CS21
Decidability and Tractability

Lecture 8

January 24, 2014

Outline

- deterministic PDAs
- deciding CFLs
- Turing Machines and variants

Deterministic PDA

- A technical detail:

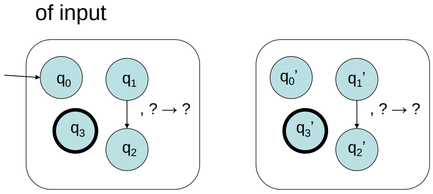
  we will give our deterministic machine the ability to detect end of input string
  - add special symbol   to alphabet
  - require input tape to contain x
- language recognized by a deterministic PDA is called a deterministic CFL (DCFL)

Deterministic PDA

Proof:
- convert machine into “normal form”
  - always reads to end of input
  - always enters either an accept state or single distinguished “reject” state
- step 1: keep track of when we have read to end of input
- step 2: eliminate infinite loops

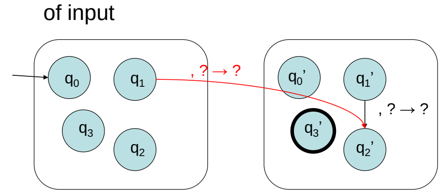
Deterministic PDA

step 1: keep track of when we have read to end of input



Deterministic PDA

step 1: keep track of when we have read to end of input

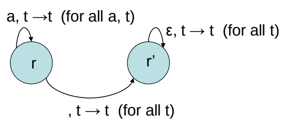


for accept state q’: replace outgoing “ε, ? → ?” transition with self-loop with same label

Deterministic PDA

step 2: eliminate infinite loops

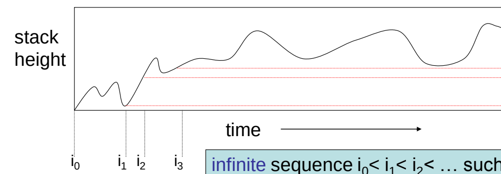
– add new "reject" states



Deterministic PDA

step 2: eliminate infinite loops

– on input x, if infinite loop, then:



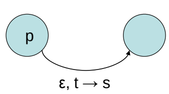
infinite sequence $i_0 < i_1 < i_2 < \ldots$ such that for all k, stack height never decreases below ht(i_k) after time i_k

Deterministic PDA

step 2: eliminate infinite loops

– infinite seq. $i_0 < i_1 < \ldots$ such that for all k, stack height never decreases below ht(i_k) after time i_k

– infinite subsequence $j_0 < j_1 < j_2 < \ldots$ such that same transition is applied at each time j_k



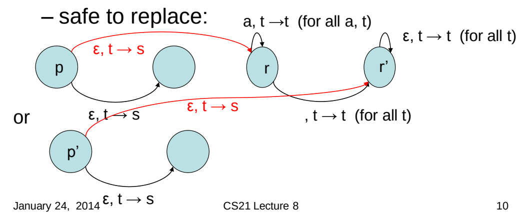
- never see any stack symbol below t from j_k on
- we are in a periodic, deterministic sequence of stack operations independent of the input

Deterministic PDA

step 2: eliminate infinite loops

– infinite subsequence $j_0 < j_1 < j_2 < \ldots$ such that same transition is applied at each time j_k

– safe to replace:



Deterministic PDA

– finishing up…

– have a machine M with no infinite loops

– therefore it always reads to end of input

– either enters an accept state q', or enters "reject" state r'

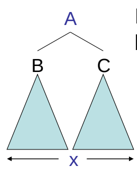
– now, can swap: make r' unique accept state to get a machine recognizing complement of L

Deciding CFLs

- Useful to have an efficient algorithm to decide whether string x is in given CFL
  – e.g. programming language often described by CFG. Determine if string is valid program.
- If CFL recognized by deterministic PDA, just simulate the PDA.
  – but not all CFLs are (homework)…
- Can simulate NPDA, but this takes exponential time in the worst case.

Deciding CFLs

- Convert CFG into Chomsky Normal form.
- parse tree for string x generated by nonterminal A:



If $A \Rightarrow^k x$ $(k > 1)$ then there must be a way to split x:

x = yz

- $A \rightarrow BC$ is a production and
- $B \Rightarrow^i y$ and $C \Rightarrow^j z$ for i, j < k

Deciding CFLs

- An algorithm:

IsGenerated(x, A)

if |x| = 1, then return YES if $A \rightarrow x$ is a production, else return NO
for all n-1 ways of splitting x = yz
for all ≤ m productions of form $A \rightarrow BC$
if IsGenerated(y, B) and IsGenerated(z, C), return YES
return NO

- worst case running time?

Deciding CFLs

- worst case running time exp(n)
- Idea: avoid recursive calls
  - build table of YES/NO answers to calls to IsGenerated, in order of length of substring
  - example of general algorithmic strategy called dynamic programming
  - notation: x[i,j] = substring of x from i to j
  - table: T(i, j) contains
    {A: A nonterminal such that $A \Rightarrow^* x[i,j]$}

Deciding CFLs

IsGenerated($x = x_1x_2x_3\ldots x_n$, G)

for i = 1 to n
T[i, i] = {A: "$A \rightarrow x_i$" is a production in G}
for k = 1 to n - 1
for i = 1 to n - k
for k splittings x[i, i+k] = x[i,i+j]x[i+j+1, i+k]
T[i, i+k] = {A: "$A \rightarrow BC$" is a production in G and $B \in T[i,i+j]$ and $C \in T[i+j+1,i+k]$ }
output "YES" if $S \in T[1, n]$, else output "NO"

Deciding CFLs

IsGenerated($x = x_1x_2x_3\ldots x_n$, G)

O(nm) steps

for i = 1 to n
T[i, i] = {A: "$A \rightarrow x_i$" is a production in G}
for k = 1 to n - 1
for i = 1 to n - k
for k splittings x[i, i+k] = x[i,i+j]x[i+j+1, i+k]
T[i, i+k] = {A: "$A \rightarrow BC$" is a production in G and $B \in T[i,i+j]$ and $C \in T[i+j+1,i+k]$ }

$O(n^3m^3)$ steps

output "YES" if $S \in T[1, n]$, else output "NO"

Summary

- Nondeterministic Pushdown Automata (NPDA)
- Context-Free Grammars (CFGs) describe Context-Free Languages (CFLs)
  - terminals, non-terminals
  - productions
  - yields, derivations
  - parse trees

Summary

- grouping determined by grammar
- ambiguity
- Chomsky Normal Form (CNF)

- NDPAs and CFGs are equivalent
- CFL Pumping Lemma is used to show certain languages are not CFLs

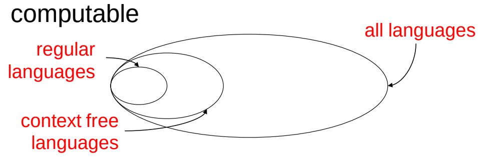
Summary

- deterministic PDAs recognize DCFLs
- DCFLs are closed under complement

- there is an efficient algorithm (based on dynamic programming) to determine if a string x is generated by a given grammar G

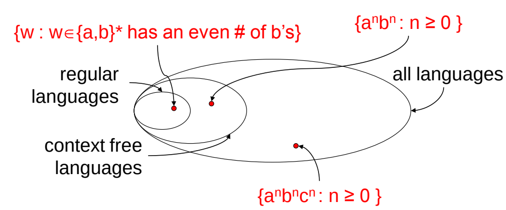
So far…

- several models of computation
  - finite automata
  - pushdown automata
- fail to capture our intuitive notion of what is computable



So far…

- We proved (using constructions of FA and NPDAs and the two pumping lemmas):



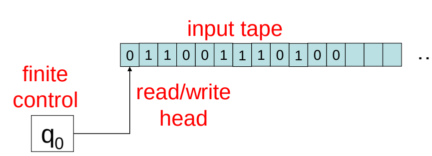
A more powerful machine

- limitation of NPDA related to fact that their memory is stack-based (last in, first out)

- What is the simplest alteration that adds general-purpose "memory" to our machine?

- Should be able to recognize, e.g., {$a^nb^nc^n$: n ≥ 0 }

Turing Machines



- New capabilities:
  - infinite tape
  - can read OR write to tape
  - read/write head can move left and right

Turing Machine

- Informal description:
  - input written on left-most squares of tape
  - rest of squares are blank
  - at each point, take a step determined by
    - current symbol being read
    - current state of finite control
  - a step consists of
    - writing new symbol
    - moving read/write head left or right
    - changing state

Example Turing Machine

language L = {w#w : w ∈ {0,1}*}