**Mapping Mirror Coordinates to Target Plane Coordinates**

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Figure 1: Mirror coordinate system [1]

P0: incoming beam origin

n0: incoming beam direction (unit vector)

M: mirror center

C: center of rotation

d: distance between M and C

nm: mirror normal vector

P1: reflected beam origin

n1: reflected beam direction (unit vector)

D: distance between mirror center(C) and target plane

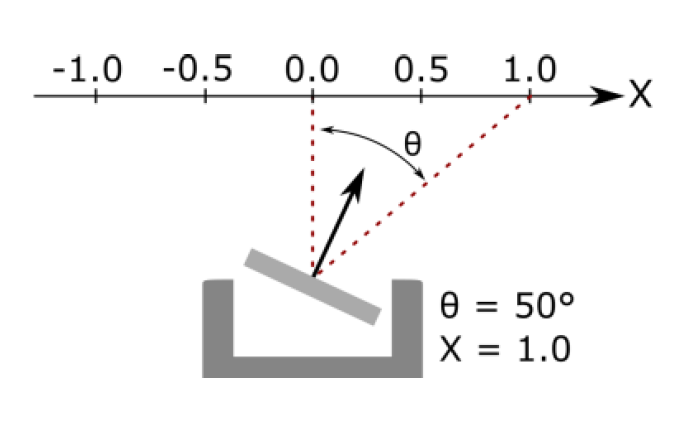
nt: target plane normal vector

P2: the point reflected beam hits the target plane

Using a mirror to steer a laser beam requires a mapping from 3D world coordinate system to mirror coordinate system. Without the conversion reflection from the mirror causes a distortion as in Figure.



Mirror coordinate system is defined in [1] as in following figure.



The coordinate system is a Cartesian coordinate system with X and Y axes. The range of both axes are [-1, 1] interval. Mirror has maximum deflection angle of 25 degrees. Θ = +50° corresponds to +1 and Θ = -50° corresponds to -1. X and Y values are required to be inside a unit circle in order to be a valid point accessible by the mirror.

Alpha: rotation degree

A\_TI = np.array([[ 1, 0 ,0],

[ 0, np.cos(alpha), -np.sin(alpha)],

[ 0, np.sin(alpha), np.cos(alpha)]])

n\_0 = self.normalize(np.array([0,-1,1]))

Assumptions:

* 𝑃1 = 𝑀 = 𝐶
* 𝑂 = 𝑃1 = 𝑀 = 𝐶
* d=0
* Incoming ray is coming in y-z plane.

Steps to convert target coordinates (x\_t, y\_t ) to mirror coordinates (x, y):

1) Compute n\_m

* n\_t = np.dot(A\_IT, np.array([0,0,1]))
* r\_OT = np.dot(A\_IT, np.array([0,0,-D]))
* T\_r\_TP\_2 = np.array([x\_t, y\_t, 0])
* I\_r\_TP\_2 = np.dot(A\_IT, T\_r\_TP\_2)
* r\_OP\_2 = r\_OT + I\_r\_TP\_2
* n\_1 = self.normalize(r\_OP\_2)
* n\_m = self.normalize(n\_1-n0)

2) Compute x,y

* r\_C = np.array([0, 0, d]) # center of rotation
* # xy coordinate system defined with respect to 90° incidence angle
* n\_0 = normalize(np.array([0,0,1]))
* r\_OP\_0 = np.array([0,0, -1])
* n\_t = np.array([0,0,1])
* r\_OT = np.array([0,0,-D])
* scaling = D\*np.tan(np.deg2rad(50))
* r\_OP\_2 = self.getSpotOnTargetPlane(n, r\_C, d, n\_0, r\_OP\_0, r\_OT, n\_t)
* x = r\_OP\_2[0]/scaling
* y = r\_OP\_2[1]/scaling

After the coordinate conversion process, the distortion caused by reflection from the mirror is eliminated.

**Measurement System**

Measurements are done with a 2-dimensional servo motor system with a PM400 power meter attached.

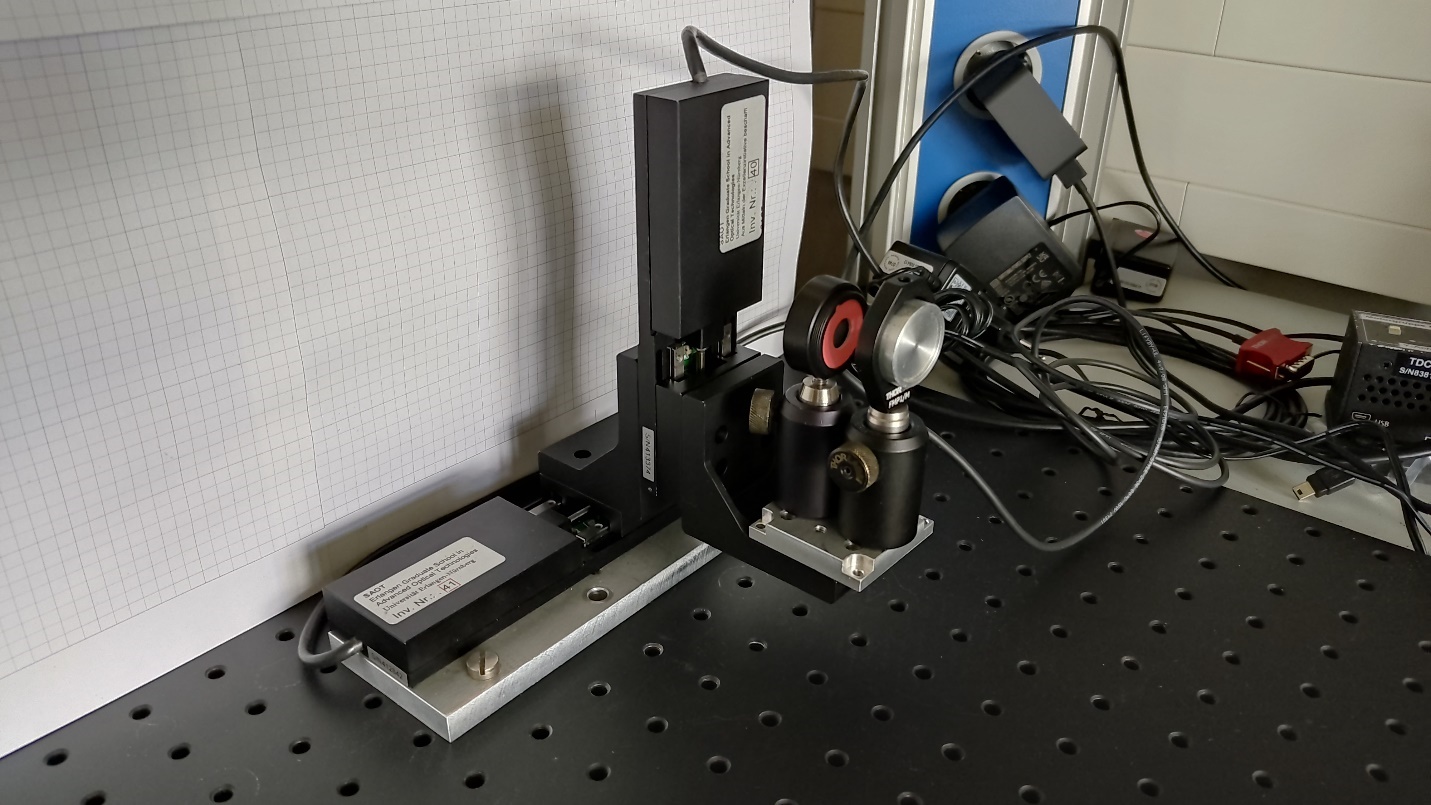


Figure 2: Position error test setup

Measurements are performed by pointing the laser to a specified target position and then sweeping the power meter in a linear trajectory. Power recordings are recorded with recording times and plotted to find the instance with maximum power. The point with maximum power gives the center position of the laser. Power measurements are performed by a Python script which has a loop with a period of around 0.01s. This sampling period is not constant throughout the measurements and might deviate from 0.01s. For this reason, after the measurement, cubic interpolation is performed to get a uniformly sampled signal.

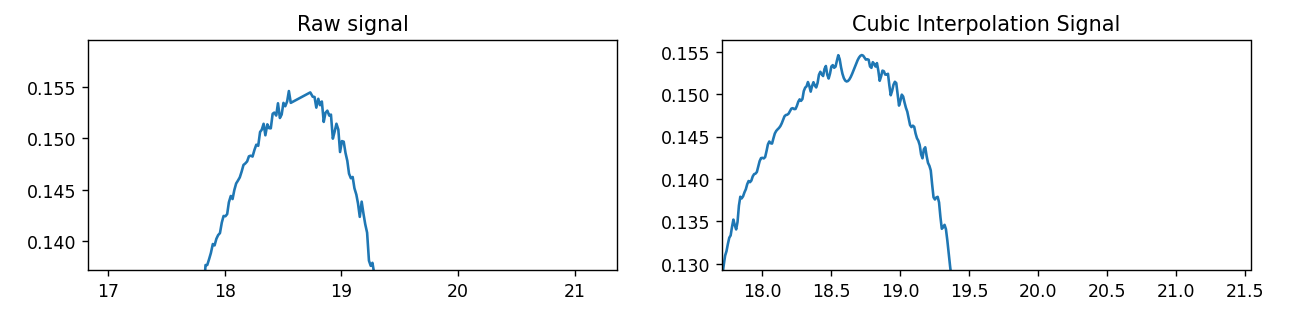
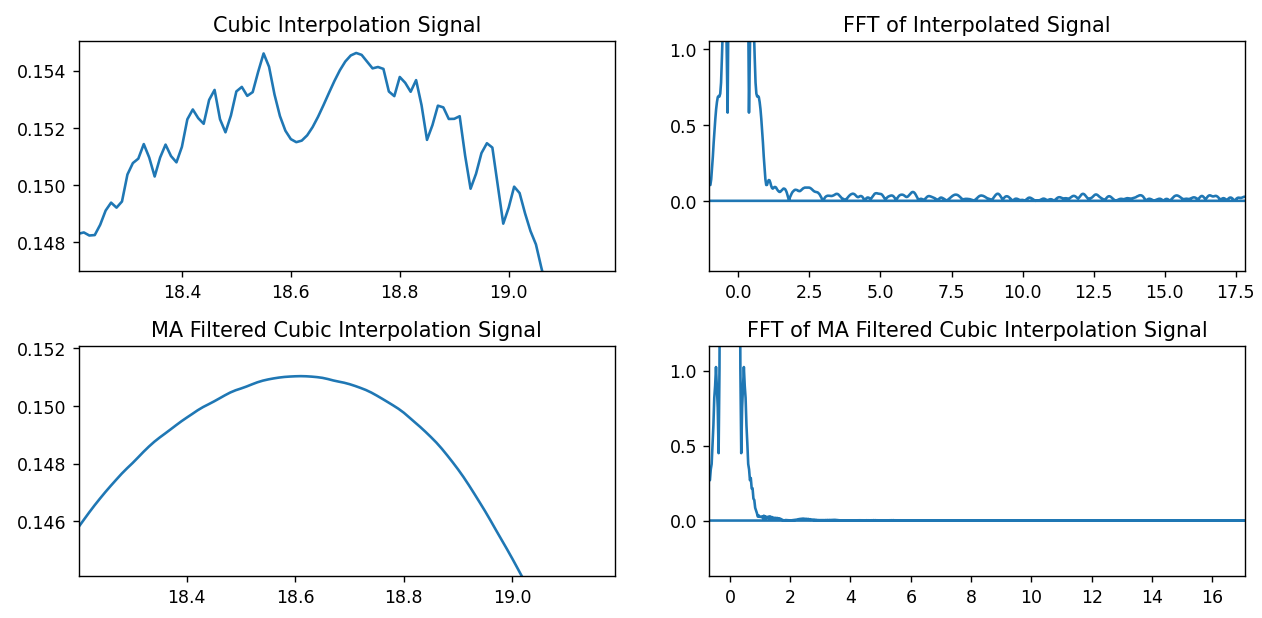
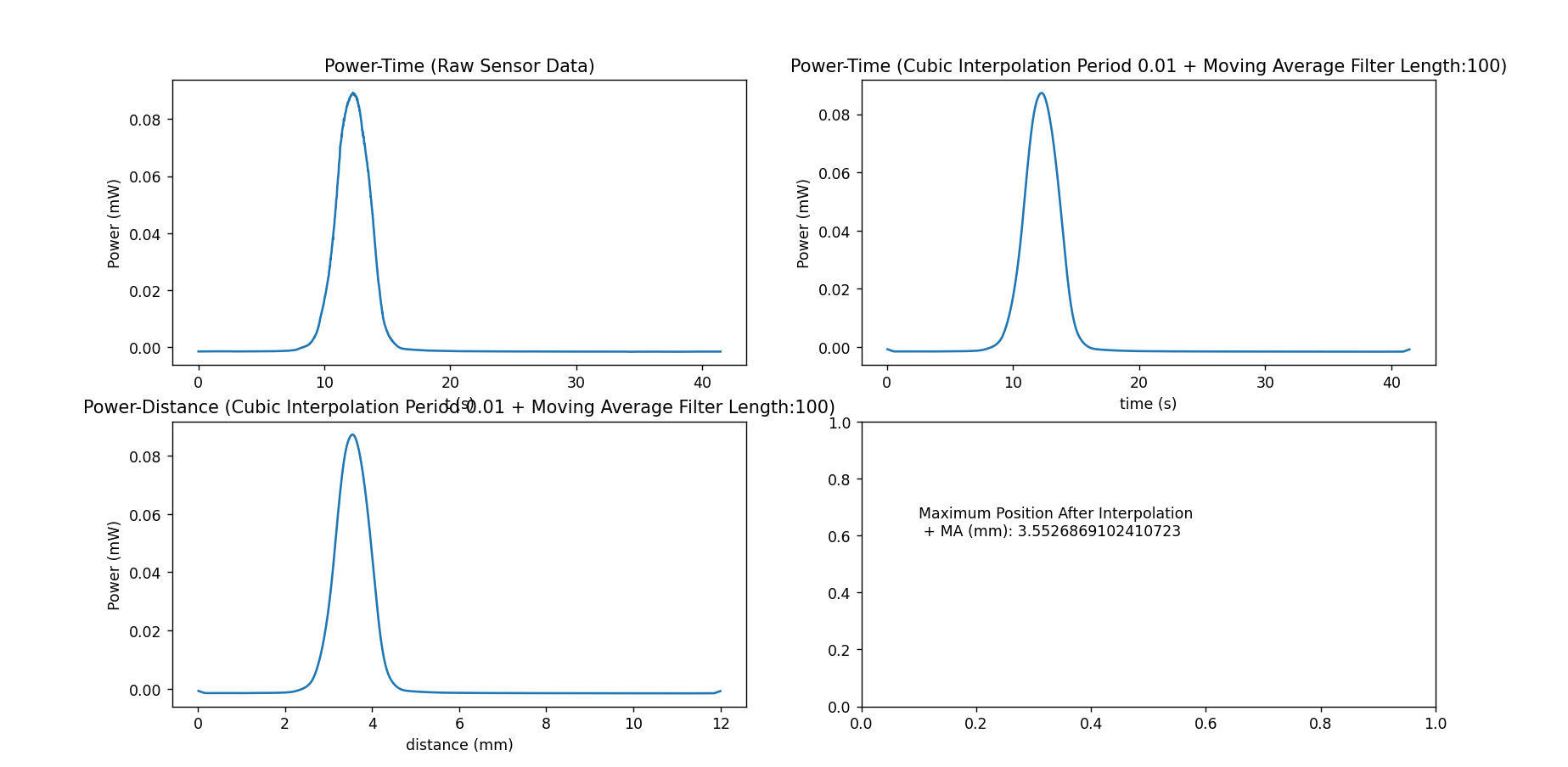
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Figure 3: Power(mW) - Position(mm) graphs for raw signal and cubic interpolation signal

Interpolation solves the nonuniform sampling problem. Obtained signal still has noise in it which might alter the maximum position. To remove the noise a low pass filter in the form of a moving average filter is applied. The figure below depicts the signal before and after the low pass filter. High-frequency noise components are removed from the signal while keeping the original signal mostly intact. This operation produced a smoother signal which is more suitable for peak finding operation.

****The recorded power signal is plotted with respect to distance in order to find the position of the laser’s center. This method gives the position of the laser relative to the initial position of the servo motor. The figure below shows the power levels with respect to time and position. Position with maximum power is written in the fourth graph in millimeters.



Measurements are performed with different parameters such as the distance between the mirror and target plane, different sweep locations.

**Horizontal Measurements D=410mm (actual distance 425mm)**

Table 1: Distance between mirror and target plane: 410mm. Mirror input y coordinate: -5mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input x (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | -5 | 2.322 | -3.096 |
| -2.5 | -5 | 2.853 | -2.565 |
| -2 | -5 | 3.372 | -2.046 |
| -1.5 | -5 | 3.868 | -1.550 |
| -1 | -5 | 4.392 | -1.026 |
| -0.5 | -5 | 4.905 | -0.513 |
| 0 | -5 | 5.418 | 0 |
| 0.5 | -5 | 5.921 | 0.503 |
| 1 | -5 | 6.446 | 1.028 |
| 1.5 | -5 | 6.923 | 1.505 |
| 2 | -5 | 7.427 | 2.009 |
| 2.5 | -5 | 7.937 | 2.519 |
| 3 | -5 | 8.452 | 3.034 |

Table 2: Distance between mirror and target plane: 410mm. Mirror input y coordinate: 0mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | 0 | 2.278 | -3.089 |
| -2.5 | 0 | 2.792 | -2.575 |
| -2 | 0 | 3.311 | -2.056 |
| -1.5 | 0 | 3.829 | -1.538 |
| -1 | 0 | 4.327 | -1.040 |
| -0.5 | 0 | 4.860 | -0.507 |
| 0 | 0 | 5.367 | 0 |
| 0.5 | 0 | 5.878 | 0.511 |
| 1 | 0 | 6.379 | 1.012 |
| 1.5 | 0 | 6.880 | 1.513 |
| 2 | 0 | 7.392 | 2.025 |
| 2.5 | 0 | 7.880 | 2.513 |
| 3 | 0 | 8.390 | 3.023 |

Table 3: Distance between mirror and target plane: 410mm. Mirror input y coordinate: 5mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | 5 | 2.391 | -3.097 |
| -2.5 | 5 | 2.909 | -2.579 |
| -2 | 5 | 3.424 | -2.064 |
| -1.5 | 5 | 3.949 | -1.539 |
| -1 | 5 | 4.468 | -1.020 |
| -0.5 | 5 | 4.974 | -0.514 |
| 0 | 5 | 5.488 | 0 |
| 0.5 | 5 | 6.013 | 0.525 |
| 1 | 5 | 6.534 | 1.046 |
| 1.5 | 5 | 7.042 | 1.554 |
| 2 | 5 | 7.547 | 2.059 |
| 2.5 | 5 | 8.052 | 2.564 |
| 3 | 5 | 8.561 | 3.073 |

**Horizontal Measurements D=425mm (actual distance 425mm)**

Table 4: Distance between mirror and target plane: 425mm. Mirror input y coordinate: -5mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input x (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | -5 | 6.723 | -2.999 |
| -2.5 | -5 | 7.225 | -2.497 |
| -2 | -5 | 7.727 | -1.995 |
| -1.5 | -5 | 8.218 | -1.504 |
| -1 | -5 | 8.716 | -1.007 |
| -0.5 | -5 | 9.228 | -0.494 |
| 0 | -5 | 9.722 | 0.0 |
| 0.5 | -5 | 10.212 | 0.49 |
| 1 | -5 | 10.695 | 0.973 |
| 1.5 | -5 | 11.191 | 1.469 |
| 2 | -5 | 11.673 | 1.951 |
| 2.5 | -5 | 12.151 | 2.429 |
| 3 | -5 | 12.653 | 2.931 |

Table 5: Distance between mirror and target plane: 425mm. Mirror input y coordinate: 0mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | 0 | 6.788 | -2.956 |
| -2.5 | 0 | 7.275 | -2.469 |
| -2 | 0 | 7.777 | -1.967 |
| -1.5 | 0 | 8.289 | -1.455 |
| -1 | 0 | 8.758 | -0.986 |
| -0.5 | 0 | 9.248 | -0.496 |
| 0 | 0 | 9.744 | 0.0 |
| 0.5 | 0 | 10.225 | 0.481 |
| 1 | 0 | 10.711 | 0.967 |
| 1.5 | 0 | 11.207 | 1.463 |
| 2 | 0 | 11.666 | 1.922 |
| 2.5 | 0 | 12.156 | 2.412 |
| 3 | 0 | 12.654 | 2.91 |

Table 6: Distance between mirror and target plane: 425mm. Mirror input y coordinate: 5mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | 5 | 6.849 | -2.978 |
| -2.5 | 5 | 7.364 | -2.463 |
| -2 | 5 | 7.867 | -1.959 |
| -1.5 | 5 | 8.359 | -1.468 |
| -1 | 5 | 8.843 | -0.984 |
| -0.5 | 5 | 9.34 | -0.487 |
| 0 | 5 | 9.826 | 0.0 |
| 0.5 | 5 | 10.308 | 0.481 |
| 1 | 5 | 10.824 | 0.998 |
| 1.5 | 5 | 11.304 | 1.478 |
| 2 | 5 | 11.793 | 1.967 |
| 2.5 | 5 | 12.279 | 2.453 |
| 3 | 5 | 12.781 | 2.954 |

**Horizontal Measurements D=210mm (actual distance 225mm)**

Table 7: Distance between mirror and target plane: 210mm. Mirror input y coordinate: -5mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | -5 | 3.387 | -3.230 |
| -2.5 | -5 | 3.913 | -2.704 |
| -2 | -5 | 4.454 | -2.163 |
| -1.5 | -5 | 5.000 | -1.617 |
| -1 | -5 | 5.533 | -1.084 |
| -0.5 | -5 | 6.082 | -0.535 |
| 0 | -5 | 6.617 | 0 |
| 0.5 | -5 | 7.170 | 0.553 |
| 1 | -5 | 7.700 | 1.083 |
| 1.5 | -5 | 8.233 | 1.616 |
| 2 | -5 | 8.765 | 2.148 |
| 2.5 | -5 | 9.296 | 2.679 |
| 3 | -5 | 9.843 | 3.226 |

Table 8: Distance between mirror and target plane: 210mm. Mirror input y coordinate: 0mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | 0 | 3.359 | -3.204 |
| -2.5 | 0 | 3.888 | -2.675 |
| -2 | 0 | 4.416 | -2.147 |
| -1.5 | 0 | 4.954 | -1.609 |
| -1 | 0 | 5.490 | -1.073 |
| -0.5 | 0 | 6.026 | -0.537 |
| 0 | 0 | 6.563 | 0 |
| 0.5 | 0 | 7.104 | 0.541 |
| 1 | 0 | 7.647 | 1.084 |
| 1.5 | 0 | 8.179 | 1.616 |
| 2 | 0 | 8.692 | 2.129 |
| 2.5 | 0 | 9.221 | 2.658 |
| 3 | 0 | 9.754 | 3.191 |

Table 9: Distance between mirror and target plane: 210mm. Mirror input y coordinate: 5mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | 5 | 3.553 | -3.189 |
| -2.5 | 5 | 4.077 | -2.665 |
| -2 | 5 | 4.604 | -2.138 |
| -1.5 | 5 | 5.158 | -1.584 |
| -1 | 5 | 5.679 | -1.063 |
| -0.5 | 5 | 6.212 | -0.529 |
| 0 | 5 | 6.742 | 0.0 |
| 0.5 | 5 | 7.27 | 0.528 |
| 1 | 5 | 7.807 | 1.065 |
| 1.5 | 5 | 8.343 | 1.601 |
| 2 | 5 | 8.873 | 2.131 |
| 2.5 | 5 | 9.404 | 2.662 |
| 3 | 5 | 9.934 | 3.192 |

**Horizontal Measurements D=225mm (actual distance 225mm)**

Table 10: Distance between mirror and target plane: 225mm. Mirror input y coordinate: -5mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | -5 | 4.985 | -3.044 |
| -2.5 | -5 | 5.503 | -2.526 |
| -2 | -5 | 6.007 | -2.023 |
| -1.5 | -5 | 6.505 | -1.524 |
| -1 | -5 | 7.034 | -0.995 |
| -0.5 | -5 | 7.536 | -0.494 |
| 0 | -5 | 8.029 | 0.0 |
| 0.5 | -5 | 8.539 | 0.509 |
| 1 | -5 | 9.045 | 1.016 |
| 1.5 | -5 | 9.546 | 1.517 |
| 2 | -5 | 10.045 | 2.015 |
| 2.5 | -5 | 10.529 | 2.499 |
| 3 | -5 | 11.009 | 2.98 |

Table 11: Distance between mirror and target plane: 225mm. Mirror input y coordinate: 0mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | 0 | 4.989 | -2.992 |
| -2.5 | 0 | 5.493 | -2.488 |
| -2 | 0 | 5.999 | -1.982 |
| -1.5 | 0 | 6.49 | -1.491 |
| -1 | 0 | 6.998 | -0.984 |
| -0.5 | 0 | 7.459 | -0.523 |
| 0 | 0 | 7.981 | 0.0 |
| 0.5 | 0 | 8.461 | 0.479 |
| 1 | 0 | 8.952 | 0.971 |
| 1.5 | 0 | 9.485 | 1.503 |
| 2 | 0 | 9.976 | 1.994 |
| 2.5 | 0 | 10.464 | 2.483 |
| 3 | 0 | 10.956 | 2.975 |

Table 12: Distance between mirror and target plane: 225mm. Mirror input y coordinate: 5mm

|  |  |  |  |
| --- | --- | --- | --- |
| **Mirror Input (mm)** | **Mirror Input y (mm)** | **Peak Power Position (mm)** | **Relative Distance (mm)** |
| -3 | 5 | 5.156 | -2.982 |
| -2.5 | 5 | 5.664 | -2.475 |
| -2 | 5 | 6.153 | -1.986 |
| -1.5 | 5 | 6.649 | -1.49 |
| -1 | 5 | 7.154 | -0.985 |
| -0.5 | 5 | 7.64 | -0.498 |
| 0 | 5 | 8.138 | 0.0 |
| 0.5 | 5 | 8.634 | 0.496 |
| 1 | 5 | 9.126 | 0.988 |
| 1.5 | 5 | 9.635 | 1.497 |
| 2 | 5 | 10.126 | 1.988 |
| 2.5 | 5 | 10.609 | 2.47 |
| 3 | 5 | 11.103 | 2.965 |

**Mapping Camera Coordinates to Laser Coordinates**

The coordinates that are used in the laser and the coordinates that are recorded by the camera are rotated and translated versions of each other. In the figure below, the laser is pointed at 30 different positions in a plane 500 cm away from the mirror center. Each point is extracted from the color images and their 3D coordinates are extracted and plotted. Laser points are named and camera points are named . Both matrices have the shape 3xN. To find the mapping between these points, the optimal rotation matrix and optimal translation vector should be found. The mapping is modeled as:

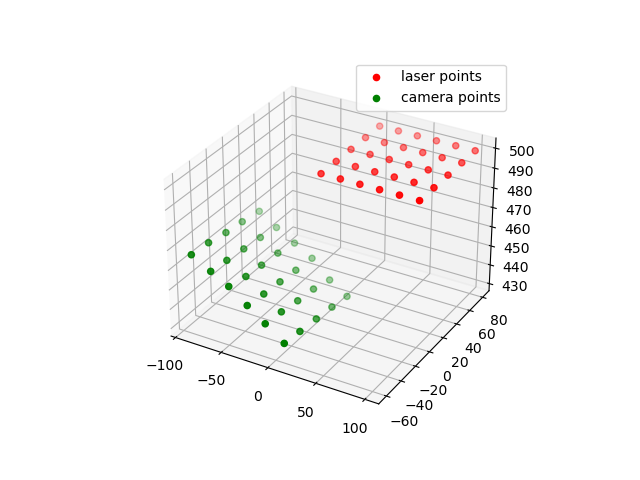


Figure 4: Recording of the same points in laser and camera coordinate systems

Optimal and t are found by the following algorithm [2] [3] [4]:

To find the rotation matrix R

To find the translation vector t

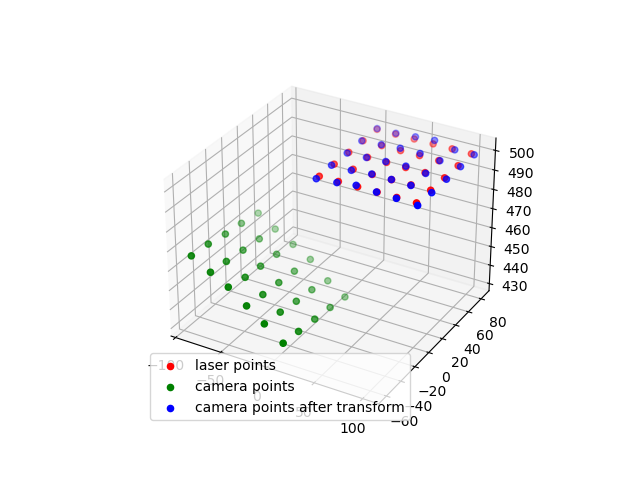


Figure 5: Camera points after the transformation

**Calibration Procedure**

Calibration of the system is done in order to coordinate depth camera and mirror together. Both mirror and depth camera have their own coordinate systems. The relation between these coordinate systems is unknown which makes them unable to perform together. The main objective of the calibration is to find the rotation and translation of the coordinate systems relative to each other.

**1) Visible Laser**

**-steps**

**2) IR Laser**

**-steps**

**Detection Algorithms:**

Position estimation is the first step to tracking the position of the body. One of the most common approaches to this problem is usage of visual markers. A visual marker can be distinguished from the environment without too much difficulty due to its characteristics. For the project 3 different algorithms are tested: ArUco, Hough Circle Detector, and WHYCon. Detection algorithms are evaluated based on their processing times in order to choose the best option for the system.

**ArUco**

ArUco is a popular binary square fiducial marker used for absolute pose estimation. 

Figure 6: ArUco marker with ids 0 upto 3 [5]

**Hough Circle Detector**

**WHYCon**

|  |  |
| --- | --- |
| **Algorithm** | **Runtime (ms)** |
| ARuCO | 4-30 |
| WHYCon | 2-3 |
| Hough Circle Detector | 8-10 |

|  |  |
| --- | --- |
| **Parts of Code** | **Elapsed Time (ms)** |
| Capturing Color Image | 3 |
| Marker Detection | 2 |
| Transforming Depth Image to Color Camera´s Coordinate System | 15 |
| 3D to mirror coordinate conversion | 1 |
| Mirror Adjustment | 2 |
| **Total time** | **23** |

**Maximum distance of detection with different marker sizes:**

**-To do**

**Overall system schema**

**-Todo**

# References

|  |  |
| --- | --- |
| [1] | Optotune, "MR-E-2 Development Kit Operation Manual," Dietikon, 2019. |
| [2] | C. Simon, "simonensemble," [Online]. Available: https://simonensemble.github.io/posts/2018-10-27-orthogonal-procrustes/. |
| [3] | N. Ho, "FINDING OPTIMAL ROTATION AND TRANSLATION BETWEEN CORRESPONDING 3D POINTS," [Online]. Available: https://nghiaho.com/?page\_id=671. |
| [4] | "Kabsch algorithm," [Online]. Available: https://en.wikipedia.org/wiki/Kabsch\_algorithm. |