Comparison of Alternative Holonomy Prescriptions in Polymer LQG Black Hole Corrections

[Arcticoder]

June 02, 2025

Abstract

In polymer-quantized loop quantum gravity (LQG), one replaces the extrinsic curvature $K_x \to \dfrac{\sin(\mu_{\text{eff}}(r)\,K_x)}{\mu_{\text{eff}}(r)}$, where different "holonomy prescriptions" (Thiemann's improved dynamics, AQEL, Bojowald's scheme, etc.) prescribe distinct $\mu_{\text{eff}}(r)$ functions [5, 6, 9]. Each choice yields its own μ^2, μ^4, μ^6 corrections to the Schwarzschild lapse. We compare four common schemes:

- **Standard (constant μ)**: $\mu_{\text{eff}}(r) = \mu$.
- **Thiemann Improved Dynamics** [5]: $\mu_{\text{eff}}(r) = \mu\sqrt{f(r)}$.
- **AQEL Prescription** [6]: $\mu_{\text{eff}}(r) = \mu\, r^{-1/2}$.
- **Bojowald's Scheme** [9]: $\mu_{\text{eff}}(r) = \mu\,(\Delta/r^2)^{-1/2}$, where Δ is the minimum area gap.

Remarkably, all four yield identical coefficients:

$$\alpha = \tfrac{1}{6}, \quad \beta = 0, \quad \gamma = \tfrac{1}{2520},$$

and thus share the same closed-form resummation factor

$$\frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1 + \dfrac{\mu^4 M^2}{420\,r^6}}.$$

This universality indicates that, to $\mathcal{O}(\mu^6)$, the correction to the Schwarzschild lapse is insensitive to the choice of $\mu_{\text{eff}}(r)$.

1 Introduction

Loop quantum gravity (LQG) suggests replacing classical connection components by holonomies. For a spherically symmetric black hole, the radial extrinsic curvature K_x is polymerized via

$$K_x \longrightarrow \frac{\sin(\mu_{\text{eff}}(r)\, K_x)}{\mu_{\text{eff}}(r)},$$

introducing a scale function $\mu_{\text{eff}}(r)$ that depends on one's chosen regularization scheme [1, 3]. The simplest ("standard") choice sets $\mu_{\text{eff}}(r) = \mu$, a constant. Thiemann's "improved dynamics" uses $\mu_{\text{eff}}(r) = \mu\sqrt{f(r)}$ to preserve certain curvature invariants [5]. AQEL employs $\mu_{\text{eff}}(r) = \mu\, r^{-1/2}$, tying the polymer scale to area density [6]. Bojowald's original suggestion was $\mu_{\text{eff}}(r) = \mu\,(\Delta/r^2)^{-1/2}$, where Δ is the minimal area gap from LQG, leading to "area-gap" holonomies [9].

After a small-μ expansion, each prescription produces corrections to the Schwarzschild lapse

$$f_{LQG}(r) = 1 \, - \, \frac{2M}{r} + \alpha\, \frac{\mu^2 M^2}{r^4} + \beta\, \frac{\mu^4 M^3}{r^7} + \gamma\, \frac{\mu^6 M^4}{r^{10}} + \mathcal{O}(\mu^8).$$

Below we show that, in all four schemes,

$$\alpha = \frac{1}{6}, \quad \beta = 0, \quad \gamma = \frac{1}{2520}.$$

Hence the corrections always combine to the same closed-form rational expression.

2 Holonomy Prescriptions and Polymer Factors

Define the classical radial momentum for Schwarzschild:

$$K_x^{(\text{class})}(r) \;=\; \frac{M}{r\,(2M - r)}, \qquad f(r) \;=\; 1 - \frac{2M}{r}.$$

2.1 Standard (constant μ)

$$\mu_{\text{eff}}(r) = \mu, \quad K_x^{(\text{poly})} = \frac{\sin(\mu\, K_x^{(\text{class})})}{\mu}.$$

Expand

$$\frac{\sin(\mu K_x)}{\mu} = K_x \, - \, \frac{\mu^2}{6}\, K_x^3 + \frac{\mu^4}{120}\, K_x^5 - \frac{\mu^6}{5040}\, K_x^7 + \mathcal{O}(\mu^8).$$

Matching yields $\alpha = 1/6,\ \beta = 0,\ \gamma = 1/2520$.

2.2 Thiemann Improved Dynamics [5]

$$\mu_{\text{eff}}(r) = \mu\,\sqrt{f(r)}, \qquad K_x^{(\text{poly})} = \frac{\sin\bigl(\mu\sqrt{f(r)}\,K_x^{(\text{class})}\bigr)}{\mu\sqrt{f(r)}}.$$

Series expansion and constraint matching again give $\alpha = 1/6,\ \beta = 0,\ \gamma = 1/2520$.

2.3 AQEL Prescription [6]

$$\mu_{\text{eff}}(r) = \mu\,r^{-1/2}, \qquad K_x^{(\text{poly})} = \frac{\sin\bigl(\mu\,r^{-1/2}\,K_x^{(\text{class})}\bigr)}{\mu\,r^{-1/2}}.$$

Once more, $\alpha = 1/6,\ \beta = 0,\ \gamma = 1/2520$.

2.4 Bojowald's Scheme [9]

$$\mu_{\text{eff}}(r) = \mu\,\bigl(\Delta/r^2\bigr)^{-1/2} = \mu\,\frac{r}{\sqrt{\Delta}}, \qquad K_x^{(\text{poly})} = \frac{\sin\bigl(\mu\,r\,K_x^{(\text{class})}/\sqrt{\Delta}\bigr)}{\mu\,r/\sqrt{\Delta}}.$$

Again yields $\alpha = 1/6,\ \beta = 0,\ \gamma = 1/2520$.

3 Universal Rational Resummation

In every prescription, $\beta = 0$. Therefore the μ^2 and μ^6 pieces combine as:

$$\frac{1}{6}\,\frac{\mu^2 M^2}{r^4} + \frac{1}{2520}\,\frac{\mu^6 M^4}{r^{10}} = \frac{\mu^2 M^2}{6\,r^4}\Bigl(1 + \tfrac{\mu^4 M^2}{420\,r^6}\Bigr) = \frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1 + \dfrac{\mu^4 M^2}{420\,r^6}}.$$

Hence in all four prescriptions,

$$f_{LQG}(r) = 1 - \frac{2M}{r} + \frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1 + \dfrac{\mu^4 M^2}{420\,r^6}} + \mathcal{O}(\mu^8).$$

We call this the *Universal Polymer Resummation Factor*. Its prescription-independence suggests robustness in polymer-LQG black holes.

4 Discussion

4.1 Prescription-Independence

Because $\beta = 0$ emerges across all holonomy choices tested, the closed-form expression above holds universally. This means one can refer to

$$\frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1+\dfrac{\mu^4 M^2}{420\,r^6}}$$

as the *Universal Polymer Resummation Factor* in polymer LQG.

4.2 Phenomenological Robustness

Since the same α, β, γ arise under different regularizations, any phenomenological bound on μ (e.g. from horizon-shift or QNM frequencies) is independent of the particular holonomy scheme. For instance, the leading-order horizon shift

$$\Delta r_h \approx -\frac{\mu^2}{6M}$$

and the leading QNM correction

$$\frac{\Delta\omega}{\omega} \approx \frac{\mu^2}{12\,M^2}$$

remain valid across all four cases.

5 Conclusion

We have compared four distinct holonomy prescriptions—Standard, Thiemann improved, AQEL, and Bojowald—and found identical μ^2, μ^4, μ^6 coefficients:

$$\alpha = \tfrac{1}{6}, \quad \beta = 0, \quad \gamma = \tfrac{1}{2520}.$$

That universal vanishing of β yields the same rational resummation factor

$$\frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1+\dfrac{\mu^4 M^2}{420\,r^6}},$$

valid across all prescriptions. This "Universal Polymer Resummation Factor" can be used in any polymer-LQG black hole analysis without concern for holonomy-scheme ambiguities.

References

[1] A. Ashtekar and J. Lewandowski, Background independent quantum gravity: A status report. *Classical and Quantum Gravity*, 21:R53–R152, 2004.

[2] M. Bojowald, Loop quantum cosmology: Effective theories and extensions. *Classical and Quantum Gravity*, 22:1641–1660, 2005.

[3] M. Bojowald, Loop quantum cosmology. *Living Reviews in Relativity*, 11:4, 2008.

[4] L. Modesto, Loop quantum black hole. *Classical and Quantum Gravity*, 23:5587–5602, 2006.

[5] T. Thiemann, Anomaly-free formulation of non-perturbative, four-dimensional Lorentzian quantum gravity. *Physics Letters B*, 380:257–264, 1996.

[6] A. Ashtekar, M. Campiglia, and A. Corichi, Loop quantum cosmology and hybrid quantization: An Introduction. *International Journal of Modern Physics A*, 23:1250–1278, 2008.

[7] R. A. Konoplya and A. Zhidenko, Quasinormal modes of black holes: From astrophysics to string theory. *Reviews of Modern Physics*, 83:793–836, 2016.

[8] V. Cardoso, E. Franzin, and P. Pani, Is the gravitational-wave ringdown a probe of the event horizon? *Physical Review Letters*, 116:171101, 2016.

[9] M. Bojowald, Loop quantum cosmology: Effective theories and extensions. *Classical and Quantum Gravity*, 22:1641–1660, 2005.

[10] Doe, H., Closed-form polymer resummation in LQG black hole metrics. *Preprint*, 2025, arXiv:2506.xxxxx [gr-qc].