verb)]} \\ 1_{[(a=Verb)\wedge(b=Start)]} \\ 1_{[(a=Verb)\wedge(b=Noun)]} \\ 1_{[(a=Verb)\wedge(b=Verb)]} \end{bmatrix}$$

$$F(y=(N,V), x="Fish\ Sleep") = w_1^T \varphi_1^1(N,x) + w_2^T \varphi_2(N,Start) + w_1^T \varphi_1^2(V,x) + w_2^T \varphi_2(V,N)$$
$$= w_{1,1} + w_{2,1} + w_{1,4} + w_{2,5} = 2+1+0+1 = 4$$

Prediction: $\underset{y}{\operatorname{argmax}} F(y,x)$

y	F(y,x)
(N,N)	2+1+1-2 = 2
(N,V)	2+1+0+1 = 4
(V,N)	1-1+1+2 = 3
(V,V)	1-1+0-2 = -1

Structured Prediction

- Complex output spaces
  - All possible Part-of-Speech sequences
- Naïve prediction is often exponential time:

$$\operatorname*{argmax}_{y} F(y,x)$$

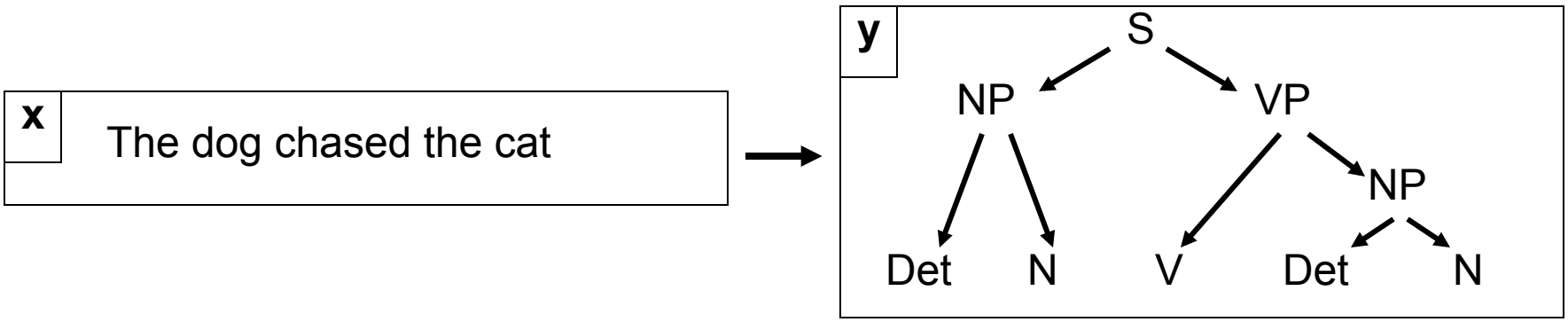
- Evaluation is also multivariate:

$$\ell\left(y,x,F\right)=\sum_{j=1}^{M}1_{\left[h(x)^{j}\neq y^{j}\right]}$$

E.g., Hamming Loss

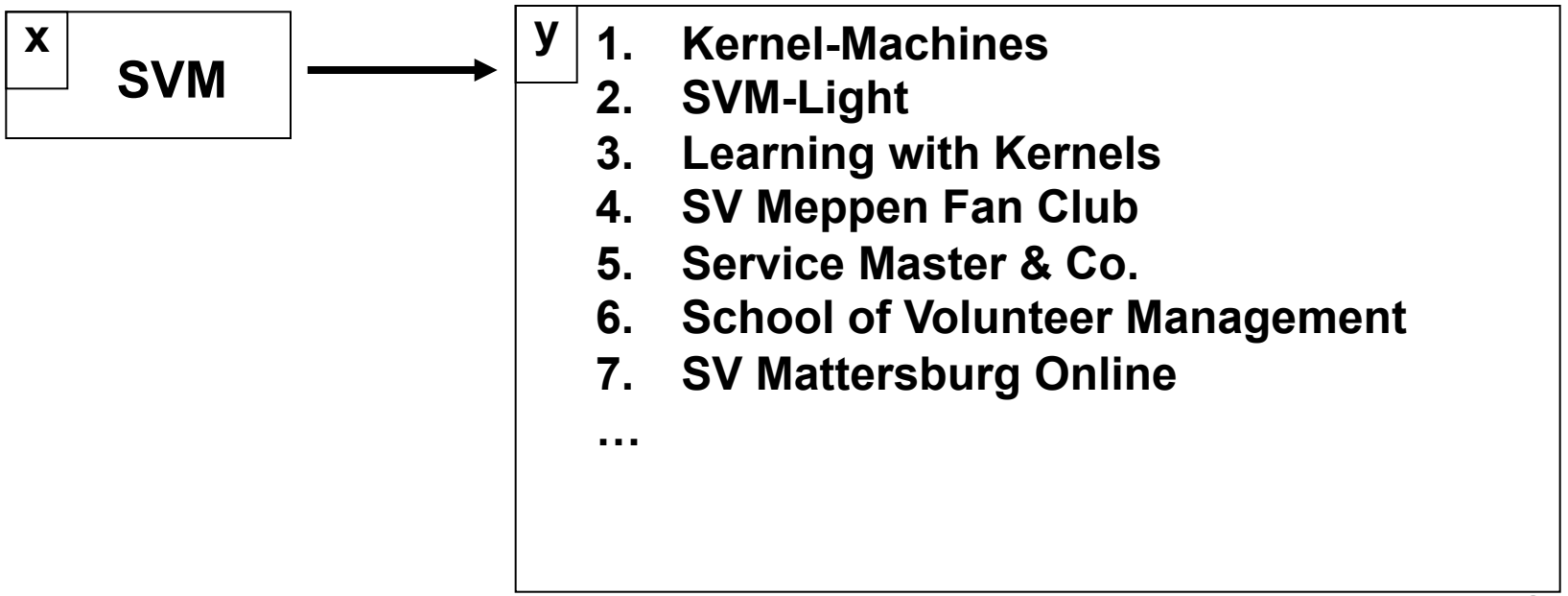
Examples of Complex Output Spaces

- Natural Language Parsing
  - Given a sequence of words *x*, predict the parse tree *y.*
  - Dependencies from structural constraints, since *y* has to be a tree.

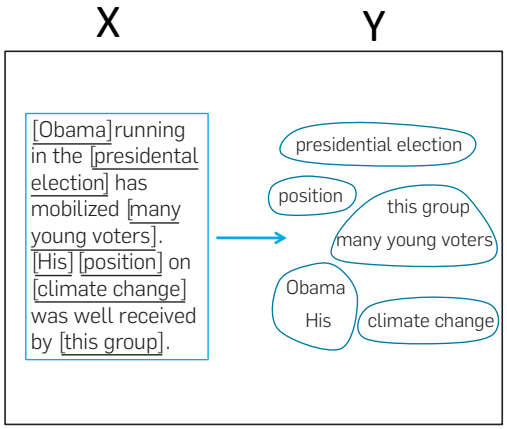


Examples of Complex Output Spaces

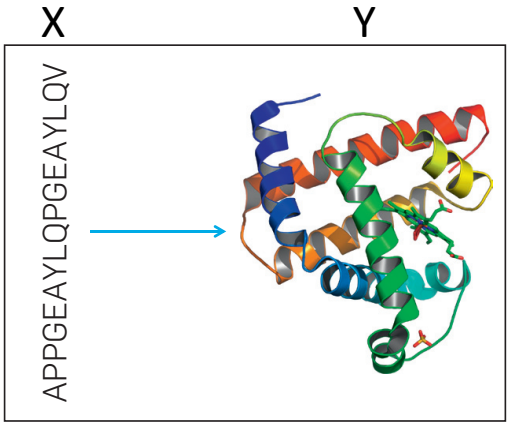
- Information Retrieval
  - Given a query x, predict a ranking *y*.
  - Dependencies between results (e.g. avoid redundant hits)
  - Loss function over rankings (e.g. Average Precision)



Examples of Complex Output Spaces



Co-reference Resolution



Protein Folding

Stereo (binocular) Depth Detection

Will see examples later in lecture.

Training Structured Prediction Models

- General Form:

$$\underset{w}{\operatorname{argmin}} \frac{\lambda}{2}\|w\|^2 + \sum_{i=1}^{N} \ell\left(y_i, x_i, F\right)$$

- Or equivalently:

$$\underset{w}{\operatorname{argmin}} \frac{1}{2}\|w\|^2 + \frac{C}{N}\sum_{i=1}^{N} \ell\left(y_i, x_i, F\right)$$

- Require continuous surrogate of evaluation measure.

Structural SVM

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

Sometimes normalize by M

"Slack"

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i \qquad \forall i : \xi_i \geq 0$$

Consider: $y' = \underset{y}{\operatorname{argmax}} F(y, x)$ ➔ $F(y_i, x_i) - F(y', x_i) \leq 0$

Prediction of Learned Model

Slack is continuous upper bound on Hamming Loss!

$y' \neq y_i$ ➔ $\xi_i \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]}$

$y' = y_i$ ➔ $\xi_i \geq 0$

Example 1

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i \qquad \forall i : \xi_i \geq 0$$

x_i = "Fish Sleep"

y_i = (N,V)

$\xi_i = 0$

y'	F(y',x_i)	F(y_i,x_i) – F(y',x_i)	Loss
(N,N)	2	2	1
(N,V)	4	0	0
(V,N)	1	3	2
(V,V)	1	3	1

Example 2

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i \qquad \forall i : \xi_i \geq 0$$

x_i = "Fish Sleep"

y_i = (N,V)

$\xi_i = 2$

y'	F(y',x_i)	F(y_i,x_i) – F(y',x_i)	Loss
(N,N)	4	-1	1
(N,V)	3	0	0
(V,N)	0	3	2
(V,V)	1	2	1

Example 3

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i \qquad \forall i : \xi_i \geq 0$$

x_i = "Fish Sleep"

y_i = (N,V)

$\xi_i = 1$

y'	F(y',x_i)	F(y_i,x_i) – F(y',x_i)	Loss
(N,N)	2	2	1
(N,V)	4	0	0
(V,N)	3	1	2
(V,V)	1	3	1

When is Slack Positive?

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i \qquad \forall i : \xi_i \geq 0$$

- Whenever margin not big enough!

$$\xi_i > 0 \iff \exists y' : \; F(y_i, x_i) - F(y', x_i) < \sum_j 1_{\left[y'^j \neq y_i^j\right]}$$

$$\xi_i = \max_{y'} \left\{ \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \left(F(y_i, x_i) - F(y', x_i) \right) \right\} = \ell(y_i, x_i, F)$$

Verify that above definition ≥0

When is Slack Positive?

$$\underset{w}{\operatorname{argmin}} \frac{1}{2}\|w\|^2 + \frac{C}{N}\sum_{i=1}^{N} \ell\left(y_i, x_i, F\right)$$

Hamming Hinge Loss

- Whenever margin not big enough!

$$\xi_i > 0 \;\leftrightarrow\; \exists y' :\; F(y_i, x_i) - F(y', x_i) < \sum_j 1_{\left[y'^j \neq y_i^j\right]}$$

$$\xi_i = \max_{y'} \left\{ \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \left(F(y_i, x_i) - F(y', x_i)\right) \right\} = \ell(y_i, x_i, F)$$

Verify that above definition ≥0

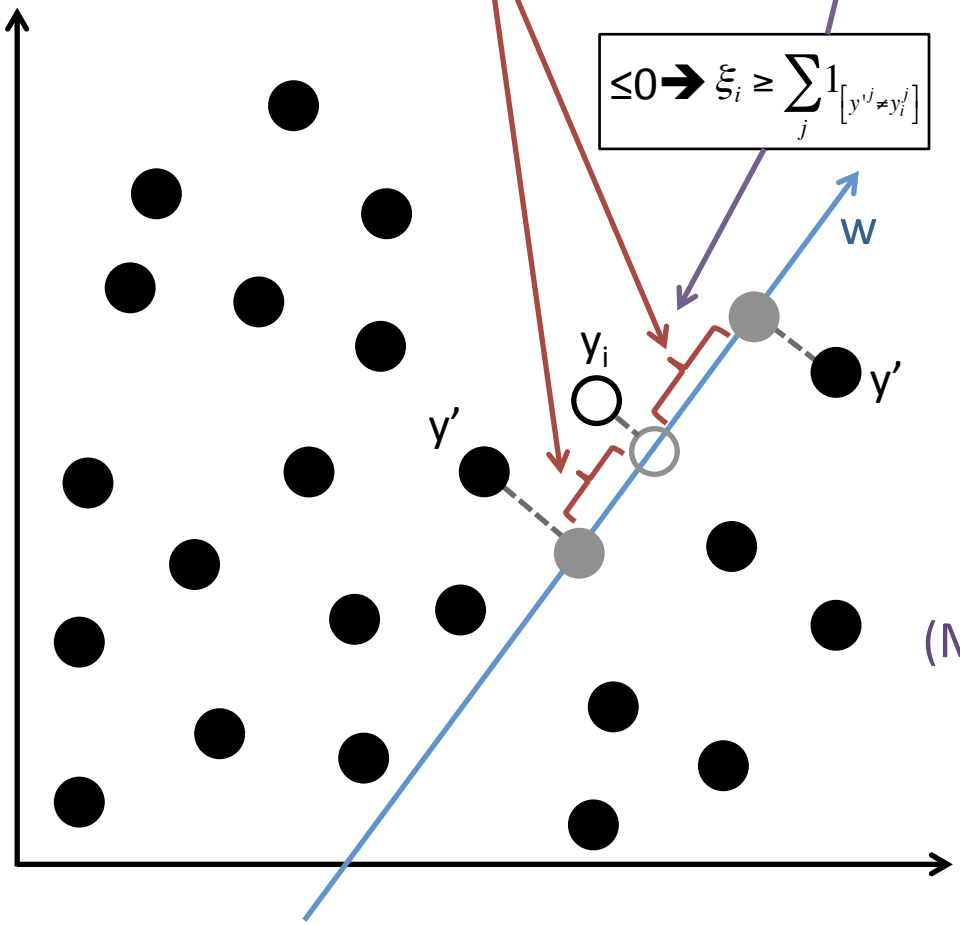
Structural SVM
Geometric Interpretation

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i \qquad \forall i : \xi_i \geq 0$$

$$F(y,x) \equiv w^T \sum_{j=1}^{M} \varphi^j(y^j, y^{j-1} \mid x)$$

High Dimensional Point



Size of Margin
vs
Size of Margin Violations
(C controls trade-off)
(Margin scaled by Hamming Loss)

Structural SVM Training

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i \qquad \forall i : \xi_i \geq 0$$

Often Exponentially Many!

- Strictly convex optimization problem
  - Same form as standard SVM optimization
  - Easy right?
- **Intractable # of constraints!**

Structural SVM Training

$$\forall y' : \quad F(y_i, x_i) \geq F(y', x_i) + \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i$$

- The trick is to not enumerate all constraints.
- Only solve the SVM objective over a small subset of constraints (**working set**).
  - Efficient!
- But some constraints might be violated.

Example

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i \qquad \forall i : \xi_i \geq 0$$

x_i = "Fish Sleep"

y_i = (N,V)

$\xi_i = 0$

y'	$F(y', x_i)$	$F(y_i, x_i) - F(y', x_i)$	Loss
(N,N)	2	2	1
(N,V)	4	0	0
(V,N)	3	1	2
(V,V)	1	3	1

Approximate Hinge Loss

- Choose tolerate ε>0:

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon \qquad \forall i : \xi_i \geq 0$$

Consider: $y' = \underset{y}{\operatorname{argmax}} F(y, x)$ ➔ $F(y_i, x_i) - F(y', x_i) \leq 0$

Prediction of Learned Model

Slack is continuous upper bound on Hamming Loss - ε!

$y' \neq y_i$ ➔ $\xi_i \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \varepsilon$

$y' = y_i$ ➔ $\xi_i \geq 0$

Example

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon \qquad \forall i : \xi_i \geq 0$$

x_i = "Fish Sleep"

y_i = (N,V)

$\xi_i = 0$

$\varepsilon = 1$

y'	$F(y', x_i)$	$F(y_i, x_i) - F(y', x_i)$	Loss	
(N,N)	2	2	1	✓
(N,V)	4	0	0	✓
(V,N)	3	1	2	✗
(V,V)	1	3	1	✓

Structural SVM Training

- **STEP 0:** Specify tolerance ε
- **STEP 1:** Solve SVM objective function using only working set of constraints **W** (initially empty). The trained model is w.
- **STEP 2:** Using w, find the y' whose constraint is most violated.
- **STEP 3:** If constraint is violated by more than ε, add it to **W**.

Constraint Violation Formula:

$$\left(\frac{1}{M_i}\sum_j 1_{\left[y'^j \neq y_i^j\right]} + \xi_i\right) - \left(F(y_i, x_i) - F(y', x_i)\right) \geq \varepsilon$$

- **Repeat STEP 1-3** until no additional constraints are added. Return most recent model w trained in STEP 1.

*This is known as a "cutting plane" method.

Example

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

Choose ε=0.1

$$\forall i, y' \in W_i : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon \qquad \forall i : \xi_i \geq 0$$

Init: $W_i = \varnothing$

Solve: $\xi_i = 0$

x_i = "Fish Sleep"

y_i = (N,V)

y'	$F(y',x_i)$	$F(y_i,x_i) - F(y',x_i)$	Loss	Viol.	
(N,N)	0	0	1	1	✗
(N,V)	0	0	0	0	✓
(V,N)	0	0	2	2	✗
(V,V)	0	0	1	1	✗

Constraint Violation: Loss – Slack – (F(y,x)-F(y',x)) = Viol

Example

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

Choose ε=0.1

$$\forall i, y' \in W_i : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon \qquad \forall i : \xi_i \geq 0$$

Update: $W_i = \{(V, N)\}$

Solve: $\xi_i = 0$

x_i = "Fish Sleep"

y_i = (N,V)

y'	$F(y',x_i)$	$F(y_i,x_i) - F(y',x_i)$	Loss	Viol.	
(N,N)	0	0	1	1	✗
(N,V)	0	0	0	0	✓
(V,N)	0	0	2	2	✗
(V,V)	0	0	1	1	✗

Constraint Violation: Loss – Slack – (F(y,x)-F(y',x)) = Viol

Example

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

Choose ε=0.1

$$\forall i, y' \in W_i : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon \qquad \forall i : \xi_i \geq 0$$

Update: $W_i = \{(V, N)\}$

Solve: $\xi_i = 0.5$

x_i = "Fish Sleep"

y_i = (N,V)

y'	$F(y',x_i)$	$F(y_i,x_i) - F(y',x_i)$	Loss	Viol.	
(N,N)	0.7	0.2	1	0.2	✗
(N,V)	0.9	0	0	0	✓
(V,N)	-0.6	1.5	2	0	✓
(V,V)	0	0.9	1	0.4	✗

Constraint Violation: Loss – Slack – (F(y,x)-F(y',x)) = Viol

Example

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

Choose ε=0.1

$$\forall i, y' \in W_i : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon \qquad \forall i : \xi_i \geq 0$$

Update: $W_i = \{(V,N),(N,N)\}$ **Solve:** $\xi_i = 0.5$

x_i = "Fish Sleep"

y_i = (N,V)

y'	F(y',x_i)	F(y_i,x_i) – F(y',x_i)	Loss	Viol.	
(N,N)	0.7	0.2	1	0.2	✗
(N,V)	0.9	0	0	0	✓
(V,N)	-0.6	1.5	2	0	✓
(V,V)	0	0.9	1	0.4	✗

Constraint Violation: Loss – Slack – (F(y,x)-F(y',x)) = Viol

Example

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

Choose ε=0.1

$$\forall i, y' \in W_i : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon \qquad \forall i : \xi_i \geq 0$$

Update: $W_i = \{(V,N),(N,N)\}$ **Solve:** $\xi_i = 0.55$

x_i = "Fish Sleep"

y_i = (N,V)

y'	F(y',x_i)	F(y_i,x_i) – F(y',x_i)	Loss	Viol.	
(N,N)	0.55	0.45	1	0	✓
(N,V)	1	0	0	0	✓
(V,N)	-0.65	1.65	2	0	✓
(V,V)	-0.05	0.95	1	0.05	✗

Constraint Violation: Loss – Slack – (F(y,x)-F(y',x)) = Viol

Example

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' \in W_i : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon \qquad \forall i : \xi_i \geq 0$$

Choose ε=0.1

Solve: $\xi_i = 0.55$

No constraint is violated by more than **ε**

			Loss	Viol.	
			1	0	✓
			0	0	✓
			2	0	✓
(V,V)	-0.05	0.95	1	0.05	✗

Constraint Violation: Loss – Slack – (F(y,x)-F(y',x)) = Viol

Geometric Interpretation

Scoring y corresponds to dot product of high dimensional point.

$$F(y,x) \equiv w^T \sum_{j=1}^{M} \left[\varphi^j(y^j, y^{j-1} \mid x) \right]$$

High Dimensional Point

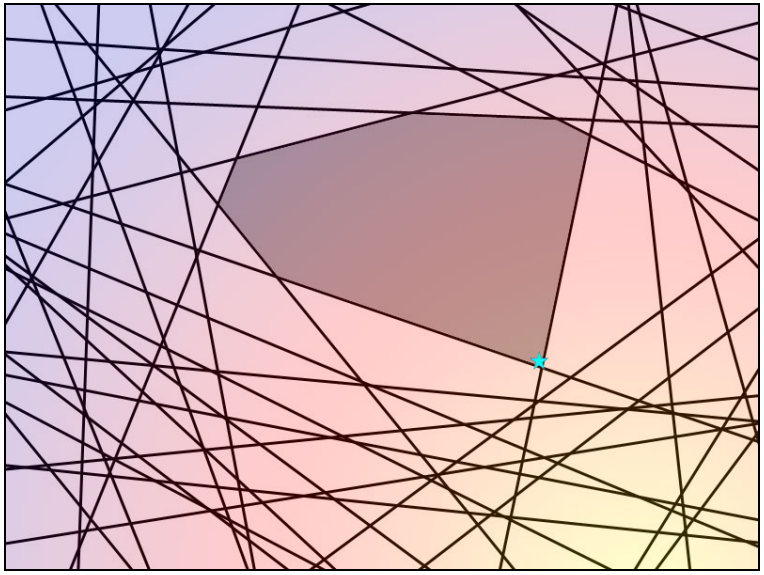
$$\Psi_{i,y'} = \sum_{j=1}^{M} \left[\varphi^j(y_i^j, y_i^{j-1} \mid x_i) - \varphi(y'^j, y'^{j-1} \mid x_i) \right]$$

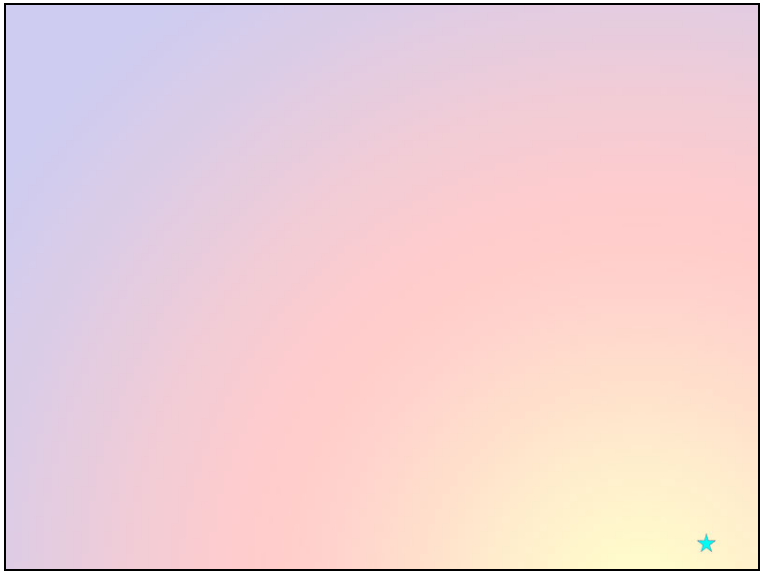
$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

Quad. Optimization Problem w/ Linear Constraints!

$$\forall i, y' : w^T \Psi_{i,y'} \geq \Delta_{i,y'} - \xi_i \qquad \forall i : \xi_i \geq 0$$

Geometric Example





Naïve SVM Problem

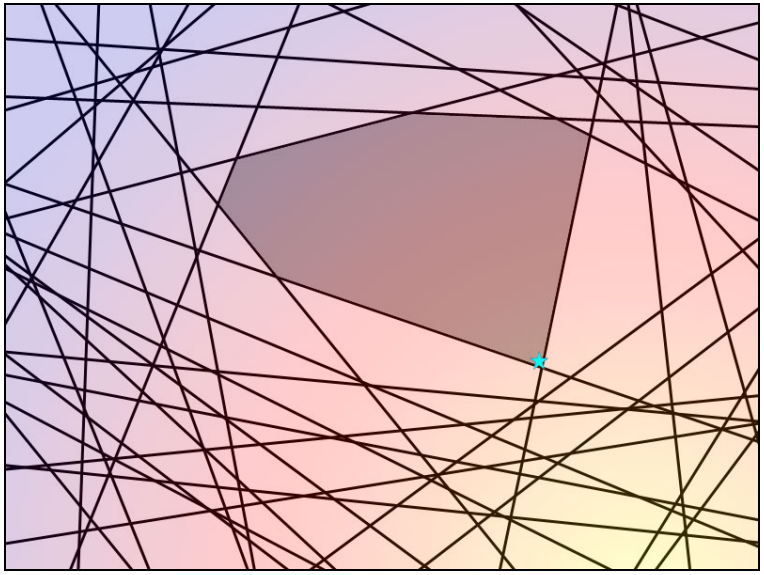
- Exponential constraints
- Most are dominated by a small set of "important" constraints

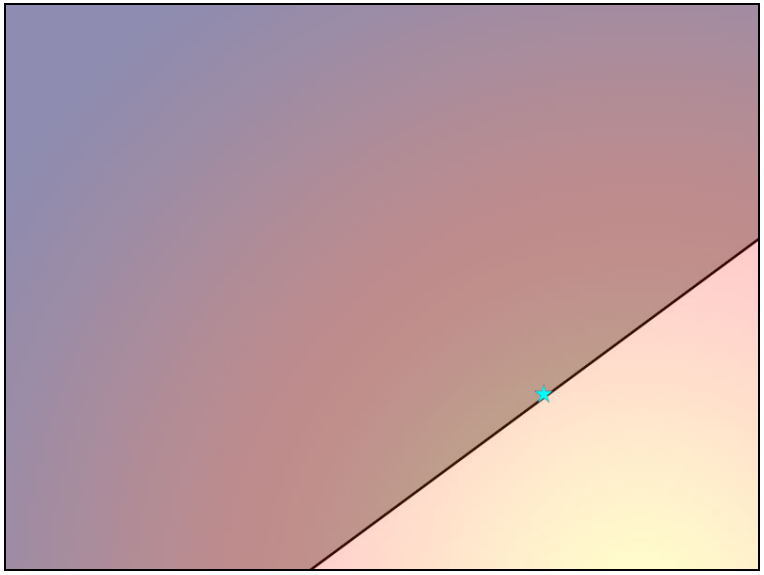
Structural SVM Approach

- Repeatedly finds the next most violated constraint…
- …until set of constraints is a good approximation.

*This is known as a "cutting plane" method.

Geometric Example





Naïve SVM Problem

- Exponential constraints
- Most are dominated by a small set of "important" constraints

Structural SVM Approach

- Repeatedly finds the next most violated constraint…
- …until set of constraints is a good approximation.

*This is known as a "cutting plane" method.

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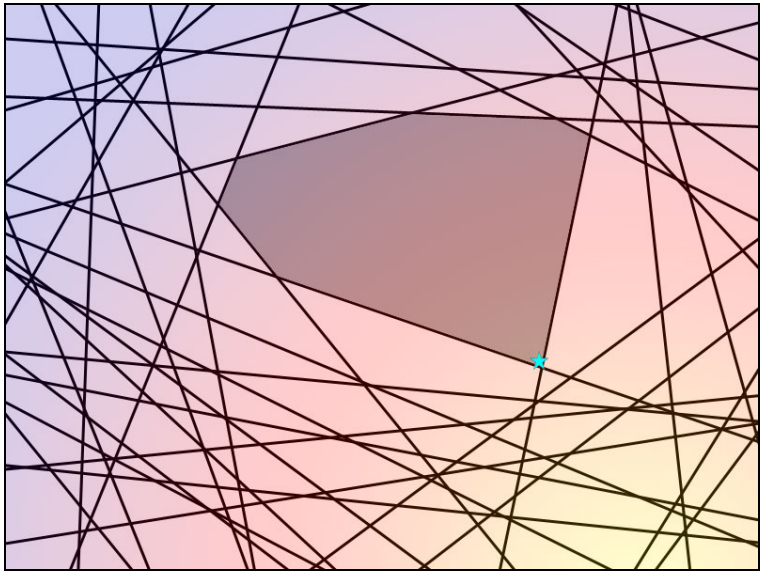

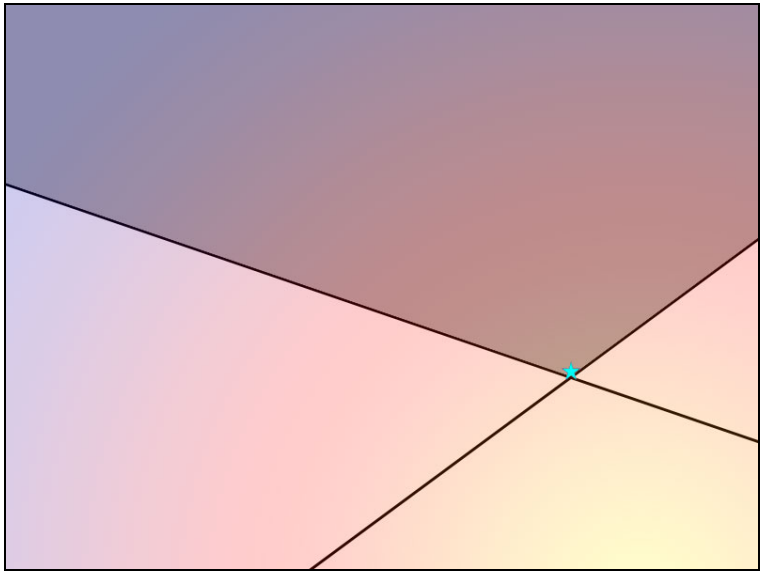

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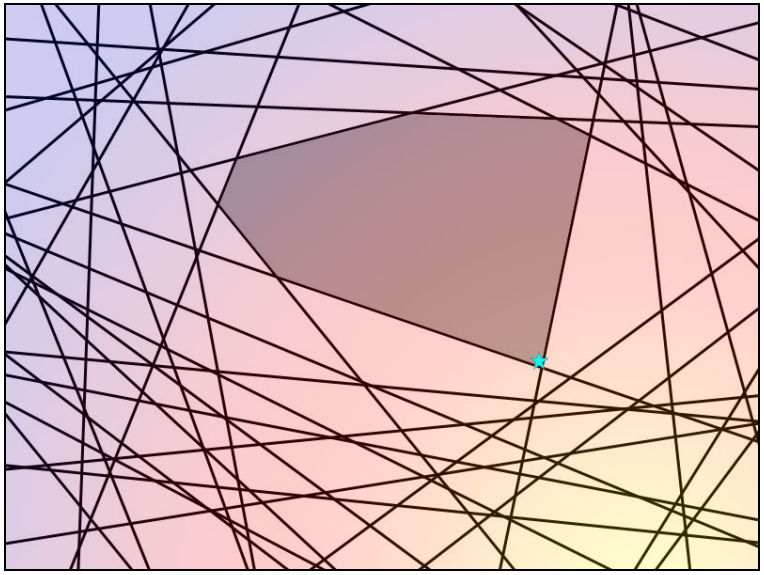

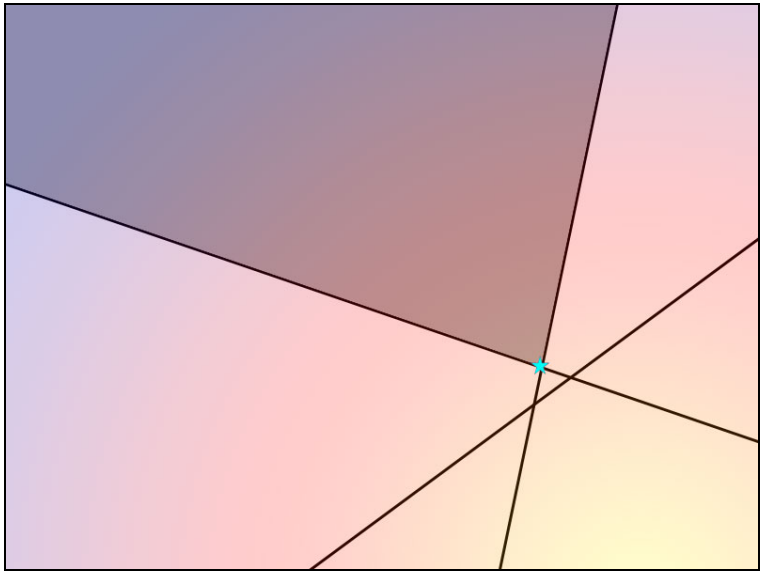

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Linear Convergence Rate

- Guarantee for any ε>0:

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i - \varepsilon$$

$$\forall i : \xi_i \geq 0$$

- Terminates after #iterations: $O\left(\frac{1}{\varepsilon}\right)$

Proof found in:
http://www.cs.cornell.edu/people/tj/publications/joachims_etal_09a.pdf

Finding Most Violated Constraint

- A constraint is violated when:

$$F(y', x_i) - F(y_i, x_i) + \sum_j 1_{\left[y'^j \neq y_i^j\right]} - \xi_i > 0$$

- Finding most violated constraint reduces to

$$\operatorname*{argmax}_{y'} F(y', x_i) + \sum_j 1_{\left[y'^j \neq y_i^j\right]}$$

"Loss augmented inference"

- Highly related to prediction:

$$\operatorname*{argmax}_{y} F(y, x_i)$$

"Augmented" Scoring Function

$$F(y,x_i) \equiv \sum_{j=1}^{M}\left[w^T\varphi^j(y^j,y^{j-1}\mid x_i)\right]$$

Goal:

$$\underset{y'}{\operatorname{argmax}}\, F(y',x_i) + \sum_j 1_{\left[y'^j \neq y_i^j\right]}$$

Solve Using Viterbi!

$$\tilde{F}(y,x_i,y_i) \equiv \sum_{j=1}^{M}\left[\tilde{w}^T\tilde{\varphi}^j(y^j,y^{j-1}\mid x_i,y_i)\right]$$

$$\tilde{\varphi}^j(a,b\mid x_i,y_i) = \begin{bmatrix} \varphi^j(a,b\mid x_i) \\ 1_{\left[a\neq y_i^j\right]} \end{bmatrix}$$

Additional Unary Feature!

$$\tilde{w} = \begin{bmatrix} w \\ 1 \end{bmatrix}$$

Goal: $\underset{y'}{\operatorname{argmax}}\, \tilde{F}(y',x_i,y_i)$

Recap: Structural SVM

- Define structured scoring function: $F(y,x)$
  - E.g., 1st order sequential model
  - Efficient prediction algorithm
- Define error function: $\Delta(y,y')$
  - E.g., Hamming Loss: $\Delta(y,y') = \sum_j 1_{\left[y'^j \neq y^j\right]}$
- Train by iteratively finding most violated constraint:

$$\operatorname*{argmax}_{y'} F(y',x_i) + \Delta(y_i,y')$$

  - Requires efficient algorithm (often same as prediction)

Structural SVMs vs CRFs

- SVM Objective:

$$\underset{w,\xi}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i \xi_i$$

$$\forall i, y' : F(y_i, x_i) - F(y', x_i) \geq \Delta(y_i, y') - \xi_i \qquad \forall i : \xi_i \geq 0$$

SVM only cares about y' that violates margin the most!

- Scales margin by loss of y'

- CRF Objective:

$$\underset{w}{\operatorname{argmin}} \frac{1}{2} w^T w + \frac{C}{N} \sum_i -\log P(y_i \mid x_i)$$

$$-\log P(y_i \mid x_i) = F(y_i, x_i) - \log\left(\sum_{y'} \exp\{F(y', x_i\} \right)$$

CRF cares about all y' so that:

- Incorrect P(y'|x) is minimized
- Correct P(y|x) is maximized

http://svmlight.joachims.org/svm_struct.html

http://mallet.cs.umass.edu/

General Structured Prediction

More Elaborate Scoring Functions

- Structure encoded by linear scoring function:

$$F(y,x)$$

- 2nd Order Sequential Model:

$$F(y,x) \equiv \sum_{j=1}^{M} \left[w^T \varphi^j (y^j, y^{j-1}, y^{j-2} \mid x) \right]$$

- Classification Model:

$$F(y,x) \equiv w^T \varphi(y \mid x)$$

- Efficient Prediction:

$$\operatorname*{argmax}_y F(y,x)$$

More Elaborate Scoring Functions

- Structure encoded by linear scoring function:

$$F(y, x)$$

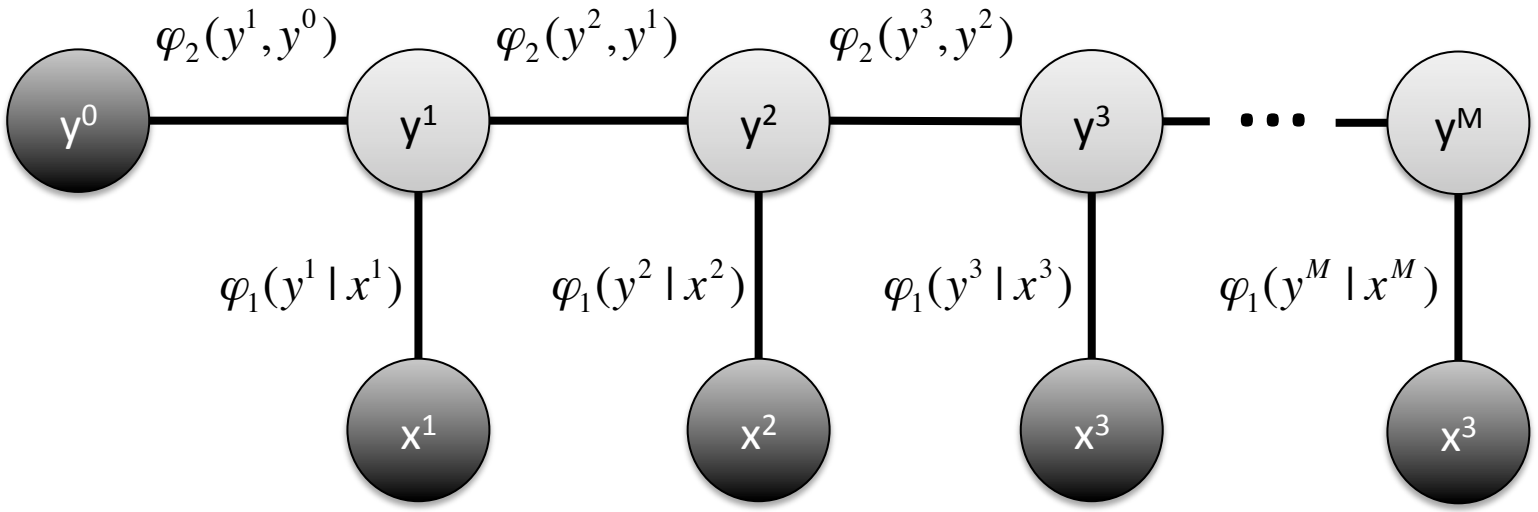
Remainder of Lecture:
Tour of Structured Prediction Models
Some Might be Interesting to You…

- Efficient Prediction:

$$\operatorname*{argmax}_{y} F(y, x)$$

Graphical Models

$$\varphi^j(a,b \mid x) = \begin{bmatrix} \varphi_1(a \mid x^j) \\ \varphi_2(a,b) \end{bmatrix}$$



Graph structure encodes structural dependencies between y^j!

https://piazza.com/cornell/fall2013/btry6790cs6782/resources

http://www.cs.cmu.edu/~guestrin/Class/10708/

https://www.coursera.org/course/pgm

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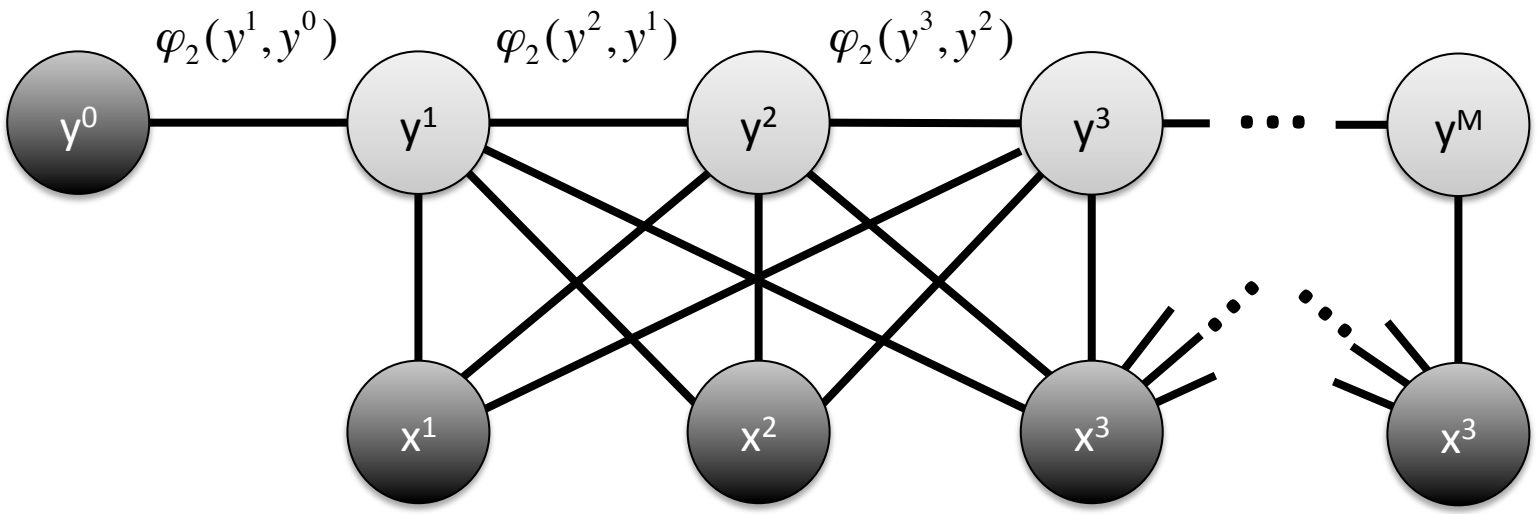

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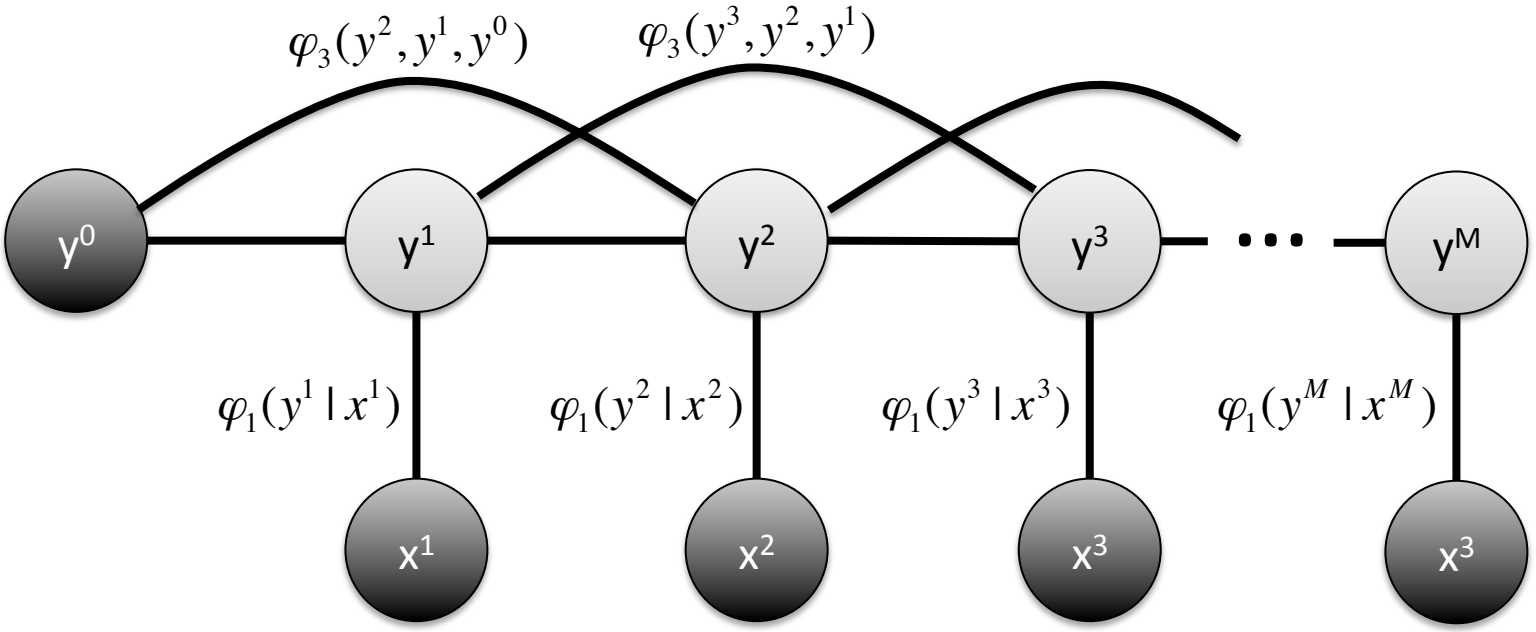
Graphical Models

$$\varphi^j(a,b,c \mid x) = \begin{bmatrix} \varphi_1(a \mid x^j) \\ \varphi_3(a,b,c) \end{bmatrix}$$



Graph structure encodes structural dependencies between y^j!

Features depend on cliques in graphical model representation.

https://piazza.com/cornell/fall2013/btry6790cs6782/resources

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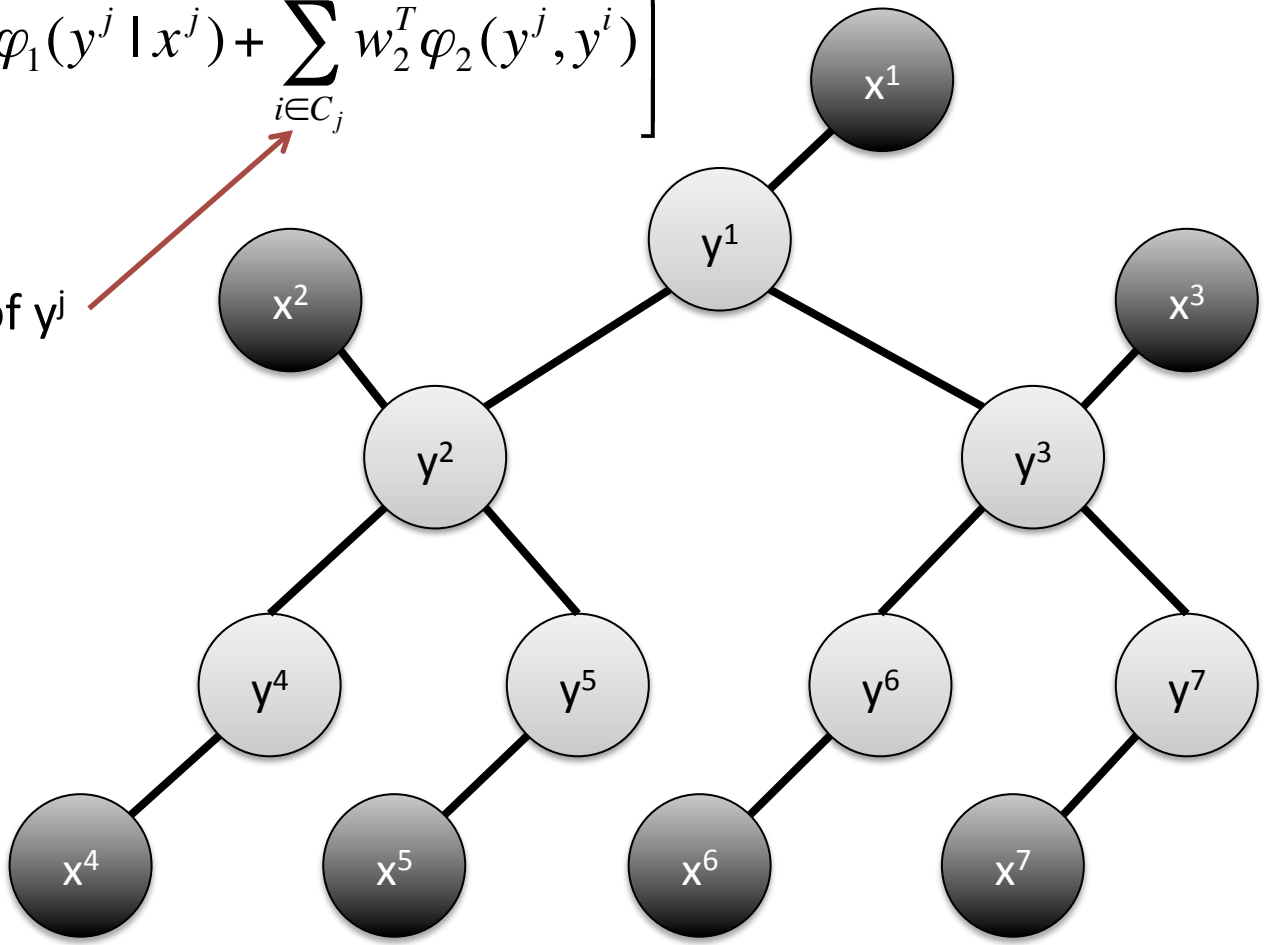
Tree Structured Models

$$F(y,x) \equiv \sum_{j=1}^{M} \left[w_1^T \varphi_1(y^j \mid x^j) + \sum_{i \in C_j} w_2^T \varphi_2(y^j, y^i) \right]$$

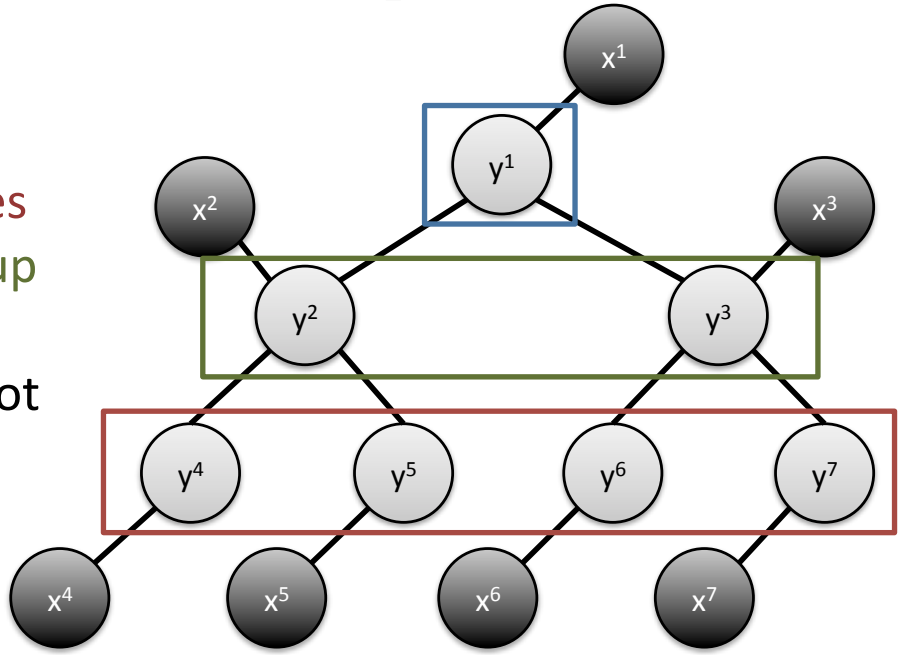
Child nodes of y^j



Prediction via Dynamic Programming

$$F(y,x) \equiv \sum_{j=1}^{M} \left[w_1^T \varphi_1(y^j \mid x^j) + \sum_{i \in C_j} w_2^T \varphi_2(y^j, y^i) \right]$$

1. Solve partial solutions of Leaves
2. Solve partial sol. of next level up
3. Repeat Step 2 until Root
4. Pick best partial solution of Root



*Max-Product Algorithm for Tree Graphical Models

*Viterbi = Max-Product for Linear Chain Graphical Models

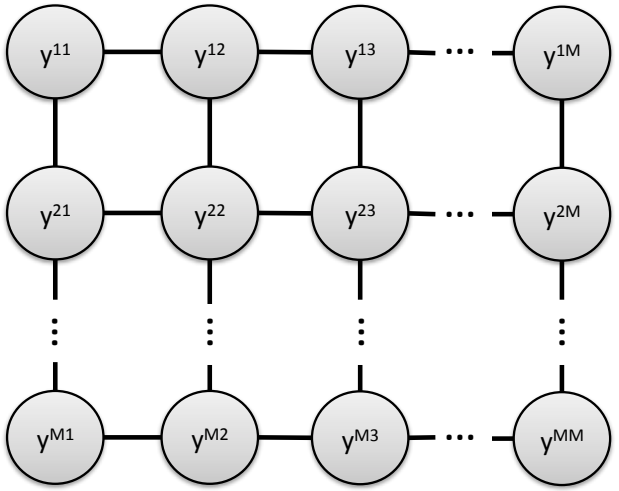
Loopy Graphical Models

Stereo (binocular) Depth Detection

- Each y^{ij} is depth of pixel
- Neighbor pixels are similar
- Features over pairs of pixels

- "Loopy" Graphical Model
- Prediction is NP-Hard!

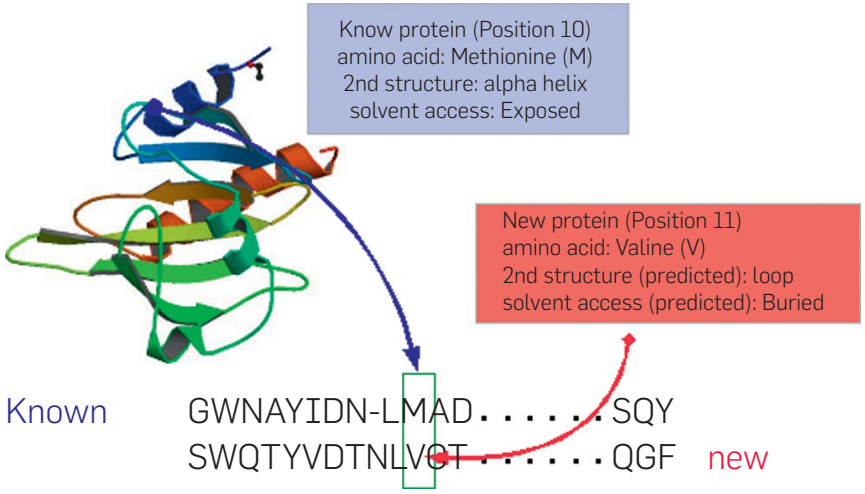
$$\underset{y}{\operatorname{argmax}} F(y, x)$$



x suppressed for brevity

http://vision.middlebury.edu/MRF/
http://www.seas.upenn.edu/~taskar/pubs/mmamn.pdf
http://www.cs.cornell.edu/~rdz/Papers/SZ-visalg99.pdf

String Alignment



x = pair of strings (one from **D**)
y = alignment

Predict Folding Structure
& Function of Protein

Database **D** of Known Proteins
(very well studied)

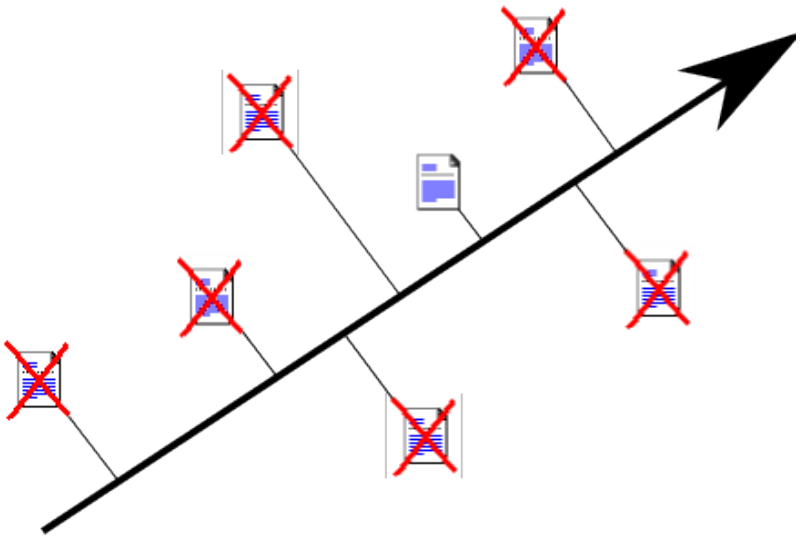
Larger Database **G** of Homologies
(proteins w/ known similarities to **D**)

Train on **G**: learn how to align any amino acid seq to proteins in **D**

$F(y,x)$ encodes score of different types of substitutions, insertions & deletions

http://www.cs.cornell.edu/People/tj/publications/yu_etal_06a.pdf
See Also: http://journals.plos.org/ploscompbiol/article?id=10.1371/journal.pcbi.1000173

Ranking



Find w that predicts best ranking of search results.

Every relevant result should be above every non-relevant result.

$$y^{ij} \in \{-1, +1\}$$

$$F(y, x) = \sum_{i,j} y^{ij} \left[w^T \varphi(x^i) - w^T \varphi(x^j) \right]$$

$$\underset{y}{\operatorname{argmax}} F(y, x) = \operatorname{sort} \left\{ w^T \varphi(x^j) \right\}_j$$

x = query & set of results
y = ranking

http://www.cs.cornell.edu/People/tj/publications/joachims_05a.pdf
http://research.microsoft.com/en-us/um/people/cburges/tech_reports/MSR-TR-2010-82.pdf
http://www.yisongyue.com/publications/sigir2007_svmmap.pdf

Summary: Structured Prediction

- Very general setting
  - Applicable to prediction made jointly over multiple y's
  - Prediction in Graphical Models
- Many learning algorithms for structured prediction
  - CRFs, SSVMs, Structured Perceptron, Learning Reductions
- Topic for Entire Class!

http://www.nowozin.net/sebastian/cvpr2011tutorial/

http://www.cs.cmu.edu/~nasmith/sp4nlp/

http://www.cs.cornell.edu/Courses/cs778/2006fa/

https://www.sites.google.com/site/spflodd/

http://www.cs.cornell.edu/People/tj/publications/joachims_06b.pdf

Next Week

- Decision Trees
- Bagging
- Random Forests
- Boosting
- Ensemble Selection

- Often the most accurate methods in practice.
  - (Hint: try them for the Kaggle mini-project)