CS 21 Decidability and Tractability **Winter 2014**

Problem Set 4

Out: February 12 *Due:* **February 19**

Reminder: you are encouraged to work in groups of two or three; however you must turn in your own write-up and note with whom you worked. You may consult the course notes and the text (Sipser). The full honor code guidelines can be found in the course syllabus.

Please attempt all problems. **To facilitate grading, please turn in each problem on a separate sheet of paper and put your name on each sheet. Do not staple the separate sheets.**

1. A graph G is called *k-colorable* if there is a way to assign a color to each vertex so that no edge has both endpoints assigned the same color, using at most k distinct colors. Show that the language

$$\text{2-COLORABLE} = \{G : G \text{ is 2-colorable}\}$$

is in P by reducing it to a problem known to be in P.

2. Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there is a bijection $f : V_1 \to V_2$ such that $(u, v) \in E_1 \Leftrightarrow (f(u), f(v)) \in E_2$. For a given graph H, define the following language:

$$\text{CONTAINS}_H = \{G : G \text{ contains a subgraph isomorphic to } H\}.$$

Here by "subgraph" we mean a subset of G's vertices together with all of G's edges on that subset of vertices – often called an "induced subgraph." Prove that for every H, CONTAINS$_H$ is in P.

3. Show that the following problem is in P:

$$\text{UNARY SUBSET SUM} = \left\{(1^B, x_1, x_2, \ldots, x_n) : \exists \text{ a multiset } I \text{ of } [n] \text{ for which } \sum_{i \in I} x_i = B\right\}.$$

Here the x_i are all positive integers, as is B, and $[n]$ is shorthand for the set $\{1, 2, 3, \ldots, n\}$. The notation 1^B means a string of B ones, which is the representation of B in unary. Hint: solve the problem for all $B' \leq B$.