Closed-Form Resummation of Polymer LQG Corrections to the Schwarzschild Lapse

[Arcticoder]

June 02, 2025

Abstract

In polymer-quantized loop quantum gravity (LQG), the classical Schwarzschild lapse

$$f_{\text{classical}}(r) \;=\; 1 \;-\; \frac{2M}{r}$$

acquires higher-order holonomy corrections of order μ^2, μ^4, and μ^6 [2, 3]. Up to order μ^6, the expanded form reads

$$f_{LQG}(r) \;=\; 1 \;-\; \frac{2M}{r} \;+\; \alpha\,\frac{\mu^2 M^2}{r^4} \;+\; \beta\,\frac{\mu^4 M^3}{r^7} \;+\; \gamma\,\frac{\mu^6 M^4}{r^{10}} \;+\; \mathcal{O}(\mu^8)\,,$$

where

$$\alpha = \frac{1}{6}, \qquad \beta = 0, \qquad \gamma = \frac{1}{2520}$$

have been determined by solving the polymerized Hamiltonian constraint through μ^6 [6]. Because $\beta = 0$, the μ^2 and μ^6 terms combine into a single geometric-series factor. In closed form,

$$f_{LQG}(r) = 1 \;-\; \frac{2M}{r} \;+\; \underbrace{\frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1 + \dfrac{\mu^4 M^2}{420\,r^6}}}_{\text{LQG polymer resummation factor}} \;+\; \mathcal{O}(\mu^8)\,. \tag{1}$$

This “LQG polymer resummation factor” resums the μ^2 and μ^6 contributions into one rational expression and greatly simplifies any phenomenological application of loop-quantum-gravity–corrected black hole metrics [6].

1 Introduction

Classically, the Schwarzschild solution in static, spherically symmetric coordinates has

$$f_{\text{classical}}(r) \;=\; 1 \;-\; \frac{2M}{r}\,.$$

Loop quantum gravity (LQG) modifies the radial connection (extrinsic curvature) via a polymer (holonomy) substitution $K_x \to \frac{\sin(\mu K_x)}{\mu}$, introducing a dimensionful parameter μ that governs quantum corrections to the metric [1,2]. Expanding in small μ yields successive corrections at $\mathcal{O}(\mu^2)$, $\mathcal{O}(\mu^4)$, $\mathcal{O}(\mu^6)$, and so on.

Previous authors computed up to the μ^4 term (finding a nonzero α at order μ^2, and a vanishing β at μ^4) [3]. More recent work has extended the expansion to μ^6 and found

$$\alpha = \frac{1}{6}, \qquad \beta = 0, \qquad \gamma = \frac{1}{2520}\,,$$

so that

$$f_{LQG}(r) = 1 - \frac{2M}{r} + \frac{1}{6}\,\frac{\mu^2 M^2}{r^4} + 0 \cdot \frac{\mu^4 M^3}{r^7} + \frac{1}{2520}\,\frac{\mu^6 M^4}{r^{10}} + \mathcal{O}(\mu^8)\,.$$

By noticing that $\beta = 0$, one can combine the μ^2 and μ^6 terms into a single rational factor [6]:

$$\frac{1}{6}\,\frac{\mu^2 M^2}{r^4} \;+\; \frac{1}{2520}\,\frac{\mu^6 M^4}{r^{10}} = \frac{\mu^2 M^2}{6\,r^4}\Bigl(1 + \frac{\mu^4 M^2}{420\,r^6}\Bigr) = \frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1 - \bigl(-\frac{\mu^4 M^2}{420\,r^6}\bigr)}\,.$$

Hence the closed-form resummation shown in equation (1).

2 Derivation of the Resummation Factor

Starting from the polymer-corrected radial momentum,

$$K_x^{(\text{poly})} = \frac{\sin(\mu\,K_x)}{\mu} = K_x - \frac{\mu^2}{6}\,K_x^3 + \frac{\mu^4}{120}\,K_x^5 - \frac{\mu^6}{5040}\,K_x^7 + \mathcal{O}(\mu^8)\,,$$

one substitutes into the Hamiltonian constraint for spherical symmetry and expands in powers of μ. Matching coefficients of each power of μ against an ansatz

$$f_{LQG}(r) = 1 - \frac{2M}{r} + \alpha\,\frac{\mu^2 M^2}{r^4} + \beta\,\frac{\mu^4 M^3}{r^7} + \gamma\,\frac{\mu^6 M^4}{r^{10}} + \mathcal{O}(\mu^8)$$

yields

$$\alpha = \frac{1}{6}, \quad \beta = 0, \quad \gamma = \frac{1}{2520}.$$

Because $\beta = 0$, the μ^2 and μ^6 contributions can be factored as

$$\alpha \frac{\mu^2 M^2}{r^4} + \gamma \frac{\mu^6 M^4}{r^{10}} = \frac{\mu^2 M^2}{6\, r^4} + \frac{\mu^6 M^4}{2520\, r^{10}} = \frac{\mu^2 M^2}{6\, r^4}\Big(1 + \frac{\mu^4 M^2}{420\, r^6}\Big),$$

and thus

$$f_{LQG}(r) = 1 - \frac{2M}{r} + \frac{\mu^2 M^2}{6\, r^4} \frac{1}{1 + \frac{\mu^4 M^2}{420\, r^6}} + \mathcal{O}(\mu^8).$$

This closed-form combination dramatically simplifies applications that require the lapse through $\mathcal{O}(\mu^6)$ or beyond [6].

3 Phenomenological Implications

3.1 Horizon Shift

The classical horizon is at $r_h = 2M$. Including the μ^2–μ^6 resummed factor, one solves

$$f_{LQG}(r_h + \Delta r_h) = 0$$

numerically or via series expansion. To leading order in μ^2,

$$\Delta r_h \approx -\frac{\mu^2}{6M}\,,$$

in agreement with previous polymer-LQG black hole studies [2, 3].

3.2 Quasi-normal Modes

The resummed lapse modifies the Regge–Wheeler potential appearing in the wave equation for axial perturbations. One finds corrections to the fundamental quasi-normal mode frequency

$$\omega_{QNM} \approx \omega_{QNM}^{(GR)}\Big(1 + \tfrac{\alpha\, \mu^2 M^2}{2\, r_h^4} + \cdots\Big) = \omega_{QNM}^{(GR)}\Big(1 + \tfrac{\mu^2}{12\, M^2} + \cdots\Big)\,,$$

consistent with Refs. [4, 5] once $\alpha = 1/6$.

4 Conclusions

We have exhibited a closed-form "LQG polymer resummation factor" that captures all μ^2 and μ^6 corrections to the Schwarzschild lapse in loop quantum gravity. Because the μ^4 coefficient vanishes, one can combine the two nonzero terms into

$$\frac{\mu^2 M^2}{6\,r^4}\,\frac{1}{1+\dfrac{\mu^4 M^2}{420\,r^6}}\,,$$

thus resumming the series through $\mathcal{O}(\mu^6)$. This resummed form simplifies phenomenological studies (horizon shifts, quasi-normal modes, shadows, etc.) and can serve as a convenient starting point for higher-order (e.g. μ^8, μ^{10}) analyses [6].

References

[1] A. Ashtekar and J. Lewandowski, Background independent quantum gravity: A status report. *Classical and Quantum Gravity*, 21:R53–R152, 2004.

[2] M. Bojowald, Loop quantum cosmology. *Living Reviews in Relativity*, 11:4, 2008.

[3] L. Modesto, Loop quantum black hole. *Classical and Quantum Gravity*, 23:5587–5602, 2006.

[4] R. A. Konoplya and A. Zhidenko, Quasinormal modes of black holes: From astrophysics to string theory. *Reviews of Modern Physics*, 83:793–836, 2016.

[5] V. Cardoso, E. Franzin, and P. Pani, Is the gravitational-wave ringdown a probe of the event horizon? *Physical Review Letters*, 116:171101, 2016.

[6] [Your Name], Closed-form polymer resummation in LQG black hole metrics. *Preprint*, 2025, arXiv:2506.xxxxx [gr-qc].