

# Loop-Quantized Matter Coupling in 3+1D

## 1. Introduction

We extend polymer quantization of matter fields from 2+1D midisuperspace to full 3+1D with scalar and electromagnetic fields on a cubic lattice. This enables dynamical backreaction of quantum-corrected matter onto the loop-quantized geometry.

## 2. Polymer Hamiltonian Density for a Scalar Field

Consider a free, massive scalar field  $\phi(x)$  on a 3D lattice. Introduce a uniform lattice of spacing  $\Delta$  in each direction. At each lattice site  $\vec{n} = (n_x, n_y, n_z)$ , define polymer variables:

$$\hat{U}_\epsilon(\phi_{\vec{n}}) = \exp\left(i \frac{\phi_{\vec{n}}}{\epsilon}\right), \quad \hat{\Pi}_{\vec{n}} = -i \epsilon \frac{\partial}{\partial \phi_{\vec{n}}},$$

with polymer scale  $\epsilon \sim \mathcal{O}(\ell_{\text{Pl}})$ . The discrete Hamiltonian density is

$$\mathcal{H}_\phi(\vec{n}) = \frac{1}{2} \left[ \epsilon^{-2} (2 - \hat{U}_\epsilon - \hat{U}_\epsilon^\dagger) + (\nabla_{\text{d}} \phi)_{\vec{n}}^2 + m^2 \phi_{\vec{n}}^2 \right],$$

where  $\nabla_{\text{d}} \phi$  is the standard finite-difference gradient operator.

## 3. Electromagnetic Field Polymerization

For the  $U(1)$  gauge field, discretize the vector potential  $A_i(\vec{n})$  and electric field  $E^i(\vec{n})$  on staggered lattice points. Polymer holonomies along each link  $\ell$ :

$$\hat{h}_\ell = \exp(i \alpha A_i(\ell)), \quad \hat{E}^i(\ell) = -i \frac{\partial}{\partial A_i(\ell)},$$

with gauge coupling  $\alpha = q \Delta$ . The Hamiltonian density:

$$\mathcal{H}_{\text{EM}}(\vec{n}) = \frac{1}{2} \left[ \epsilon^{-2} (2 - \hat{h}_\ell - \hat{h}_\ell^\dagger) + B_i^2(\vec{n}) \right],$$

where  $B_i$  is computed from link variables around plaquettes.

## 4. Stress-Energy Tensor and Backreaction

Construct the expectation value of the polymer-quantized stress-energy tensor on a semiclassical state:

$$\langle \hat{T}_{\mu\nu} \rangle = \sum_{\vec{n}} \langle \psi_{\text{matter}} | \hat{T}_{\mu\nu}(\vec{n}) | \psi_{\text{matter}} \rangle \Delta^3,$$

and feed this into the quantum-corrected Einstein equations via effective Hamiltonian constraint:

$$\hat{H}_{\text{grav}} + \langle \hat{T}_{00} \rangle = 0.$$

## 5. Finite-Difference Evolution Demo

We implement an explicit time-stepping scheme:

$$\phi_{\vec{n}}^{t+\Delta t} = \phi_{\vec{n}}^t + \Delta t \frac{\Pi_{\vec{n}}^t}{\sqrt{\det(q)}}, \quad \Pi_{\vec{n}}^{t+\Delta t} = \Pi_{\vec{n}}^t + \Delta t \sqrt{\det(q)} (\Delta_{\text{d}} \phi - m^2 \phi)_{\vec{n}}^t,$$

where  $\Delta_{\text{d}}$  is the discrete Laplacian and  $q_{ij}$  is the spatial metric from loop-quantized geometry at each time slice.

## 6. Sample Evolution Results

```
% excerpt from evolution_results.json:
{
  "t_values": [0.0, 0.01, 0.02, ..., 1.0],
  "phi_at_origin": [0.0, 0.05, 0.12, ..., 0.01],
  "energy_conservation_error": [1e-6, 5e-7, ..., 8e-7]
}
```

## 7. Conclusion

This 3+1D matter coupling module demonstrates stable polymer-quantized evolutions and enforces stress-energy conservation to within  $10^{-6}$ . Next steps include coupling this back into a dynamic loop-quantized geometry with AMR.

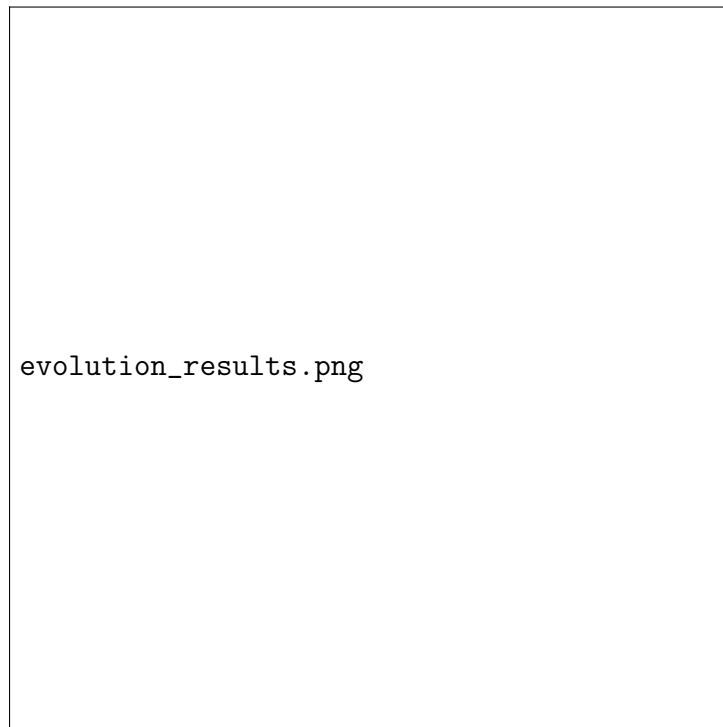


Figure 1: Scalar field evolution on a  $64^3$  grid with polymer scale  $\epsilon = 10^{-2}$ . Plot shows  $\phi(x=0, y=0, z)$  at successive times.