CS 24: Introduction to Computing Systems

Spring 2014

Lecture 2

Last Time

- Began exploring the concepts behind a simple programmable computer
- Construct the computer using Boolean values (a.k.a. "bits") and logic gates to process them
- Represent unsigned and signed integers as vectors of bits

$$\mathrm{B2U}_w(\boldsymbol{x}) = \sum_{i=0}^{w-1} x_i \, 2^i$$

$$\mathrm{B2T}_w(\boldsymbol{x}) = -x_{w-1}2^{w-1} + \sum_{i=0}^{w-2} x_i \, 2^i$$

- Briefly explored how to construct more complex computations using gates
  - e.g. unsigned and signed arithmetic

Signed Integer Representation (2)

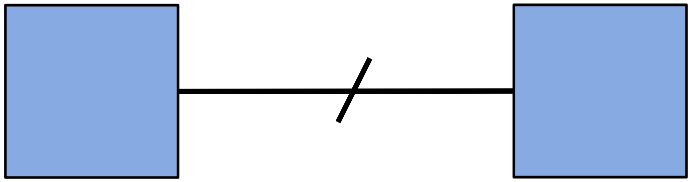
- Easy trick for converting an integer to its two's complement representation:
  - Invert the bits, then add one
- Example:
  - Find two's complement representation for -42
  - Unsigned representation for 42 is 00101010_2
  - Invert the bits: 11010101_2
  - Add one: 11010110_2
- Converting back, following $\text{B2T}_{w=8}$ function:
  - $= -1\times2^7 + 1\times2^6 + 1\times2^4 + 1\times2^2 + 1\times2^1$
  - $= -128 + 64 + 16 + 4 + 2$
  - $= -42$

Functional Components of a Simple Processor

- Can use our logic gates to construct various components to use in a processor
  - Already saw how to implement addition with logic
- Minimal components for a simple processor:
- Signal Buses
  - Ability to route signals within our processor
- Arithmetic/Logic Unit (ALU)
  - Performs various arithmetic and logical operations on data inputs, based on control inputs
- Memory
  - Addressable locations to store and retrieve values

Buses

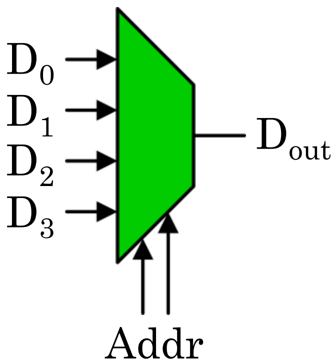
- A bus is a set of wires that transfer signals from one component to another
  - Transmits values of a fixed bit-width, e.g. 32 bits
- Common uses for buses in a computer:
  - Transfer data between CPU and memory
  - Transfer data between CPU and peripherals
- Buses often drawn as a single line with a slash across it



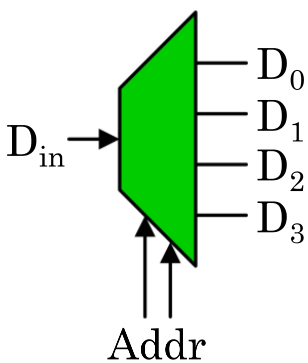
- Individual signals drawn as a line with no slash

Routing Buses

- Multiplexers and demultiplexers (decoders) are used to route buses between multiple components
- Example: a 4-input multiplexer (MUX)
  - Has two address inputs
  - Address selects one of 4 data inputs
  - Corresponding data input is fed to the data output

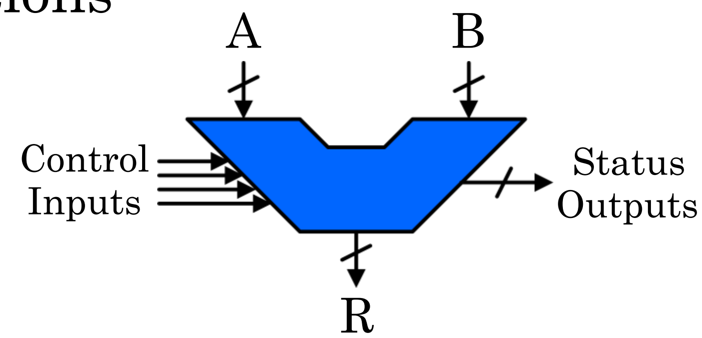


- A 4-input demultiplexer (DEMUX)
  - Again, two address inputs
  - Address selects one of 4 data outputs
  - Single data input fed to corresponding data output



ARITHMETIC/LOGIC UNIT

- A component that can perform various arithmetic and logic functions
- Symbol:



- Given two w-bit inputs and a set of control inputs
  - Control inputs specify the operation to perform
- Produces a w-bit result, and status outputs
  - Example status outputs:
    - sign flag (topmost bit of R)
    - carry-out flag (unsigned overflow)
    - zero flag (is R == 0?)
    - overflow flag (signed overflow)

Example ALU Operations

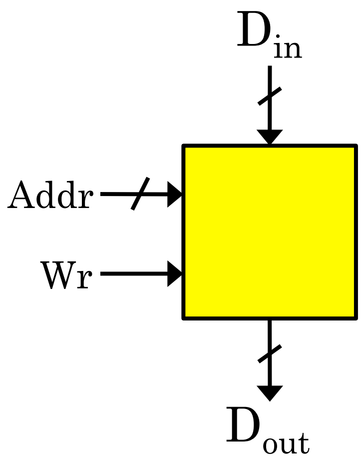
- Control signals specify what operation to perform
- Example: for our contrived ALU

Control	Operation		
0001	ADD	A	B
0011	SUB	A	B
0100	NEG	A	
1000	AND	A	B
1001	OR	A	B
1010	XOR	A	B
1011	INV	A	
1100	SHL	A	
1110	SHR	A	

- By feeding appropriate control and data signals to ALU in sequence, can perform computations
- Some operations require only one argument
  - Second argument ignored

Memory

- Need a component to store both instructions and data to feed to the ALU
- Memory:
  - An array of linearly addressable locations
  - Each location has its own address
  - Each location can hold a single *w*-bit value
- Inputs and outputs:
  - Address of location to read or write
  - Read/Write control signal
  - Data-input bus
  - Data-output bus

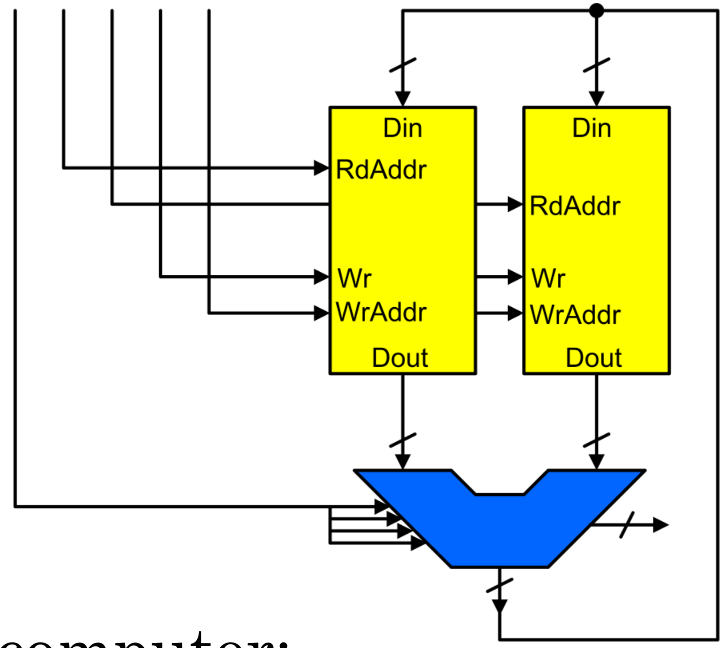


Aside: CPU Components and Gates

- It is a big claim that we can construct all of these components entirely from logic gates…
- Unfortunately, beyond the scope of CS24 to explore all the ways such components can be constructed, different approaches, etc.
- If you are curious how these things work, see the primer on the CS24 Moodle
  - Shows some *basic* ways these components can be constructed
- **Don't need to know this material in depth!**
  - For CS24, really only need to understand the basics of how to implement logic equations with gates

ASSEMBLING THE COMPUTER

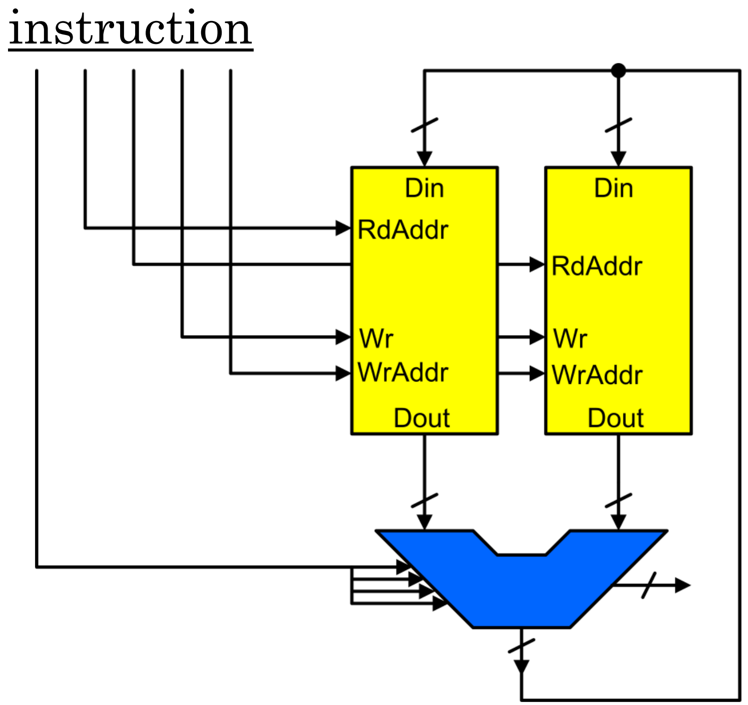
- Hook these components together, like this:



- Simplifications in our computer:
  - Two memory banks; identical copies of each other
  - Don't care about ALU status outputs

INSTRUCTING THE COMPUTER

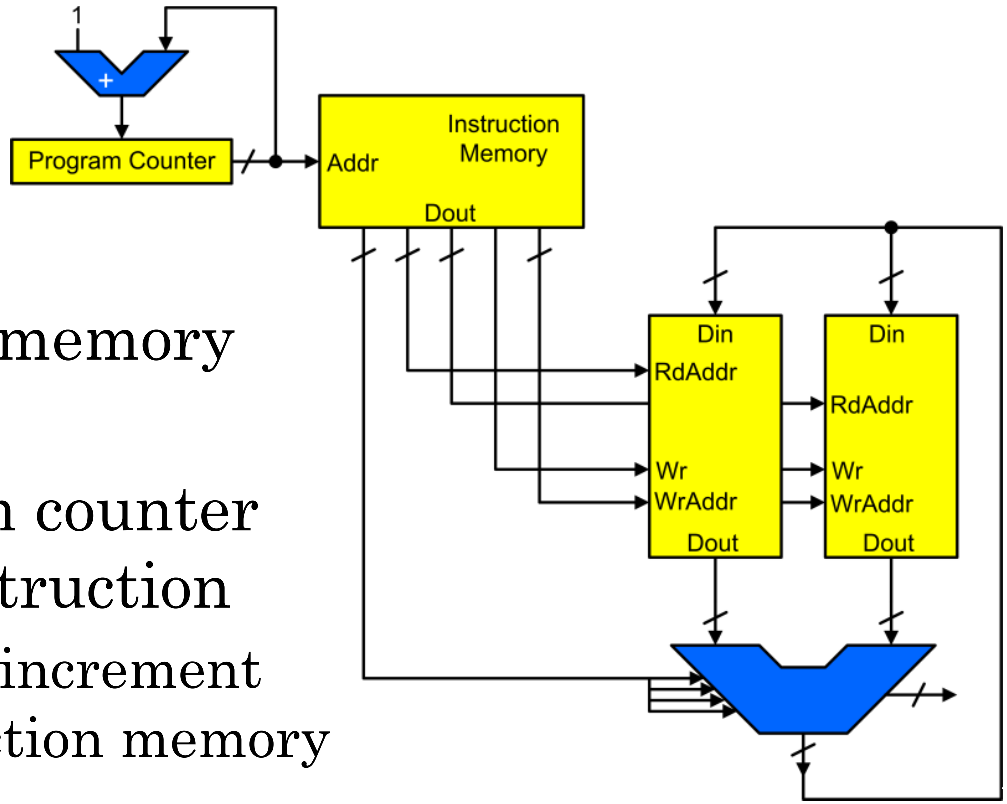
- This set of inputs forms an instruction
- Consists of:
  - The operation the ALU should perform
  - Addresses of two input values
  - Whether result should be stored
  - If so, what address to store the result at



- To program our computer:
  - Devise a set of instructions to implement our desired computation

Instructing the Computer (2)

- Need a way to feed instructions to our computer



- Add an instruction memory to our system
- Also, add a program counter to track current instruction
  - Configured to auto-increment through the instruction memory

Writing a Program

- Instructions for the processor are very limited
  - Can only compute one value, from one or two values
- Usually can't implement a program in only one instruction
- Instead:
  - String together a sequence of instructions to implement the computation
  - Instructions will communicate via memory locations
- Computation we will implement:
  - $C = (A - 2B) \& 00001111_2$
  - Given inputs A and B
  - Multiply B by 2, subtract result from A, then bitwise-AND with a mask

IMPLEMENTING OUR COMPUTATION (1)

- Computation: $C = (A - 2B)\ \&\ 00001111_2$

- Step 1: Assign locations for inputs and outputs
- Inputs:
  - A and B (obvious)
  - Also, our mask: 00001111_2
  - Program needs to include our constants, too
- Output:
  - C
- Givens:
  - Our memory has 8 locations
  - Memory addresses are 3 bits wide
  - Data values are 8 bits wide ($w = 8$)

Assigning Data Locations

- Locations for our initial and final data values:

Address	Value
0	A
1	B
2	
3	
4	00001111_2
5	
6	
7	C

Implementing our Computation (2)

- Step 2: decompose our program into instructions the processor can actually handle
- Program:
  - $C = (A - 2B) \ \& \ 00001111_2$
- Need to know processor's operations for this step.

Control	Operation		
0001	ADD	A	B
0011	SUB	A	B
0100	NEG	A	
1000	AND	A	B
1001	OR	A	B
1010	XOR	A	B
1011	INV	A	
1100	SHL	A	
1110	SHR	A	

- Steps:
  - Perform 2B first, as B + B
  - Then, subtract previous result from A
  - Finally, bitwise-AND this with mask to produce C

Implementing our Computation (3)

- Step 3: need to assign locations to these intermediate values!

- Result of B + B = location 2
- Result of A – 2B = location 3
- Result of bitwise-AND stored in location 7
  - This is our result

Address	Value
0	A
1	B
2	2B
3	A – 2B
4	00001111_2
5	
6	
7	C

Implementing our Computation (4)

- Step 4: Translate our program into instructions!
- Need to know form of instructions:
  - Operation Rd1Addr Rd2Addr Wr WrAddr
- Also need our memory layout and operation codes

Control	Operation		
0001	ADD	A	B
0011	SUB	A	B
0100	NEG	A	
1000	AND	A	B
1001	OR	A	B
1010	XOR	A	B
1011	INV	A	
1100	SHL	A	
1110	SHR	A	

Address	Value
0	A
1	B
2	2B
3	A – 2B
4	00001111_2
5	
6	
7	C

Implementing our Computation (5)

Control	Operation		
0001	ADD	A	B
0011	SUB	A	B
0100	NEG	A	
1000	AND	A	B
1001	OR	A	B
1010	XOR	A	B
1011	INV	A	
1100	SHL	A	
1110	SHR	A	

Address	Value
0	A
1	B
2	2B
3	A – 2B
4	00001111_2
5	
6	
7	C

- Operation Rd1Addr Rd2Addr Wr WrAddr
- Writing our program:
  - Slot 2 = 2B
  - Slot 3 = A – 2B
  - Slot 7 = (…) & mask

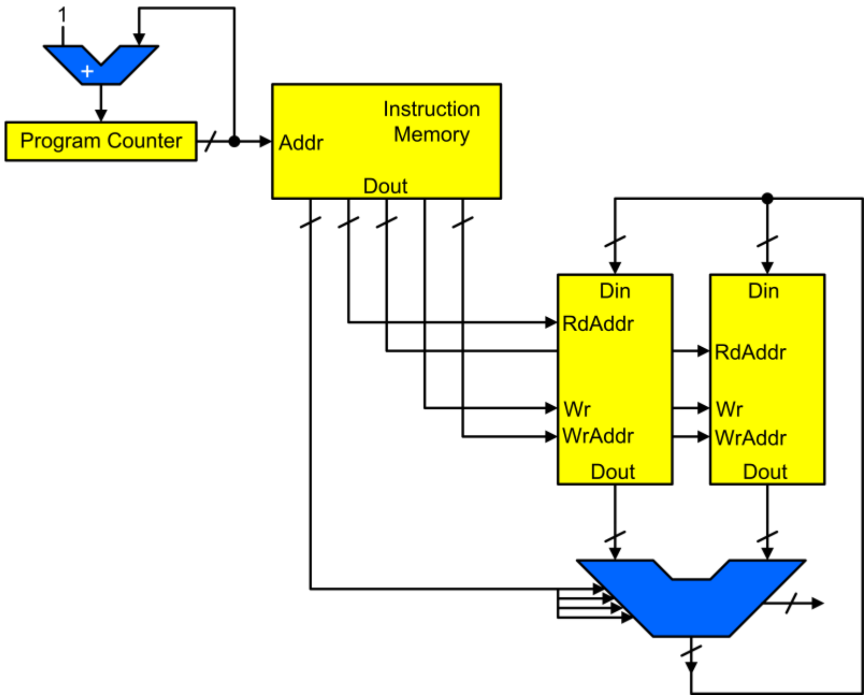
```
000:  0001 001 001 1 010
001:  0011 000 010 1 011
010:  1000 011 100 1 111
```

Running our Program

- To run our program:
  - Load instructions into instruction memory
  - Load initial data into data memory
  - Start program counter at 0
- Each instruction executes in sequence, updating memory locations
  - Uses results of previous instructions
- State of our computer:
  - Instruction memory
  - Data memory
  - Program counter

Running our Program: Initial State

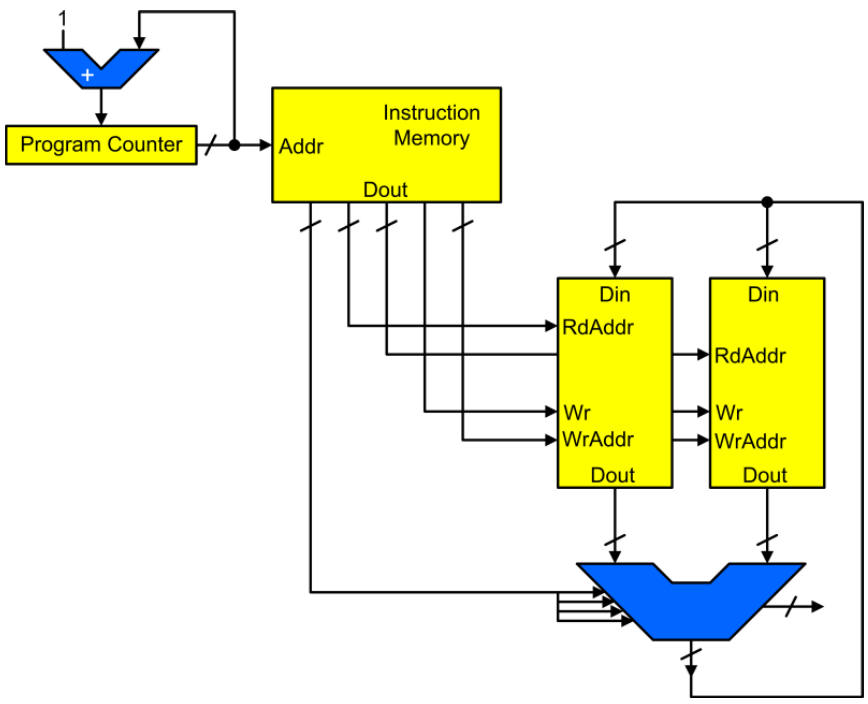
- Instruction memory:
  - 000: 0001 001 001 1 010
  - 001: 0011 000 010 1 011
  - 010: 1000 011 100 1 111
- Data memory:
  - 0: A
  - 1: B
  - 2: ???
  - 3: ???
  - 4: 00001111_2
  - 5: ???
  - 6: ???
  - 7: ???



Program Counter: 000

Step 1: Slot 2 = B + B

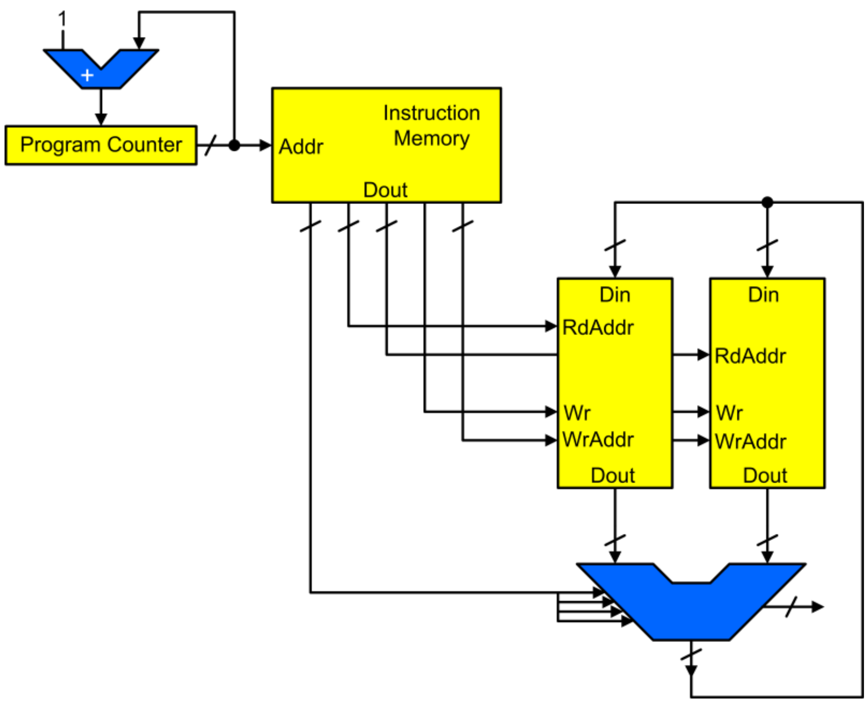
- Instruction memory:
  000: 0001 001 001 1 010
  001: 0011 000 010 1 011
  010: 1000 011 100 1 111
- Data memory:
  0: A
  1: B
  2: 2B = B + B
  3: ???
  4: 00001111_2
  5: ???
  6: ???
  7: ???



Program Counter: 000

STEP 1: UPDATE PROGRAM COUNTER

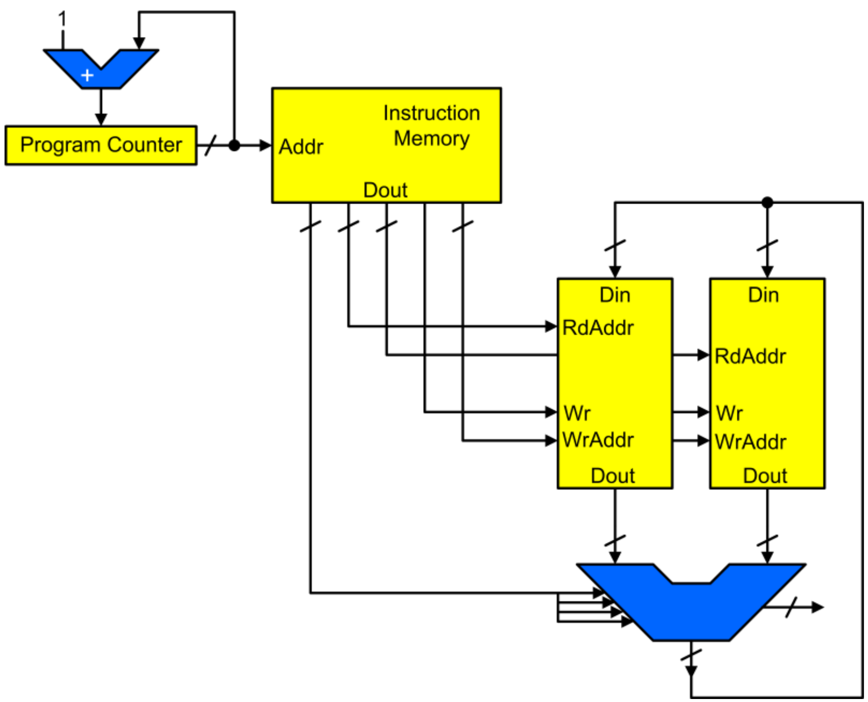
- Instruction memory:
  - 000: 0001 001 001 1 010
  - 001: 0011 000 010 1 011
  - 010: 1000 011 100 1 111
- Data memory:
  - 0: A
  - 1: B
  - 2: 2B
  - 3: ???
  - 4: 00001111_2
  - 5: ???
  - 6: ???
  - 7: ???



Program Counter: 001

Step 2: Subtract 2B from A

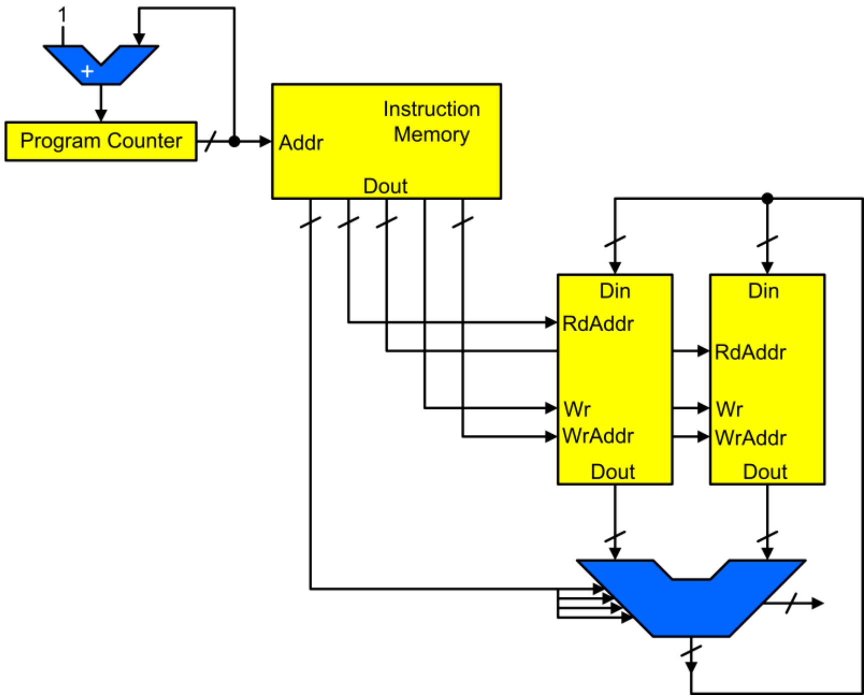
- Instruction memory:
  - 000: 0001 001 001 1 010
  - **001: 0011 000 010 1 011**
  - 010: 1000 011 100 1 111
- Data memory:
  - 0: A
  - 1: B
  - 2: 2B
  - 3: A – 2B
  - 4: 00001111_2
  - 5: ???
  - 6: ???
  - 7: ???



Program Counter: 001

STEP 2: UPDATE PROGRAM COUNTER

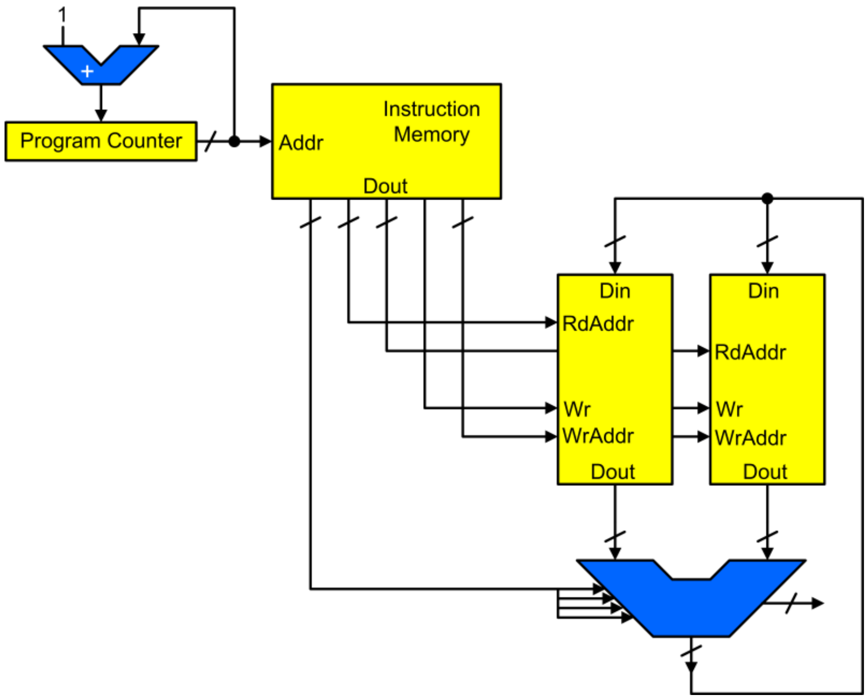
- Instruction memory:
  - 000: 0001 001 001 1 010
  - 001: 0011 000 010 1 011
  - 010: 1000 011 100 1 111
- Data memory:
  - 0: A
  - 1: B
  - 2: 2B
  - 3: A – 2B
  - 4: 00001111_2
  - 5: ???
  - 6: ???
  - 7: ???



Program Counter: 010

Step 3: $C = (A - 2B)$ & 00001111_2

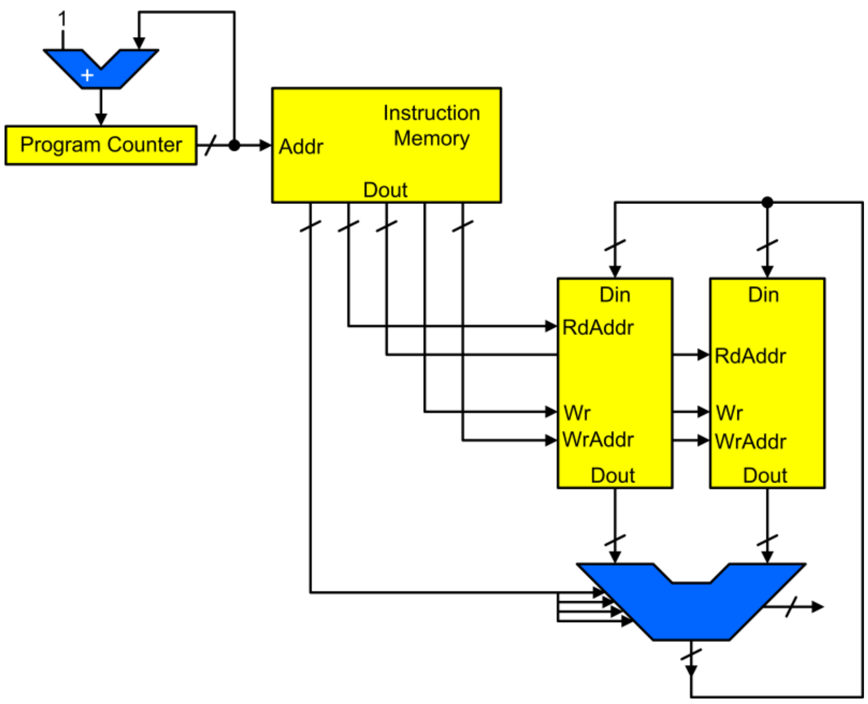
- Instruction memory:
  - 000: 0001 001 001 1 010
  - 001: 0011 000 010 1 011
  - **010: 1000 011 100 1 111**
- Data memory:
  - 0: A
  - 1: B
  - 2: 2B
  - 3: A – 2B
  - 4: 00001111_2
  - 5: ???
  - 6: ???
  - 7: (A – 2B) & 00001111_2



Program Counter: 010

Running Our Program: Final Result

- Instruction memory:
  - 000: 0001 001 001 1 010
  - 001: 0011 000 010 1 011
  - 010: 1000 011 100 1 111
- Data memory:
  - 0: A
  - 1: B
  - 2: 2B
  - 3: A – 2B
  - 4: 00001111_2
  - 5: ???
  - 6: ???
  - 7: (A – 2B) & 00001111_2



A Programmable Computer!

- Using our basic functional components, we are able to build a simple programmable computer!
- Implemented a computation using our processor's instruction set:
  1. Assigned memory locations to inputs and output
  2. Decomposed computation into processor instructions
  3. Assigned memory locations for intermediate values
  4. Encoded sequence of instructions for our program
- By feeding instructions to computer in sequence, we can perform our computation
  - Individual instructions communicate by reading and writing various memory locations

Machine Code, Assembly Language

- Our program:

  000: 0001 001 001 1 010

  001: 0011 000 010 1 011

  010: 1000 011 100 1 111

- This is called machine code
  - The actual data values that comprise the program
  - Hard to read and write!
- Humans normally use assembly language
  - A more human-readable language that is translated into machine code using an assembler

    ```
    ADD  R1, R1, R2    # R2 = 2B
    SUB  R0, R2, R3    # R3 = A – 2B
    AND  R3, R4, R7    # C = R3 & 00001111
    ```
  - Allows human-readable names, operations, comments

C Logical and Bitwise Operations

- Before going forward, need to review what C offers for logical and bitwise operations
- C uses integers to represent Boolean values
  - 0 = false; any nonzero value = true
- Logical Boolean operators:
  - Logical AND: **`a && b`**
  - Logical OR: **`a || b`**
  - Logical NOT: **`!a`**
  - Result is 1 if true, 0 if false
- **`&&`** and **`||`** are short-circuit operators
  - Evaluated left-to-right
  - For **`&&`**, if LHS is false then RHS is not evaluated
  - For **`||`**, if LHS is true then RHS is not evaluated

C Logical and Bitwise Operations (2)

- C also has many bit-manipulation operations
- Given a = 00010100_2 (20_{10}), b = 00110010_2 (50_{10})
  - `a & b` = 00010000 Bitwise AND
  - `a | b` = 00110110 Bitwise OR
  - `~a` = 11101011 Bitwise negation (invert)
  - `a ^ b` = 00100110 Bitwise XOR
- Note:
  - C has no way of specifying base-2 literals
  - Normal approach: use *hexadecimal* literals instead
- Hexadecimal: base-16 numbers
  - Digits are 0..9, A..F (or a..f, makes no difference)
  - A = 10, B = 11, …, F = 15

Hexadecimal Values and Bit-Masks

- Example: **`0x0F`** is a hexadecimal literal in C
  - Each digit of a hexadecimal value represents 4 bits – a compact, simple way to write bit-fields
  - **`0x0F`** = 0000 1111 (also **`0x0f`**)
  - **`0x03C7`** = 0000 0011 1100 0111 (also **`0x03c7`**)
- Use bitwise AND to mask out or clear specific bits
  - **`a & 0x0F`**
    - Clears high nibble of **`a`**; retains low nibble of **`a`**
- Use bitwise OR to set specific bits
  - **`a = a | 0x28`**
    - Sets bits 3 and 5 of value in **`a`** (**`0x28`** = 0010 1000)
    - Other bits in **`a`** remain unchanged
- Use bitwise XOR to toggle specific bits
  - **`a = a ^ 0x28`**
    - Toggles bits 3 and 5 of value in **`a`**; other bits are left unchanged

C Bit-Shifting Operations

- C also includes bit-shifting operations
- Shift bits in **`a`** left by **`n`** bits: **`a << n`**
  - New bits on right are 0
  - Shifting left by n bits is identical to multiplying by 2^n

```
a = 42;       /* a = 00101010 = 42 */
a = a << 1;   /* a = 01010100 = 84 */
```

- Shift bits in **`a`** right by **`n`** bits: **`a >> n`**
  - Shifting right by n bits is identical to dividing by 2^n
- Question: What should new bits on left be?
  - Depends on whether **`a`** is signed or unsigned!

```
a = -24;      /* Two's complement:  11101000  */
a = a >> 1;   /* Should be -12 (11110100) now */
```

  - Leftmost bit represents sign
  - Preserve sign by using same value as original sign-bit

ARITHMETIC VS. LOGICAL SHIFT-RIGHT

- Distinguish between arithmetic shift-right and logical (i.e. bitwise) shift-right
  - Arithmetic shift-right preserves the value's sign
  - Logical shift-right always adds 0-bits to left of value
- Some languages make this distinction
  - Java: **>>** is arithmetic, **>>>** is logical
  - IA32 assembly: **SAR** is arithmetic, **SHR** is logical
- In C:
  - If argument is signed, shift-right is arithmetic

```
char a = -24;           /*        -24 = 11101000 */
printf("%d", a >> 1);   /* Prints -12 = 11110100 */
```

  - If argument is unsigned, shift-right is logical

```
/* Prints 116 = 01110100; topmost bit is 0 */
printf("%d", (unsigned char) a >> 1);
```

Bit-Shifts and Masks

- Can use bit-shifts with masks to extract sub-byte values
- **`(a >> 4) & 0x0F`**
  - Retrieves high nibble of **`a`**
- Does it matter if **`a`** is signed or unsigned?
  - Nope. We chop off the sign bit after we shift.

Multiplication?

- Our processor's instruction set:
- Hmm, no multiply instruction.
- No problem; implement multiply with addition and shifting

$$\mathrm{mul}_w(\boldsymbol{a}, \boldsymbol{b}) = \sum_{i=0}^{w-1} a_i \, \boldsymbol{b} \, 2^i$$

```
int mul(int a, int b) {
    int p = 0;
    while (a != 0) {
        if (a & 1 == 1)
            p = p + b;
        a = a >> 1;
        b = b << 1;
    }
    return p;
}
```

Control	Operation
0001	ADD A B
0011	SUB A B
0100	NEG A
1000	AND A B
1001	OR A B
1010	XOR A B
1011	INV A
1100	SHL A
1110	SHR A

Multiplication ???

- Our multiply program:

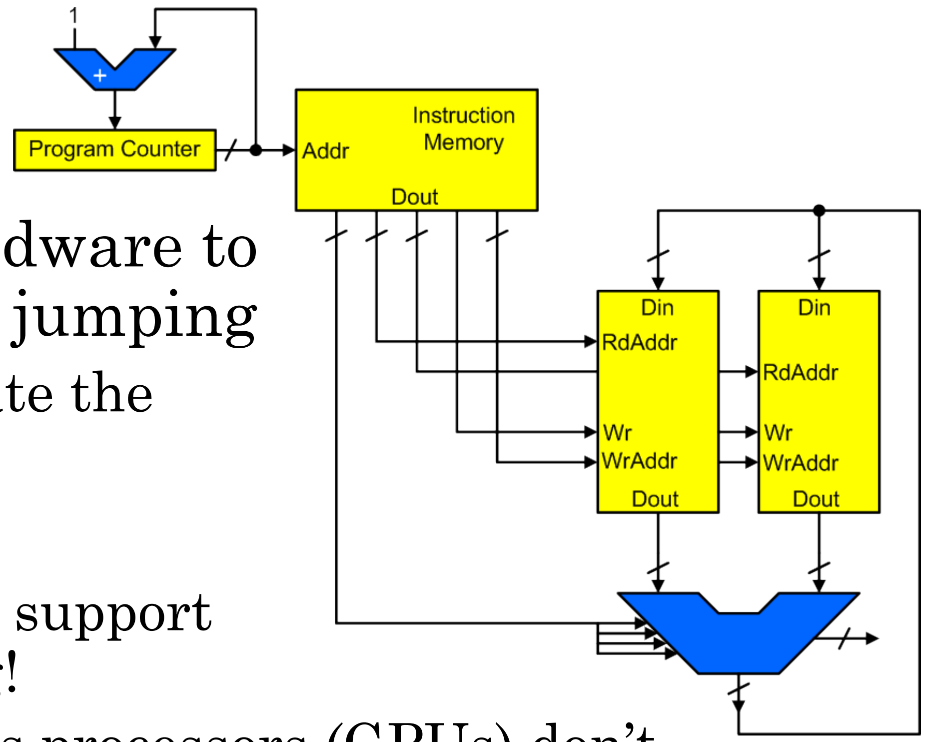
```
int mul(int a, int b) {
    int res = 0;
    while (a != 0) {
        if (a & 1 == 1)
            p = p + b;
        a = a >> 1;
        b = b << 1;
    }
    return p;
}
```

Control	Operation		
0001	ADD	A	B
0011	SUB	A	B
0100	NEG	A	
1000	AND	A	B
1001	OR	A	B
1010	XOR	A	B
1011	INV	A	
1100	SHL	A	
1110	SHR	A	

- Can we write a program to execute this code?
  - NO! ☹
  - Our processor doesn't support any branching or jumping operations

Branching Support

- Our current processor architecture *can't* support branching or jumping!
  - Can only execute code in sequential order
- Need to extend the hardware to support branching and jumping
  - Need to be able to update the Program Counter field
- Note:
  - Not always essential to support branching and jumping!
  - Most dedicated graphics processors (GPUs) don't support looping or branching at all
  - However, is essential for a general-purpose processor



Summary

- We designed a simple programmable computer!
  - Assembled functional components so we can perform a variety of simple computations
  - Feed instructions into our processor in sequence, from instruction memory
  - Instructions communicate by reading and writing various memory locations
- But, our computer has substantial limitations…
  - Can't even implement a simple loop yet. ☹
  - Need to extend our processor to support branching
- Also, our computer only has 8 bytes of memory
  - Need to examine impact of increasing memory size