Ma/CS 6a: Problem Set 10*

Due noon, Thursday, December 11

1. Let E_n denote the number of partitions of n with an even number of even parts, and let O_n denote the number of partitions of n with an odd number of even parts. Prove that $E_n - O_n$ is the number of self-conjugate partitions of n (hint: find two generating functions).

2. (NO COLLABORATION) Consider a directed graph $G = (V, E)$, a weight function $f : E \to R$ under which there are no negative cycles in G, and two vertices $s, t \in V$. Given a path P from s to t, we denote by $\ell(P)$ the number of edges in P, and by $d[P]$ the weight of the path (that is, the sum of its edge weights). Describe an algorithm for finding a path from s to t that minimizes $\ell(P) + d[P]$ (hint: use Bellman-Ford as a black box and change G).

3. Consider a directed graph $G = (V, E)$, a weight function $f : E \to R$ under which there are no negative cycles in G, and two vertices $s, t \in V$. Moreover, every edge of e is colored either red or blue. Describe an algorithm for finding the shortest path from s to t (shortest by weight. Not by length) among the paths that have an even number of blue edges. It is fine if there are shorter paths that have an odd number of blue edges (hint: use Bellman-Ford as a black box and change G).

4. We say that a pair of $n \times n$ Latin squares (L, L') is *good* if for every pair of symbols (s, s') there exists exactly one positive (i, j) such that $L(i, j) = s$ (that is, the j'th cell of the i'th row of L_1 contains s) and $L'(i, j) = s'$.

Let p be a prime and for every $t \in \{1, 2, \cdots, p-1\}$ let L_t be the $p \times p$ matrix defined as $L_t(i, j) = ti + j \mod p$. We set $S = \{L_t \mid 1 \le t \le p-1\}$. Prove that every element of S is a Latin square and that every two elements of S form a good pair.

*The awesome students who helped correcting this assignment: Tim Holland.