Ma/CS 6a: Problem Set 6*

Due noon, Thursday, November 13

1. (a) Given a flow network (V, E, s, t, c) and an edge $e \in E$, describe an efficient algorithm for checking whether e is in at least one minimum cut of the network.

(b) Given a 0-1 flow network (V, E, s, t, c) (i.e., a flow network where all the capacities are 1) and a number $k \in \mathbb{N}$, describe an efficient algorithm for finding a set of k edges whose removal from the network minimizes the size of the maximum flow.

2. (NO COLLABORATION) In Lecture 15 we solved a question about removing at most k edges from a graph so that an edge e is in every MST of the resulting graph. Solve the question that is obtained by changing "every MST" with "at least one MST" in the statement of this problem that is in the presentation. Prove that your solution is correct.

3. Victor, Henry, and Adam decide to quit mathematics and move to Hollywood to make a movie. They of course receive offers from m different producers, where the i'th producer suggests to give a funding of X_i. The group can accept funding from as many producers as they like. However, every producer is willing to invest in the movie only if each of her favorite actors is in it (every producer has a list of such actors, and the lists may have common actors in them). In total, the lists contain the names of n distinct actors, and the i'th actor requires a fee of Y_i. Since the group are only doing this for the money, they would like to maximize the funding gained minus the salaries paid. Describe an efficient algorithm that can help Victor, Henry, and Adam (hint: build a flow network and consider only the cuts of this network. The flows are not important.)

4. (a) Prove or disprove: Any cycle of odd length can be expressed as a composition of cycles of length three.

(b) Prove or disprove: Every even permutation can be expressed as a composition of cycles of length three (hint: consider pairs of transpositions).

5. Prove that if a permutation $\pi \in S_n$ (for $n \geq 3$) satisfies $\pi\tau = \tau\pi$ for every transposition $\tau \in S_n$, then $\pi = \text{id}$. (hint: assume for contradiction that π is a different permutation, and show that there exists τ for which $\tau\pi\tau^{-1} \neq \pi$).

*The awesome students who helped correcting this assignment: Stephanie Reyes and Leon Ding.