Measurement of Quantum States of Squeezed Light

Arunava Das

IIT Delhi

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Aim

- State of individual quantum particle/system is not observable, so an ensemble of identically prepared systems is used.
- A large no of measurements allow Quantum State Reconstruction (QSR)
- Density matrix or equivalent Wigner function and photon statistics of the quantum state is derived.
- Photon statistics and properties of the following types of squeezed states of light are measured in this study
    - Squeezed Vacuum State
    - Amplitude Squeezed State
    - Phase Squeezed State
    - Arbitrary quadrature squeezed state
- A balanced Homodyne detector is used to measure the field variances

Optical Homodyne Tomography

- The Electric Field Operator for the single mode field is given by

$$\hat{E} \;=\; \sqrt{\frac{2\hbar\omega}{\epsilon_0 V}}\left[\hat{X}\sin\zeta \;-\; \hat{Y}\cos\zeta\right] \tag{1}$$

  Where $\hat{X} \;=\; \dfrac{\hat{a}+\hat{a}^{\dagger}}{2}$ is the X quadrature operator and $\hat{Y} \;=\; \dfrac{\hat{a}-\hat{a}^{\dagger}}{2i}$ is the Y quadrature operator, $\hat{a}$ and $\hat{a}^{\dagger}$ being the regular creation and annihilation operators. [1]
- To access experimentally the electric field, which oscillates at frequency $\dfrac{\omega}{2\pi}$ (which is in 100s of Terahertz), we apply balanced homodyne detection

Technique

- Signal wave is spatially overlapped at a beam splitter with a local oscillator wave of same frequency.
- Let the emerging outputs be ε_1 and ε_2. If local oscillator field has magnitude E_{LO} and phase ϕ_{LO}, output from beam splitter are thus

$$\varepsilon_1 = \frac{1}{\sqrt{2}}\left[\left(E_{LO}\cos\phi_{LO} - E_Y\right) + i\left(E_{LO}\sin\phi_{LO} - E_X\right)\right] \quad (2)$$

$$\varepsilon_2 = \frac{1}{\sqrt{2}}\left[-\left(E_{LO}\cos\phi_{LO} - E_X\right) + i\left(E_{LO}\sin\phi_{LO} - E_Y\right)\right] \quad (3)$$

- the homodyne detection (involving finding difference in the fluxes) gives result output function

$$\mathrm{E}_O \propto 2E_{LO}\left(E_X\sin\phi_{LO} - E_Y\cos\phi_{LO}\right) (4)$$

- A large no of observations gives the probability distribution $P(\epsilon_\phi)$ as a function of ϵ_ϕ - the eigenvalues of field operator $\hat{E}$

Density Matrix

- Density operator is related to probability distribution $P(\epsilon_0)$ as

$$P(\epsilon_0) = \left\langle \epsilon_\phi \left| U^\dagger(\phi)\, \rho\, U(\phi) \right| \epsilon_\phi \right\rangle \tag{5}$$

  where $U(\phi) = \exp(-i\phi\hat{a}^\dagger\hat{a})$ is the operator for a rotation in the phase space of the local oscillator.
- As optical state evolves with ω freely, U is equivalent to the time evolution operator with $\phi = \omega t + \text{constant}$
- The evolution in ϕ is equivalent to the time dependence in position probability of the state.
- So homodyne detection is used to map time evolution of a harmonic oscillator state here

Wigner Function

- Distributions of $P(\epsilon_\phi)$ are marginals of the wigner function in rotated coordinates of ϕ

$$P(\epsilon_\phi) = \int_{-\infty}^{\infty} W\left(\epsilon_\phi \cos\phi - \lambda_\phi \sin\phi\ ,\ \epsilon_\phi \cos\phi + \lambda_\phi \sin\phi\right) d\lambda_\phi \quad (6)$$

  Where λ_ϕ is the 90 degree rotated E field of the single mode field

- $W(\epsilon, \lambda)$ can be obtained by inverse Radon Transform

$$W(\epsilon, \lambda) = \frac{1}{\pi ab}\left(-\frac{(x - e_0 \cos\phi)^2}{a^2} - \frac{(y - e_0 \sin\phi)^2}{b^2}\right) \quad (7)$$

- Here $x = \epsilon_0 \cos\phi + \epsilon_{\frac{\pi}{2}} \sin\phi$, $y = -\epsilon_0 \sin\phi + \epsilon_{\frac{\pi}{2}} \cos\phi$ are the general phase space coordinates.
- $e_0 = 2\beta E(0)$ is the state's amplitude, β is the modulation index.
- a and b are the min and max standard deviation of quadrature fluctuations.

The Detector

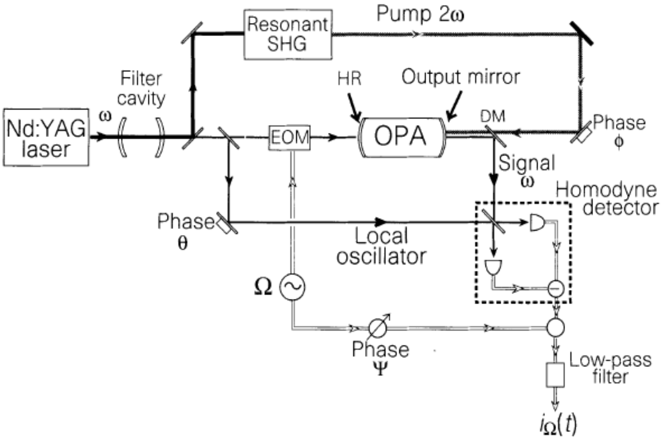

Figure 1: Homodyne detector [2]

Experimental setup (SHG method)

- Central component is a Lithium Niobate optical parametric oscillator (OPA) pumped by a frequency doubled continuous wave Nd-YaG laser at 1064 nm.
- Filter Cavity is a mode-cleaning cavity that transmits 75% of the infrared wave. It's narrow linewidth of 170 kHz supresses high-frequency technical noise of laser, gives a shot-noise limited local oscillator for light at freq > 1 MHz and in range of few mW
- Pump wave for the OPA with freq 2ω, power 20-30 mW is generated by second harmonic generator(SHG) in resonance with the laser beam.
- Operated below threshold, OPA acts as a source of squeezed vacuum state.[3].
- Field's spectral components were studied around an offset of $\frac{\Omega}{2\pi}$ frequency (around 2 MHz) from ω to avoid low freq laser noise.
- SHG basically does the reverse process of SPDC

Procedure

- OPA is operated in dual-port configuration to generate light with non-vanishing average electric field
- A weak beam of laser is phase-modulated by Ω (index $= \beta \ll 1$) using Electro-Optic Modulator (EOM) and injected into the High Reflector (HR) port of OPA
- Two bright sidebands at $\omega \mp \Omega$ are within cavity bandwidth.
- A small semi-classical analysis shows that due to small ratio of HR transmission to Output Mirror transmission, sidebands and quantum fluctuations (arising from the vacuum fluctuations entering through the output coupler) are heavily attenuated.
- By interacting with the 2ω pump wave, Quadrature fluctuations out-of-phase with the pump freq are squeezed while in-phase fluctuations are amplified.
- By changing phase θ as in the figure, any arbitrary squeezed state is generated.

Detection

- Changing power of seed wave (from laser) controls the excitation of sidebands coarsely, while varying modulation strength β of EOM changes it finely.
- Blocking OPA pump beam leaves us with a coherent state.
- final signal is analysed by homodyne detection, final output current $i_{\Omega}(\theta, t)$ is processed further in the following steps
  - modulated with the electrical local oscillator current $\sin \Omega t + \phi$ with $cos\phi = 1$
  - Phase locked to modulation frequency.
  - filtered through a LPF of freq 100KHz to avoid unnecessary signals.
- Resulting current is of the form

$$i_{\Omega} = (2\beta E_0 + X_n(\Omega, t) - X_n(-\Omega, t)) \sin \theta + (Y_n(\Omega, t) - Y_n(-\Omega, t)) \cos \theta \tag{8}$$

- X_n and Y_n are the noise fluctuations in the 100 KHz wide band centered at Ω.

Analysis of results

- By variation of the local oscillator phase θ (as in the fig), nay quadrature difference between $\omega - \Omega$ and $\omega + \Omega$ can be accessed.
- Time traces of i_Ω for coherent and squeezed states are the experimental counterpart of the theoretical depictions of squeezed states.
- At times when the local oscillator phase is approximately constant, individual time traces of states can be regarded as their quantum trajectories of a particular quadrature X_ϕ. The statistics obtained are normalised with respect to the vacuum state and probability distributions are plotted
- variances of the distributions determine the amount of squeezing/anti-squeezing.
- slight phase instabilities in the seed wave restrict the maximum squeezing limit

Phase Space distribution

- Squeezed states are commonly depicted by ellipses in phase space with half axes lengths a and b as determined in equation (7), which correspond to a horizontal section through the Wigner function.
- area of ellipse is the purity of the state

$$\text{Trace}(\rho) \;=\; 2\pi \int\int W(x,y)^2 dx\, dy \;=\; \frac{1}{ab} \tag{9}$$

- In this experimental setup, area of the ellipse was around 1 for coherent state and 0.41-0,46 fr the squeezed states. Reason for significantly mixed character are
  - Loss experienced in the cavity of OPA (escape efficiency around 0,88)
  - loss during propagation and detection (detection efficiency around 0.94)
- These losses are equivalent to convolution of the Wigner function with two Gaussian's - one for propagation & detection and one for filtering procedures

Noise traces, Wigner function plot

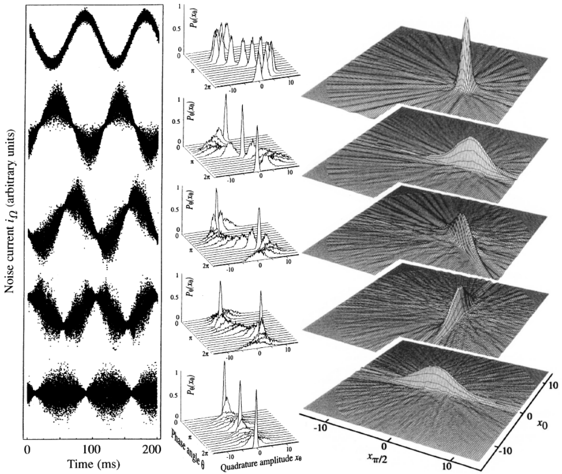

Figure 2: Noise Traces, Quadrature distributions, Reconstructed Wigner Functions. Coherent, Phase Squeezed, 48 deg quad Squeezed, Amplitude Squeezed, Squeezed vacuum in order [2]

Alternative Approach

- density matrices of the final state can be calculated in the Fock basis, where the states are described by energy eigenstates $|n>$.
- Diagonal elements ρ_{nn} of the density matrix ρ are the occupational probabilities of number states $|n>$.
- Analogously - n is the photon flux per unit bandwidth, p(n) is the probability that the counter registers n photons per second,
- From the data in figure 3, a rotation of squeezing axis changes photon distribution substantially.
- Most squeezed states are super-poissonian in distribution, as opposed to the poissonian coherent state

Photon Statistics Observed

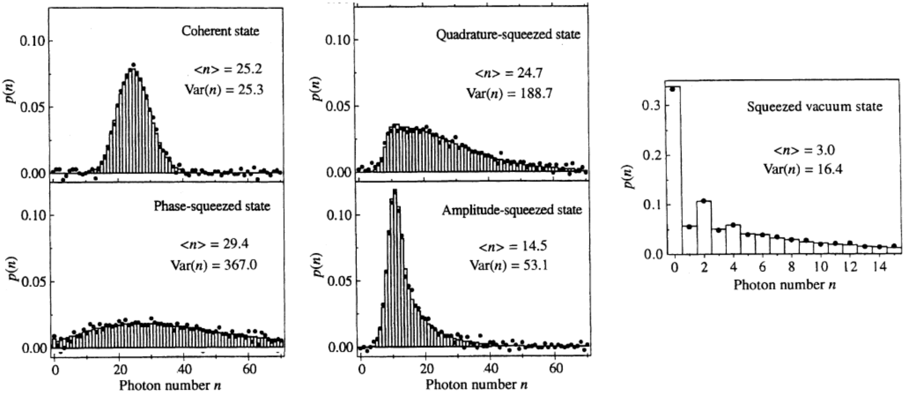

Figure 3: Solid points refer to theoretical expectations, except for coherent state being poissonian, all other states are super-poissonian (Var(n) > mean(n)) [2]

Photon Statistics

- the mean and variance of n in this experiment is given by

$$< n >= \frac{1}{4}\left(a^2 + b^2 - 2\right) + \frac{1}{2}e_0^2 \tag{10}$$

$$\mathrm{Var}(n) = \frac{1}{8}\left(a^2 + b^2 - 2\right) + \frac{1}{2}e_0^2\left(a^2 \cos\phi^2 + b^2 \sin\phi^2\right) \tag{11}$$

- For amplitude squeezed light, in the data in fig 3, the characteristics are super-poissonian, which is counter-intuitive but can be explained by the dominance of the first term in equation 10 and 11 when e_0 is small
- A final plot of density matrices are made after changing parameters (decreasing a & b and increasing e_0) to make amplitude squeezed light sub-poissonian. (n,m = 25) in figure 4

Density Matrix Plot

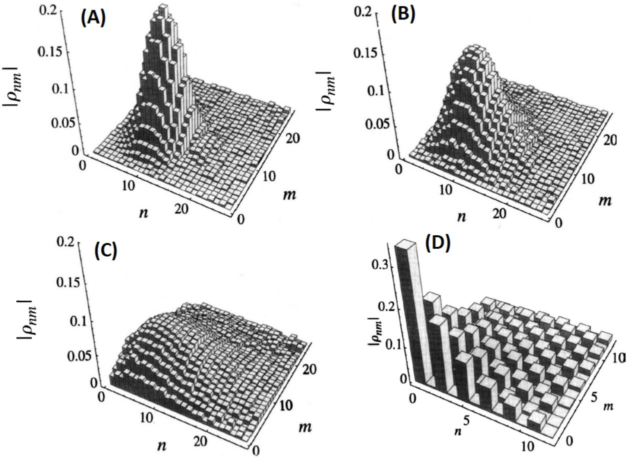

Figure 4: (A) Sub poissonian amplitude squeezed state, (B) Coherent state, (C) Phase-squeezed state, (D) Vacuum squeezed state with odd off diagonals zero showing symmetry of phase space state distribution [2]

Density Matrices

- Interesting observations from these density matrices are
  - For coherent and phase squeezed state, all elements ρ_{mn} are positive
  - Density matrix of squeezed vacuum display a chessboard pattern (alternating ups and downs). This is due to down-conversion process in OPA where photons are created in pairs.
- This concludes the study of squeezed states of light.

References

[1] Claude Fabre, Gilbert Grynberg **and** Alain Aspect. *Introduction to quantum optics. From the semi-classical approach to quantized light*. **september** 2010. ISBN: 978-0-521-55112-0. DOI: 10.1017/CBO9780511778261.

[2] G. Breitenbach, S. Schiller **and** J. Mlynek. ?Measurement of the quantum states of squeezed light? **in**: *Nature* 387.6632 (**may** 1997), **pages** 471–475. ISSN: 1476-4687. DOI: 10.1038/387471a0. URL: https://doi.org/10.1038/387471a0.

[3] Ling-An Wu **andothers**. ?Generation of Squeezed States by Parametric Down Conversion? **in**: *Phys. Rev. Lett.* 57 (20 **november** 1986), **pages** 2520–2523. DOI: 10.1103/PhysRevLett.57.2520.