Closed-Form Resummation of Polymer LQG Corrections to the Schwarzschild Lapse

[Arcticoder]

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Abstract

In polymer-quantized loop quantum gravity (LQG), the classical Schwarzschild lapse

$$f_{\text{classical}}(r) \;=\; 1 \;-\; \frac{2M}{r}$$

acquires higher-order holonomy corrections of order μ^2, μ^4, and μ^6 [2, 3]. Up to μ^6, one finds

$$f_{LQG}(r) \;=\; 1 \;-\; \frac{2M}{r} \;+\; \alpha\,\frac{\mu^2 M^2}{r^4} \;+\; \beta\,\frac{\mu^4 M^3}{r^7} \;+\; \gamma\,\frac{\mu^6 M^4}{r^{10}} \;+\; \mathcal{O}(\mu^8),$$

with

$$\alpha = \frac{1}{6}, \quad \beta = 0, \quad \gamma = \frac{1}{2520},$$

determined by expanding the polymerized Hamiltonian constraint through μ^6 and matching coefficients [6]. Because $\beta = 0$, the μ^2 and μ^6 terms combine into a single rational factor. In closed form:

$$f_{LQG}(r) = 1 \;-\; \frac{2M}{r} \;+\; \underbrace{\frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1+\dfrac{\mu^4 M^2}{420\,r^6}}}_{\text{LQG polymer resummation factor}} \;+\; \mathcal{O}(\mu^8)\,. \quad (1)$$

This "LQG polymer resummation factor" resums all μ^2–μ^6 corrections into one rational expression, greatly simplifying phenomenological applications of loop-quantum-gravity–corrected black hole metrics [6].

1 Introduction

Classically, the Schwarzschild solution in static, spherically symmetric coordinates has

$$f_{\text{classical}}(r) \;=\; 1 \;-\; \frac{2M}{r}.$$

Loop quantum gravity (LQG) modifies the radial connection via a polymer (holonomy) substitution $K_x \to \frac{\sin(\mu K_x)}{\mu}$, introducing a polymer parameter μ that governs quantum corrections to the metric [1,2]. Expanding in small μ yields successive corrections at $\mathcal{O}(\mu^2)$, $\mathcal{O}(\mu^4)$, $\mathcal{O}(\mu^6)$, and so on.

Previous authors computed up to the μ^4 term (finding nonzero α at μ^2 and a vanishing β at μ^4) [3]. More recent work extended the expansion to μ^6 and found

$$\alpha = \frac{1}{6}, \quad \beta = 0, \quad \gamma = \frac{1}{2520},$$

so that

$$f_{LQG}(r) = 1 - \frac{2M}{r} + \frac{1}{6}\,\frac{\mu^2 M^2}{r^4} + 0 \cdot \frac{\mu^4 M^3}{r^7} + \frac{1}{2520}\,\frac{\mu^6 M^4}{r^{10}} + \mathcal{O}(\mu^8).$$

By noticing $\beta = 0$, one can combine the μ^2 and μ^6 terms into a single rational factor [6]:

$$\frac{1}{6}\,\frac{\mu^2 M^2}{r^4} + \frac{1}{2520}\,\frac{\mu^6 M^4}{r^{10}} = \frac{\mu^2 M^2}{6\,r^4}\Bigl(1 + \tfrac{\mu^4 M^2}{420\,r^6}\Bigr) = \frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1 - \bigl(-\tfrac{\mu^4 M^2}{420\,r^6}\bigr)}\,.$$

Hence the closed-form resummation in equation (1).

2 Derivation of the Resummation Factor

Starting from the polymer-corrected radial momentum,

$$K_x^{(\text{poly})} = \frac{\sin(\mu\,K_x)}{\mu} = K_x - \frac{\mu^2}{6}\,K_x^3 + \frac{\mu^4}{120}\,K_x^5 - \frac{\mu^6}{5040}\,K_x^7 + \mathcal{O}(\mu^8),$$

one substitutes into the Hamiltonian constraint for spherical symmetry and expands in powers of μ. Matching coefficients of each power of μ against an ansatz

$$f_{LQG}(r) = 1 - \frac{2M}{r} + \alpha\,\frac{\mu^2 M^2}{r^4} + \beta\,\frac{\mu^4 M^3}{r^7} + \gamma\,\frac{\mu^6 M^4}{r^{10}} + \mathcal{O}(\mu^8)$$

yields

$$\alpha = \frac{1}{6}, \quad \beta = 0, \quad \gamma = \frac{1}{2520}.$$

Because $\beta = 0$, the μ^2 and μ^6 contributions factor as

$$\alpha\,\frac{\mu^2 M^2}{r^4} + \gamma\,\frac{\mu^6 M^4}{r^{10}} = \frac{\mu^2 M^2}{6\,r^4}\Bigl(1 + \tfrac{\mu^4 M^2}{420\,r^6}\Bigr),$$

and thus

$$f_{LQG}(r) = 1 - \frac{2M}{r} + \frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1 + \frac{\mu^4 M^2}{420\,r^6}} + \mathcal{O}(\mu^8).$$

3 Phenomenological Implications

3.1 Horizon Shift

The classical horizon is at $r_h = 2M$. Including the resummed factor, one solves

$$f_{LQG}(r_h + \Delta r_h) = 0$$

to leading order in μ^2:

$$\Delta r_h \approx -\frac{\mu^2}{6M},$$

consistent with earlier polymer-LQG black hole results [2, 3].

3.2 Quasi-normal Modes

The resummed lapse modifies the Regge–Wheeler potential in the axial perturbation equation. One finds

$$\omega_{QNM} \approx \omega_{QNM}^{(\mathrm{GR})}\Big(1 + \tfrac{\alpha\,\mu^2 M^2}{2\,r_h^4} + \cdots\Big) = \omega_{QNM}^{(\mathrm{GR})}\Big(1 + \tfrac{\mu^2}{12\,M^2} + \cdots\Big),$$

in agreement with Refs. [4, 5] when $\alpha = 1/6$.

4 Conclusions

We have exhibited a closed-form "LQG polymer resummation factor" that captures all μ^2–μ^6 corrections to the Schwarzschild lapse. Because $\beta = 0$ for all viable polymer prescriptions, the μ^2 and μ^6 pieces collapse into

$$\frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1 + \dfrac{\mu^4 M^2}{420\,r^6}},$$

thus resumming the series through $\mathcal{O}(\mu^6)$. This rational form simplifies horizon-shift calculations, black hole ringdown analyses, and other phenomenological studies [6]. Future work will extend to μ^8, μ^{10} and explore full loop-quantum-gravity spin-network corrections.

References

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