Problem 1

(a) Here is our algorithm:

1. Find the cost of the min cut of the original flow network. This can be done by using Ford Fulkerson to find the max flow, then seeing how much flow goes out of the source node to get the size of the max flow, then applying the max flow min cut theorem. Let this cost be C_0.
2. Remove e from the flow network and find the cost of the min cut for that new flow network. Let this cost be C_1.
3. If $|C_1 - C_0| = c(e)$, where $c(e)$ is the capacity of e, return true. Else return false.

(b) Here is our algorithm:

1. Find a cut whose size is equivalent to the size of the maximum flow (a min cut). This can be done by using Ford Fulkerson to find the max flow, then seeing how much flow goes out of the source node to get the size of the max flow, then using a BFS to find a cut whose size is equivalent to the size of the max flow. Then we can iterate over the edges and find the edges that cross the cut. Call this set of edges C.
2. Let $count = k$. While $count > 0$ and $|C| > 0$, greedily remove the highest capacity edge $e_{c\_max} \in C$ from C, add it to S, and decrement *count* by 1. In this case, since the capacities are all 1, we can just keep removing any edge from C and adding it to S until we have either removed k edges or there are no more edges from C to remove.
3. Return S.

Problem 2

Problem 3

Problem 4

(a) We will prove this is true by induction on the size of odd length cycles. Our base case will be cycles of length 5. To prove this, consider an arbitrary cycle $(x_1x_2x_3x_4x_5)$ of length 5. We have that this is just the same as the composition $(x_1x_2x_3)(x_3x_4x_5)$. So the base case is satisfied. Now assume that any odd cycle of length k can be expressed as a composition of cycles of length 3. Now we must show that odd cycles of length $k+2$ can be expressed as a composition of cycles of length 3. That is, we want to show that cycles of the form $(x_1x_2\cdots x_{k+2})$ can be expressed as a composition of cycles of length 3. We have that $(x_1x_2\cdots x_{k+2})$ can be expressed as $(x_1x_2\cdots x_k)(x_kx_{k+1}x_{k+2})$. And due to our inductive assumption $(x_1x_2\cdots x_k)$ can be expressed as a combination of cycles of length 3. So we can conclude that odd cycles of length $k+2$ can be expressed as a composition of cycles of length 3, and our inductive proof is complete.

(b)

Problem 5