Problem 1

b is the correct answer.
We want the probability bound $2Me^{2\epsilon^2 N}$ to be at most .03. So, we can solve for

$$2Me^{-2\epsilon^2 N} = .03$$

$$2e^{-2(.05)^2 N} = .03$$

$$e^{-2(.05)^2 N} = .015$$

$$N = 839.94$$

So for any N less than this value, our probability bound will be greater than .03. So the answer is 1000. We can check this by plugging in 500 and 1000 into $2Me^{2\epsilon^2 N}$. At 500, we get .164, which is too big. At 1000, we get .013, which is below the desired bound. Thus 1000 is the correct answer.

Problem 2

c is the correct answer.
We want the probability bound $2Me^{2\epsilon^2 N}$ to be at most .03. So, we can solve for

$$2Me^{-2\epsilon^2 N} = .03$$

$$20e^{-2(.05)^2 N} = .03$$

$$e^{-2(.05)^2 N} = .0015$$

$$N = 1300.46$$

So for any N less than this value, our probability bound will be greater than .03. So the answer is 1500. We can check this by plugging in 1000 and 1500 into $2Me^{2\epsilon^2 N}$. At 1000, we get .135, which is too big. At 1500, we get .011, which is below the desired bound. Thus 1500 is the correct answer.

Problem 3

d is the correct answer.
We want the probability bound $2Me^{2\epsilon^2 N}$ to be at most .03. So, we can solve for

$$2Me^{-2\epsilon^2 N} = .03$$

$$200e^{-2(.05)^2 N} = .03$$

$$e^{-2(.05)^2 N} = .00015$$

$$N = 1760.98$$

So for any N less than this value, our probability bound will be greater than .03. So the answer is 2000. We can check this by plugging in 1500 and 2000 into $2Me^{2\epsilon^2 N}$. At 1500, we get .111, which is too big. At 2000, we get .009, which is below the desired bound. Thus 2000 is the correct answer.

Problem 4

b is the correct answer.
First of all, we know this is true because of the VC dimension + 1 theory. Secondly, for 5 points in $\mathbb{R}^3$, we get one of the following situations. One situation is if all 5 points are on the same plane. Then we obviously cannot shatter this, because we cannot even shatter 4 points in the $\mathbb{R}^2$ case. Another situation is if 4 points are on the same plane and one is on a different plane. We obviously cannot shatter this either, since we could not shatter 4 points in $\mathbb{R}^2$. In our last situation, we can form a plane with 3 points that separates the 2 other points. Then, we have that the dichotomy where the 3 points in that plane are +1, the 1 point on one side of that plane is -1, and the 1 point on the other side of the plane is also -1 cannot be achieved with the Perceptron Model.

Problem 5

b is the correct answer.
We have that if there is no break point, $m_{\mathcal{H}}(N) = 2^N$, and that if there is any break point, $m_{\mathcal{H}}(N)$ is polynomial in N. So, we know that (i) and (ii) are actual growth functions, since they are the growth functions for positive rays and positive intervals and are of the form $\sum_{i=0}^{k-1} \binom{N}{i}$ (polynomial). And we have that (v) is just 2^N. Then we have that (iii) and (iv) are not polynomial in N and not 2^N. So we have our answer.

Problem 6

c is the correct answer.
We have that with the "2-invervals" learning model, we can have at most 2 distinct sets of positive points. This is enough to shatter 3 and 4 points, as it is impossible to have 3 distinct sets of positive points with just 3 or 4 points. But with 5 points, it is possible to have this: +1, -1, +1, -1, +1, and we cannot achieve this dichotomy with the "2-intervals" learning model. So the answer is 5.

Problem 7

c is the correct answer.
We get to this answer the following way. There are $\binom{N+1}{4}$ ways to place 2 distinct intervals (choosing 4 bounds in total), and $\binom{N+1}{2}$ ways to place 1 distinct inverval (choosing 2 bounds in total). Then there is just 1 dichotomy in which all the intervals are placed together so that all the points are negative.

Problem 8

d is the correct answer.
d is the only option that is consistent with the "1-interval" and "2-interval" learning models - for the "1-interval" learning model we had a break point of 3, and for the "2-interval" learning model we had a break point of 5. This also makes sense in general, as given M intervals to play with, we can have at most M distinct sets of positive points. So we are going to break once we reach $2M + 1$ points, because at this point we can have $M + 1$ distinct sets of positive points as a possible dichotomy.

Problem 9

c is the correct answer.
Basically solved this problem by doing the brute force way, trying each answer one by one. 1 point can clearly be shattered. 3 points can also clearly be shattered because a triangle is more powerful than a line. If we arrange all the points in a circle, it is clear that 5 points can be shattered as well. Trying to shatter 7 points (arranged in a circle, which should give us the most possible dichotomies since a triangle is a convex set) fails. So we have that our answer is 5.

Problem 10

e is the correct answer.
Here, the maximum number of dichotomies we can get on N points is when we put the N points on a line. When we do this, the concentric circles act as intervals, and we can then see that this problem breaks down to the "2-intervals" model. Thus our growth function for this is the same as our growth function for the single positive interval problem, $\binom{N+1}{4} + \binom{N+1}{2} + 1$. But this answer is not here, so it is none of the above.