Fourth Grade CCSS Math Vocabulary Word List

***Terms with an asterisk are meant for teacher knowledge only—students need to learn the concept but not necessarily the term.**

Acute angle The measure of an angle with a measure between 0° and 90°

Add To combine; put together two or more quantities

Addend Any number being added

***Additive comparison** a situation that compares by asking or telling how much more (how much less) one amount is than another.

Algorithm set of steps used to solve a mathematical computation

Angle is formed by two rays with a common endpoint (called the vertex).

Angle measure The size of an angle is measured in degrees

Arc a curved line that is a part of a circle

Area The number of square units that covers a shape or figure

Area model a pictorial way of representing multiplication. In the area model, the length and width of a rectangle represent factors, and the area of the rectangle represents their product.

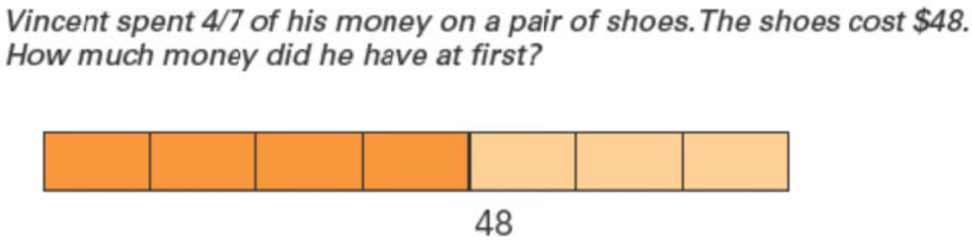
Array an orderly arrangement in rows and columns used in multiplication and division to show how multiplication can be shown as repeated addition and division can be shown as fair shares.

***Associative Property of Addition** When three or more numbers are added, the sum is the same regardless of the grouping of the addends. For example (2 + 3) + 4 = 2 + (3 + 4)

***Associative Property of Multiplication** When three or more numbers are multiplied, the product is the same regardless of the grouping of the factors. For example (2 x 3) x 4 = 2 x (3 x 4)

Attribute A characteristic of an object such as color, shape, size, etc

Bar Model a visual model used to solve word problems in the place of guess and check. Example:



Benchmark fractions common fractions that you can judge other numbers against

Capacity the amount of liquid a container can hold

Centimeter A measure of length. There are 100 centimeters in a meter

Classify to sort shapes according to the definitions of various terms

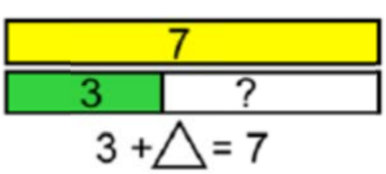
Common denominator A common multiple of the denominators of two or more fractions

***Commutative Property of Addition** When two numbers are added, the sum is the same regardless of the order of the addends. For example 4 + 2 = 2 + 4

***Commutative Property of Multiplication** When two numbers are multiplied, the product is the same regardless of the order of the factors. For example 4 x 2 = 2 x 4

Compare To decide if one number is greater than, less than, or equal to another number . Can also be used to tell how shapes are alike or different.

Comparison bars
Used to represent larger and smaller amounts in a comparison situation. Can be used to represent all four operations. Different lengths of bars are drawn to represent each number.



Compose To put together basic elements. (e.g., Numbers or geometric shapes.)

Composite number numbers which are divisible by another number other than 1 and the number.

Congruent Figures or angles that have the same size and shape

Cup a customary unit of measurement for volume equal to 8 fluid ounces

Customary system the United States standard system of measurement

Data A collection of information

Decimal the expression of a fraction in the base of ten, using a decimal point to separate whole numbers from the fractional value

Decimal fraction a fraction in which the denominator is a power of ten

Decimal notation a number containing a decimal point

Decimal point a printed or written dot in a decimal number that divides the whole numbers from the tenths, hundredths, and smaller divisions of ten

Decompose To separate into basic elements. (e.g., Numbers or geometric shapes.)

Degree (angle measure) the basic unit for measuring the size of an angle.

Denominator The bottom part of a fraction.

Digit Any of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.

Difference The result when one number is subtracted from another

***Distributive Property** multiply a sum by multiplying each addend separately and then add the products. Example:
4 x 53
(4 x 50) + (4 x 3)
200 + 12
212

Divide split into equal parts or groups

Dividend The number that is divided by another number in a division operation

Divisor The quantity by which another quantity is to be divided

Endpoint a point at which a line segment or a ray ends

Equal Having the same amount. (e.g., 4 equals 3 + 1 means that 4 is the same amount as 3 + 1.)

Equation A number sentence *with an equal sign*. The amount on one side of the equal sign has the same value as the amount on the other side.

Equivalent fractions different fractions that name the same number or amount

Estimate A close guess of the actual value, usually with some thought or calculation involved

Evaluate To substitute number values into an expression

Expanded form a way to write a number that shows the sum of values of each digit of a number. Example: the expanded form of the number 543 would be 500 + 40 + 3.

Expression A mathematical phrase *without an equal sign*.

Factor One of two or more expressions that are multiplied together to get a product

Factor pairs A set of two whole numbers when multiplied that will result in a given product. For example, the factor pairs for 6 are (2,3) and (1,6)

***Fluency** efficient, flexible and accurate methods for computing

Foot 12 inches

Formula a standard procedure for solving a class of mathematical problems

Fraction two quantities written one above the other, that shows how much of a whole is shown

Friendly or Nice numbers numbers that end in 0 or 5 and help with mental math

Function table displays the relationship between the inputs and outputs of a specified function.

Gallon A unit of volume in the U.S. Customary System, used in liquid measure, equal to 4 quarts

Gram A metric unit of mass (weight). 1,000 grams = 1 kilogram

>Greater than is used to compare two numbers when the first number is larger than the second number

Hour a period of 30 minutes

Hundredth One out of one hundred equal parts; the position of the second digit to the right of the decimal point

***Identity Property of Addition** The sum of any number and 0 is that number.

***Identity Property of Multiplication** The product of 1 and any number is that number

Improper fraction a fraction in which the number in the numerator is greater than or equal to the number in the denominator.

Inch a measure of length. There are 12 inches in a foot

Intersecting lines Where lines cross over and have one common point

Inverse operations Two operations that have the opposite effect, such as addition and subtraction.

Kilogram a unit of mass in the metric system. 1,000 grams = one kilogram

Kilometer a unit of length in the metric system. 1,000 meters = 1 kilometer

<Less than is used to compare two numbers when the first number is

smaller than the second number

Like denominators denominators in two or more fractions that are the same

Line In geometry a line is straight (no curves); has no thickness, and extends in both directions without end

Line of symmetry a line that divides a figure into two congruent parts, each of which is the mirror image of the other

Line plot shows data on a number line with x or other marks to show frequency

Line segment Two points on a line, and all the points between those two points

Line symmetric figures a figure whose that can be folded in half so that the two parts match exactly

Liter the basic unit of volume or capacity in the metric system

Lowest terms a fraction expressed in the fewest number of pieces possible (also simplest form)

Mass the quantity of matter in an object

Meter The basic unit of length (or distance) in the Metric System. The abbreviation is m

Metric system A system of measuring based on the meter for length

Mile a customary unit used for measuring length or distance

Milliliter a metric unit used to measure volume or capacity; 1,000 ml = 1 liter

Millimeter a metric unit used to measure length; 1,000 mm = 1 meter

Minute A period of 60 seconds

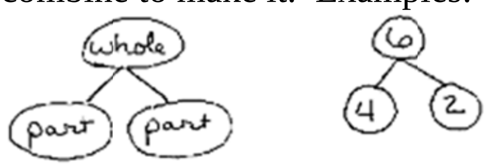
Mixed number A number that is the sum of a whole number and a proper fraction

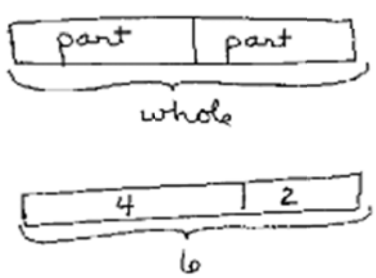
Multiple the product of that number and any other whole number. Zero is a multiple of every number

***Multiplicative comparison** a situation that compares by asking or telling how many times more (how many times less) one amount is than another.

Multiply to find the product of by multiplication

Number bond a picture of the relationship between a number and the parts that combine to make it. Examples:





Number line A line with numbers placed in their correct position

Numerator The top part of a fraction

Obtuse angle An angle between (but not including) 90 deg and 180 deg.

Open Number Line A number line with no numbers or tick marks

***Order of Operations** is a rule used to clarify which procedures should be performed first in a given mathematical expression.

Ounce a customary unit of weight; 16 oz. = 1 pound

Parallel lines distinct lines lying in the same plane and they never intersect each other

Parentheses the symbols (and) used in grouping

Pattern a set of numbers or objects in which all the members are related with each other by a specific rule

Perimeter The sum of the lengths of the sides of a polygon.

Period groups of three digits in large numbers that help determine place value

Perpendicular lines Two intersecting lines have four right angles formed at the intersection points

Pint A unit of volume or capacity in the U.S. Customary System, used in liquid measure, equal to 16 ounces

Place value The value of where the digit is in the number

Plane figure a 2-dimensional shape

Point A location in a plane or in space, having no dimensions

Pound a customary unit of weight equal to 16 ounces

Prime number a positive whole number with exactly two factors, which are one and itself

Product The result of two numbers being multiplied together

Protractor a tool used to measure the angles

Quart customary unit for measuring capacity or volume equal to two pints

Ray a part of a line that begins at a particular point (called the endpoint) and extends endlessly in one direction

Reasonableness an answer based on good number sense

Related facts addition and/or subtraction number sentences that are alike in some way

Remainder the amount left over after division when one divisor does not divide the dividend exactly

Right angle one whose measure is exactly 90 degrees

Right triangle a triangle with one right angle

Round a whole number A method of approximating a number to its nearest place value

Second an interval of time that is one sixtieth of a minute

Sequence an ordered list of numbers that has a constant difference between every two consecutive numbers

Simplest form when a fraction is expressed with the fewest number of pieces possible (also lowest term)

Simplify to express a fraction in simplest form

Square unit a unit of measurement that determines the area of a plane figure

Standard form the numerical version of a number where each number has a place value

Subtract Take away; remove; compare

Sum The answer to an addition problem

Tenth One out of one ten equal parts; the position of the first digit to the right of the decimal point

Time interval Duration of a segment of time

Two-dimensional Lying in a plane; flat

Unit fraction a fraction with a numerator of one

Unlike denominators two or more fractions that do not have the same denominator

Variable a value that may change within the scope of a given problem or set of operations

Vertex A corner of a figure. (plural - vertices)

Volume (liquid) a measurement of capacity

Whole numbers The set of numbers that includes zero and all of the natural numbers

Word form A way to write the number using words. Example: The word form of the number 9,325 is nine thousand, three hundred twenty-five.

Yard a customary unit of length equal to three feet

***Zero Property of Multiplication** The product of zero and any number is zero

100 KEY MATH CONCEPTS FOR THE ACT

NUMBER PROPERTIES

1. UNDEFINED

   On the ACT, *undefined* almost always means **division by zero.** The expression $\frac{a}{bc}$ is undefined if either b or c equals 0.

2. REAL/IMAGINARY

   A real number is a number that has a **location on the number line.** On the ACT, imaginary numbers are numbers that involve the square root of a negative number. $\sqrt{-4}$ is an imaginary number.

3. INTEGER/NONINTEGER

   Integers are **whole numbers;** they include negative whole numbers and zero.

4. RATIONAL/IRRATIONAL

   A **rational number** is a number that can be expressed as a **ratio of two integers. Irrational numbers** are real numbers—they have locations on the number line—they just **can't be expressed precisely as a fraction or decimal.** For the purposes of the ACT, the most important **irrational numbers** are $\sqrt{2}$, $\sqrt{3}$, and π.

5. ADDING/SUBTRACTING SIGNED NUMBERS

   To **add a positive and a negative,** first ignore the signs and find the positive difference between the number parts. Then attach the sign of the original number to the larger number part. For example, to add 23 and −34, first we ignore the minus sign and find the positive difference between 23 and 34—that's 11. Then we attach the sign of the number with the larger number part—in this case it's the minus sign from the −34. So, $23 + (-34) = -11$.

   Make **subtraction** situations simpler by turning them into addition. For example, think of $-17 - (-21)$ as $-17 + (+21)$.

   To **add or subtract a string of positives and negatives,** first turn everything into addition. Then combine the positives and negatives so that the string is reduced to the sum of a single positive number and a single negative number.

6. MULTIPLYING/DIVIDING SIGNED NUMBERS

   To multiply and/or divide positives and negatives, treat the number parts as usual and **attach a negative sign if there were originally**

an odd number of negatives. To multiply −2, −3, and −5, first multiply the number parts: 2 × 3 × 5 = 30. Then go back and note that there were three—an odd number—negatives, so the product is negative: (−2) × (−3) × (−5) = −30.

7. PEMDAS

When performing multiple operations, remember PEMDAS, which means **Parentheses** first, then **Exponents,** then **Multiplication** and **Division** (left to right), then **Addition** and **Subtraction** (left to right).

In the expression $9 - 2 \times (5 - 3)^2 + 6 \div 3$, begin with the parentheses: (5 − 3) = 2. Then do the exponent: $2^2 = 4$. Now the expression is: 9 − 2 × 4 + 6 ÷ 3. Next do the multiplication and division to get 9 − 8 + 2, which equals 3.

8. ABSOLUTE VALUE

Treat absolute value signs a lot like **parentheses.** Do what's inside them first and then take the absolute value of the result. Don't take the absolute value of each piece between the bars before calculating. In order to calculate $|(-12) + 5 - (-4)| - |5 + (-10)|$, first do what's inside the bars to get: $|-3| - |-5|$, which is 3 − 5, or −2.

9. COUNTING CONSECUTIVE INTEGERS

To count consecutive integers, **subtract the smallest from the largest and add 1.** To count the integers from 13 through 31, subtract: 31 − 13 = 18. Then add 1: 18 + 1 = 19.

DIVISIBILITY

10. FACTOR/MULTIPLE

The **factors** of integer *n* are the positive integers that divide into *n* with no remainder. The **multiples** of *n* are the integers that *n* divides into with no remainder. 6 is a factor of 12, and 24 is a multiple of 12. 12 is both a factor and a multiple of itself.

11. PRIME FACTORIZATION

A **prime number** is a positive integer that has exactly two positive integer factors: 1 and the integer itself. The first eight prime numbers are 2, 3, 5, 7, 11, 13, 17, and 19.

To find the prime factorization of an integer, just keep breaking it up into factors until **all the factors are prime.** To find the prime factorization of 36, for example, you could begin by breaking it into 4 × 9:

$$36 = 4 \times 9 = 2 \times 2 \times 3 \times 3$$

12. RELATIVE PRIMES

To determine whether two integers are relative primes, break them both down to their prime factorizations. For example: 35 = 5 × 7, and 54 = 2 × 3 × 3 × 3. They have **no prime factors in common,** so 35 and 54 are relative primes.

13. COMMON MULTIPLE

You can always get a common multiple of two numbers by **multiplying** them, but, unless the two numbers are relative primes, the product will not be the least common multiple. For example, to find a common multiple for 12 and 15, you could just multiply: 12 × 15 = 180.

14. LEAST COMMON MULTIPLE (LCM)

To find the least common multiple, check out the **multiples of the larger number** until you find one that's **also a multiple of the smaller.** To find the LCM of 12 and 15, begin by taking the multiples of 15: 15 is not divisible by 12; 30's not; nor is 45. But the next multiple of 15, 60, is divisible by 12, so it's the LCM.

15. GREATEST COMMON FACTOR (GCF)

To find the greatest common factor, break down both numbers into their prime factorizations and take **all the prime factors they have in common.** $36 = 2 \times 2 \times 3 \times 3$, and $48 = 2 \times 2 \times 2 \times 2 \times 3$. What they have in common is two 2s and one 3, so the GCF is $= 2 \times 2 \times 3 = 12$.

16. EVEN/ODD

To predict whether a sum, difference, or product will be even or odd, just **take simple numbers like 1 and 2 and see what happens.** There are rules—"odd times even is even," for example—but there's no need to memorize them. What happens with one set of numbers generally happens with all similar sets.

17. MULTIPLES OF 2 AND 4

An integer is divisible by 2 if the **last digit is even.** An integer is divisible by 4 if the **last two digits form a multiple of 4.** The last digit of 562 is 2, which is even, so 562 is a multiple of 2. The last two digits make 62, which is not divisible by 4, so 562 is not a multiple of 4.

18. MULTIPLES OF 3 AND 9

An integer is divisible by 3 if the **sum of its digits is divisible by 3.** An integer is divisible by 9 if the **sum of its digits is divisible by 9.** The sum of the digits in 957 is 21, which is divisible by 3 but not by 9, so 957 is divisible by 3 but not 9.

19. MULTIPLES OF 5 AND 10

An integer is divisible by 5 if the **last digit is 5 or 0.** An integer is divisible by 10 if the **last digit is 0.** The last digit of 665 is 5, so 665 is a multiple of 5 but not a multiple of 10.

20. REMAINDERS

The remainder is the whole number left over after division. 487 is 2 more than 485, which is a multiple of 5, so when 487 is divided by 5, the remainder will be 2.

FRACTIONS AND DECIMALS

21. REDUCING FRACTIONS

To reduce a fraction to lowest terms, **factor out and cancel** all factors the numerator and denominator have in common.

$$\frac{28}{36} = \frac{4 \times 7}{4 \times 9} = \frac{7}{9}$$

22. ADDING/SUBTRACTING FRACTIONS

To add or subtract fractions, first find a **common denominator,** and then add or subtract the numerators.

$$\frac{2}{15} + \frac{3}{10} = \frac{4}{30} + \frac{9}{30} = \frac{4+9}{30} = \frac{13}{30}$$

23. MULTIPLYING FRACTIONS

To multiply fractions, **multiply** the numerators and **multiply** the denominators.

$$\frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$$

24. DIVIDING FRACTIONS

To divide fractions, **invert** the second one and **multiply.**

$$\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$$

25. CONVERTING A MIXED NUMBER TO AN IMPROPER FRACTION

To convert a mixed number to an improper fraction, **multiply** the whole number part by the denominator, then **add** the numerator. The result is the new numerator (over the same denominator). To convert $7\frac{1}{3}$, first multiply 7 by 3, then add 1, to get the new numerator of 22. Put that over the same denominator, 3, to get $\frac{22}{3}$.

26. CONVERTING AN IMPROPER FRACTION TO A MIXED NUMBER

To convert an improper fraction to a mixed number, **divide** the denominator into the numerator to get a **whole number quotient with a remainder.** The quotient becomes the whole number part of the mixed number, and the remainder becomes the new numerator—with the same denominator. For example, to convert $\frac{108}{5}$ first divide 5 into 108, which yields 21 with a remainder of 3. Therefore, $\frac{108}{5} = 21\frac{3}{5}$.

27. RECIPROCAL

To find the reciprocal of a fraction, switch the numerator and the denominator. The reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$. The reciprocal of 5 is $\frac{1}{5}$. The product of reciprocals is 1.

28. COMPARING FRACTIONS

One way to compare fractions is to re-express them with a **common denominator.**

$\frac{3}{4} = \frac{21}{28}$ and $\frac{5}{7} = \frac{20}{28}$. $\frac{21}{28}$ is greater than $\frac{20}{28}$, so $\frac{3}{4}$ is greater than $\frac{5}{7}$.

Another way to compare fractions is to convert them both to **decimals.** $\frac{3}{4}$ converts to .75, and $\frac{5}{7}$ converts to approximately .714.

29. CONVERTING FRACTIONS TO DECIMALS

To convert a fraction to a decimal, **divide the bottom into the top.** To convert $\frac{5}{8}$, divide 8 into 5, yielding .625.

30. REPEATING DECIMAL

To find a particular digit in a repeating decimal, note the **number of digits in the cluster that repeats.** If there are 2 digits in that cluster, then every 2nd digit is the same. If there are 3 digits in that cluster, then every 3rd digit is the same. And so on. For example, the decimal equivalent of $\frac{1}{27}$ is .037037037. . ., which is best written $.\overline{037}$.

There are 3 digits in the repeating cluster, so every 3rd digit is the same: 7. To find the 50th digit, look for the multiple of 3 just less than 50—that's 48. The 48th digit is 7, and with the 49th digit the pattern repeats with 0. The 50th digit is 3.

31. IDENTIFYING THE PARTS AND THE WHOLE

The key to solving most fractions and percents story problems is to identify the part and the whole. Usually you'll find the **part** associated with the verb ***is/are*** and the **whole** associated with the word ***of.*** In the sentence, "Half of the boys are blonds," the whole is the boys ("*of* the boys), and the part is the blonds ("*are* blonds").

PERCENTS

32. PERCENT FORMULA

Whether you need to find the part, the whole, or the percent, use the same formula:

Part = Percent × Whole

Example: What is 12% of 25?
Setup: Part = .12 × 25

Example: 15 is 3% of what number?
Setup: 15 = .03 × Whole

Example: 45 is what percent of 9?
Setup: 45 = Percent × 9

33. PERCENT INCREASE AND DECREASE

To increase a number by a percent, **add the percent to 100%,** convert to a decimal, and multiply. To increase 40 by 25%, add 25% to 100%, convert 125% to 1.25, and multiply by 40. 1.25 × 40 = 50.

34. FINDING THE ORIGINAL WHOLE

To find the **original whole before a percent increase or decrease,** set up an equation. Think of a 15% increase over x as $1.15x$.

Example: After a 5% increase, the population was 59,346. What was the population *before* the increase?

Setup: $1.05x = 59{,}346$

35. COMBINED PERCENT INCREASE AND DECREASE

To determine the combined effect of multiple percents increase and/or decrease, **start with 100 and see what happens.**

Example: A price went up 10% one year, and the new price went up 20% the next year. What was the combined percent increase?

Setup: First year: 100 + (10% of 100) = 110. Second year: 110 + (20% of 110) = 132. That's a combined 32% increase.

RATIOS, PROPORTIONS, AND RATES

36. SETTING UP A RATIO

To find a ratio, put the number associated with the word ***of* on top** and the quantity associated with the word ***to* on the** bottom and reduce. The ratio of 20 oranges to 12 apples is $\frac{20}{12}$ which reduces to $\frac{5}{3}$.

37. PART-TO-PART AND PART-TO-WHOLE RATIOS

If the parts add up to the whole, a part-to-part ratio can be turned into two part-to-whole ratios by putting **each number in the original ratio over the sum of the numbers.** If the ratio of males to females is 1 to 2, then the males-to-people ratio is $\frac{1}{1+2} = \frac{1}{3}$ and the females-to-people ratio is $\frac{2}{1+2} = \frac{2}{3}$. Or, $\frac{2}{3}$ of all the people are female.

38. SOLVING A PROPORTION

To solve a proportion, **cross multiply:**

$$\frac{x}{5} = \frac{3}{4}$$
$$4x = 5 \times 3$$
$$x = \frac{15}{4} = 3.75$$

39. RATE

To solve a rates problem, **use the units** to keep things straight.

Example: If snow is falling at the rate of 1 foot every 4 hours, how many inches of snow will fall in 7 hours?

Setup:

$$\frac{1 \text{ foot}}{4 \text{ hours}} = \frac{x \text{ inches}}{7 \text{ hours}}$$
$$\frac{12 \text{ inches}}{4 \text{ hours}} = \frac{x \text{ inches}}{7 \text{ hours}}$$
$$4x = 12 \times 7$$
$$x = 21$$

40. AVERAGE RATE

Average rate is *not* simply the average of the rates.

$$\text{Average } A \text{ per } B = \frac{\text{Total } A}{\text{Total } B}$$
$$\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time}}$$

To find the average speed for 120 miles at 40 mph and 120 miles at 60 mph, **don't just average the two speeds.** First figure out the total distance and the total time. The total distance is $120 + 120 = 240$ miles. The times are 3 hours for the first leg and 2 hours for the second leg, or 5 hours total. The average speed, then, is $\frac{240}{5} =$ 48 miles per hour.

AVERAGES

41. AVERAGE FORMULA

To find the average of a set of numbers, **add them up and divide by the number of numbers.**

$$\text{Average} = \frac{\text{Sum of the terms}}{\text{Number of terms}}$$

To find the average of the five numbers 12, 15, 23, 40, and 40, first add them: $12 + 15 + 23 + 40 + 40 = 130$. Then divide the sum by 5: $130 \div 5 = 26$.

42. AVERAGE OF EVENLY SPACED NUMBERS

To find the average of evenly spaced numbers, just **average the smallest and the largest.** The average of all the integers from 13 through 77 is the same as the average of 13 and 77.

$$\frac{13 + 77}{2} = \frac{90}{2} = 45$$

43. USING THE AVERAGE TO FIND THE SUM

$$\text{Sum} = (\text{Average}) \times (\text{Number of terms})$$

If the average of ten numbers is 50, then they add up to 10×50, or 500.

44. FINDING THE MISSING NUMBER

To find a missing number when you're given the average, **use the sum.** If the average of four numbers is 7, then the sum of those four numbers is 4×7, or 28. Suppose that three of the numbers are 3, 5, and 8. These numbers add up to 16 of that 28, which leaves 12 for the fourth number.

POSSIBILITIES AND PROBABILITY

45. COUNTING THE POSSIBILITIES

The fundamental counting principle: if there are ***m* ways** one event can happen and ***n* ways** a second event can happen, then there are ***m* × *n* ways** for the two events to happen. For example, with 5 shirts and 7 pairs of pants to choose from, you can put together $5 \times 7 = 35$ different outfits.

46. PROBABILITY

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total possible outcomes}}$$

If you have 12 shirts in a drawer and 9 of them are white, the probability of picking a white shirt at random is $\frac{9}{12} = \frac{3}{4}$. This probability can also be expressed as .75 or 75%.

POWERS AND ROOTS

47. MULTIPLYING AND DIVIDING POWERS

To multiply powers with the same base, **add the exponents:** $x^3 \times x^4 = x^{3+4} = x^7$. To divide powers with the same base, **subtract the exponents:** $y^{13} \div y^8 = y^{13-8} = y^5$.

48. RAISING POWERS TO POWERS

To raise a power to an exponent, **multiply the exponents.** $(x^3)^4 = x^{3 \times 4} = x^{12}$.

49. SIMPLIFYING SQUARE ROOTS

To simplify a square root, **factor out the perfect squares** under the radical, unsquare them and put the result in front. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$.

50. ADDING AND SUBTRACTING ROOTS

You can add or subtract radical expressions only if the part under the radicals is the same.

$$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$$

51. MULTIPLYING AND DIVIDING ROOTS

The product of square roots is equal to the square root of the product:

$\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$. The quotient of square roots is equal to the **square root of the quotient:**

$$\frac{\sqrt{6}}{\sqrt{3}} = \frac{\sqrt{6}}{3} = \sqrt{2}.$$

ALGEBRAIC EXPRESSIONS

52. EVALUATING AN EXPRESSION

To evaluate an algebraic expression, **plug in** the given values for the unknowns and calculate according to PEMDAS. To find the value of $x^2 + 5x - 6$ when $x = -2$, plug in -2 for x:

$$(-2)^2 + 5(-2) - 6 = 4 - 10 - 6 = -12.$$

53. ADDING AND SUBTRACTING MONOMIALS

To combine like terms, **keep the variable part unchanged while adding or subtracting the coefficients.** $2a + 3a = (2 + 3)a = 5a$

54. ADDING AND SUBTRACTING POLYNOMIALS

To add or subtract polynomials, **combine like terms.**

$$(3x^2 + 5x - 7) - (x^2 + 12) =$$
$$(3x^2 - x^2) + 5x + (-7 - 12) = 2x^2 + 5x - 19$$

55. MULTIPLYING MONOMIALS

To multiply monomials, **multiply the coefficients and the variables separately.**

$$2a \times 3a = (2 \times 3)(a \times a) = 6a^2$$

56. MULTIPLYING BINOMIALS—FOIL

To multiply binomials, use **FOIL**. To multiply $(x + 3)$ by $(x + 4)$, first multiply the **F**irst terms: $x \times x = x^2$. Next the **O**uter terms: $x \times 4 = 4x$. Then the **I**nner terms: $3 \times x = 3x$. And finally the **L**ast terms: $3 \times 4 = 12$. Then add and combine like terms: $x^2 + 4x + 3x + 12 = x^2 + 7x + 12$.

57. MULTIPLYING OTHER POLYNOMIALS

FOIL works only when you want to multiply two binomials. If you want to multiply polynomials with more than two terms, make sure you **multiply each term in the first polynomial by each term in the second.**

$$(x^2 + 3x + 4)(x + 5) =$$
$$x^2(x + 5) + 3x(x + 5) + 4(x + 5) =$$
$$x^3 + 5x^2 + 3x^2 + 15x + 4x + 20 =$$
$$x^3 + 8x^2 + 19x + 20$$

FACTORING ALGEBRAIC EXPRESSIONS

58. FACTORING OUT A COMMON DIVISOR

A factor common to all terms of a polynomial can be **factored out.** All three terms in the polynomial $3x^3 + 12x^2 - 6x$ contain a factor of $3x$. Pulling out the common factor yields $3x(x^2 + 4x - 2)$.

59. FACTORING THE DIFFERENCE OF SQUARES

One of the test maker's favorite factorables is the **difference of squares.**

$$a^2 - b^2 = (a - b)(a + b)$$

$x^2 - 9$, for example, factors to $(x - 3)(x + 3)$.

60. FACTORING THE SQUARE OF A BINOMIAL

Learn to recognize polynomials that are squares of binomials:

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

For example, $4x^2 + 12x + 9$ factors to $(2x + 3)^2$, and $n^2 - 10n + 25$ factors to $(n - 5)^2$.

61. FACTORING OTHER POLYNOMIALS—FOIL IN REVERSE

To factor a quadratic expression, **think about what binomials you could use FOIL on to get that quadratic expression.** To factor $x^2 - 5x + 6$, think about what **F**irst terms will produce x^2, what **L**ast terms will produce $+6$, and what **O**uter and **I**nner terms will produce $-5x$. Common sense—and trial and error—lead you to $(x - 2)(x - 3)$.

62. SIMPLIFYING AN ALGEBRAIC FRACTION

Simplifying an algebraic fraction is a lot like simplifying a numerical fraction. The general idea is to **find factors common to the numerator and denominator and cancel them.** Thus, simplifying an algebraic fraction begins with factoring.

To simplify $\frac{x^2 - x - 12}{x^2 - 9}$ first factor the numerator and denominator: $\frac{x^2 - x - 12}{x^2 - 9} = \frac{(x - 4)(x + 3)}{(x - 3)(x + 3)}$

Canceling $x + 3$ from the numerator and denominator leaves you with $\frac{x-4}{x-3}$.

SOLVING EQUATIONS

63. SOLVING A LINEAR EQUATION

To solve an equation, do whatever is necessary to both sides to **isolate the variable.** To solve $5x - 12 = -2x + 9$, first get all the x's on one side by adding $2x$ to both sides: $7x - 12 = 9$. Then add 12 to both sides: $7x = 21$, then divide both sides by 7 to get: $x = 3$.

64. SOLVING "IN TERMS OF"

To solve an equation for one variable **in terms of** another means to **isolate the one variable on one side of the equation,** leaving an expression containing the other variable on the other side. To solve $3x - 10y = -5x + 6y$ for x in terms of y, isolate x:

$$\begin{aligned} 3x - 10y &= -5x + 6y \\ 3x + 5x &= 6y + 10y \\ 8x &= 16y \\ x &= 2y \end{aligned}$$

65. TRANSLATING FROM ENGLISH INTO ALGEBRA

To translate from English into algebra, look for the key words and systematically turn phrases into algebraic expressions and sentences into equations. Be careful about order, especially when subtraction is called for.

Example: The charge for a phone call is r cents for the first 3 minutes and s cents for each minute thereafter. What is the cost, in cents, of a call lasting exactly t minutes? ($t > 3$)

Setup: The charge begins with r, and then something more is added, depending on the length of the call. The amount added is s times the number of minutes past 3 minutes. If the total number of minutes is t, then the number of minutes past 3 is $t - 3$. So the charge is $r + s(t - 3)$.

INTERMEDIATE ALGEBRA

66. SOLVING A QUADRATIC EQUATION

To solve a quadratic equation, put it in the $ax^2 + bx + c = 0$ form, **factor** the left side (if you can), and set each factor equal to 0 separately to get the two solutions. To solve $x^2 + 12 = 7x$, first rewrite it as $x^2 - 7x + 12 = 0$. Then factor the left side:

$$\begin{aligned} (x - 3)(x - 4) &= 0 \\ x - 3 = 0 &\text{ or } x - 4 = 0 \\ x &= 3 \text{ or } 4 \end{aligned}$$

Sometimes the left side might not be obviously factorable. You can always use the **quadratic formula.** Just plug in the coefficients a, b, and c from $ax^2 + bx + c = 0$ into the formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To solve $x^2 + 4x + 2 = 0$, plug $a = 1$, $b = 4$, and $c = 2$ into the formula:

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 2}}{2 \times 1} \\ &= \frac{-4 \pm \sqrt{8}}{2} = -2 \pm \sqrt{2} \end{aligned}$$

67. SOLVING A SYSTEM OF EQUATIONS

You can solve for two variables only if you have two distinct equations. Two forms of the same equation will not be adequate. **Combine the equations in such a way that one of the variables cancels out.** To solve the two equations $4x + 3y = 8$ and $x + y = 3$, multiply both sides of the second equation by -3 to get: $-3x - 3y = -9$. Now add the equations; the $3y$ and the $-3y$ cancel out, leaving: $x = -1$. Plug that back into either one of the original equations and you'll find that $y = 4$.

68. SOLVING AN EQUATION THAT INCLUDES ABSOLUTE VALUE SIGNS

To solve an equation that includes absolute value signs, **think about the two different cases.** For example, to solve the equation $|x - 12| = 3$, think of it as two equations:

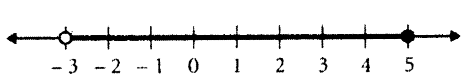
$$x - 12 = 3 \text{ or } x - 12 = -3$$
$$x = 15 \text{ or } 9$$

69. SOLVING AN INEQUALITY

To solve an inequality, do whatever is necessary to both sides to **isolate the variable.** Just remember that when you **multiply or divide both sides by a negative number,** you must **reverse the sign.** To solve $-5x + 7 < -3$, subtract 7 from both sides to get: $-5x < -10$. Now divide both sides by -5, remembering to reverse the sign: $x > 2$.

70. GRAPHING INEQUALITIES

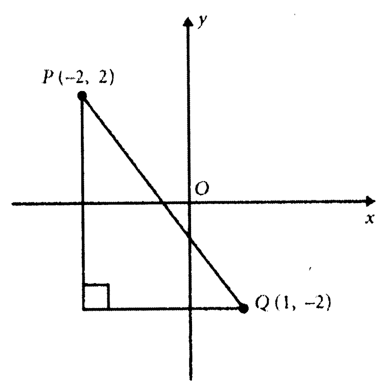
To graph a range of values, use a thick, black line over the number line, and at the end(s) of the range, use a **solid circle** if the point ***is* included** or an **open circle** if the point is ***not* included.** The figure here shows the graph of $-3 < x \leq 5$.



COORDINATE GEOMETRY

71. FINDING THE DISTANCE BETWEEN TWO POINTS

To find the distance between points, **use the Pythagorean theorem or special right triangles.** The difference between the *x*s is one leg and the difference between the *y*s is the other leg.



In the figure above, $\overline{PQ}$ is the hypotenuse of a 3-4-5 triangle, so $PQ = 5$.

You can also use the **distance formula:**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

To find the distance between $R(3, 6)$ and $S(5, -2)$:

$$d = \sqrt{(5 - 3)^2 + (-2 - 6)^2}$$
$$= \sqrt{(2)^2 + (-8)^2}$$
$$= \sqrt{68} = 2\sqrt{17}$$

72. USING TWO POINTS TO FIND THE SLOPE

In mathematics, the slope of a line is often called *m*.

$$\text{Slope} = m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{Rise}}{\text{Run}}$$

The slope of the line that contains the points $A(2, 3)$ and $B(0, -1)$ is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{0 - 2} = \frac{-4}{-2} = 2$$

73. USING AN EQUATION TO FIND THE SLOPE

To find the slope of a line from an equation, put the equation into the **slope-intercept** form:

$$y = mx + b$$

The slope is m. To find the slope of the equation $3x + 2y = 4$, reexpress it:

$$3x + 2y = 4$$
$$2y = -3x + 4$$
$$y = -\frac{3}{2}x + 2$$

The slope is $-\frac{3}{2}$.

74. USING AN EQUATION TO FIND AN INTERCEPT

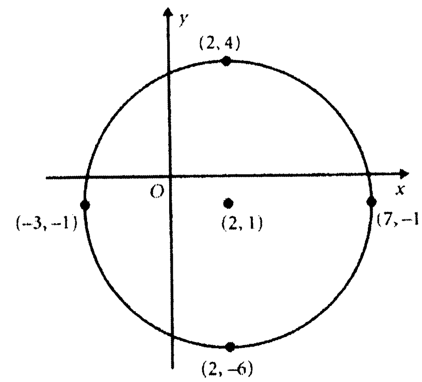
To find the y-intercept, you can either put the equation into $y = mx + b$ **(slope-intercept)** form—in which case b is the y-intercept—or you can just plug $x = 0$ into the equation and solve for y. To find the x-intercept, plug $y = 0$ into the equation and solve for x.

75. EQUATION FOR A CIRCLE

The equation for a circle of radius r and centered at (h, k) is:

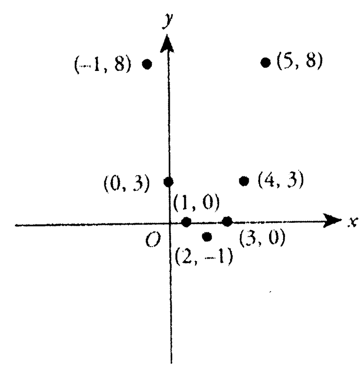
$$(x - h)^2 + (y - k)^2 = r^2$$

The following figure shows the graph of the equation $(x - 2)^2 + (y + 1)^2 = 25$:



76. EQUATION FOR A PARABOLA

The graph of an equation in the form $y = ax^2 + bx + c$ is a parabola. The figure below shows the graph of seven pairs of numbers that satisfy the equation $y = x^2 - 4x + 3$:

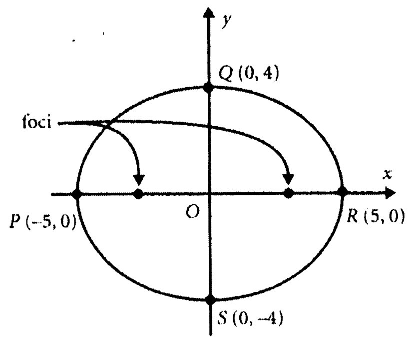


77. EQUATION FOR AN ELLIPSE

The graph of an equation in the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is an ellipse with $2a$ as the sum of the focal radii and with foci on the x-axis at $(0, -c)$ and $(0, c)$, where $c = \sqrt{a^2 - b^2}$. The following figure shows the graph of $\frac{x^2}{25} + \frac{y^2}{16} = 1$:

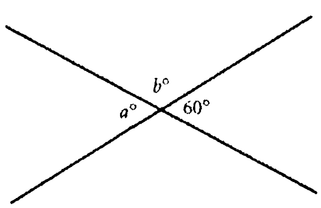


The foci are at (−3, 0) and (3, 0). $\overline{PR}$ is the **major axis,** and $\overline{QS}$ is the **minor axis.** This ellipse is symmetrical about both the x- and y-axes.

LINES AND ANGLES

78. INTERSECTING LINES

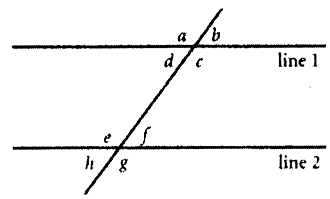
When two lines intersect, **adjacent angles are supplementary** and **vertical angles are equal.**



In the figure above, the angles marked $a°$ and $b°$ are adjacent and supplementary, so $a + b = 180$. Furthermore, the angles marked $a°$ and 60° are vertical and equal, so $a = 60$.

79. PARALLEL LINES AND TRANSVERSALS

A transversal across parallel lines forms **four equal acute angles and four equal obtuse angles.**



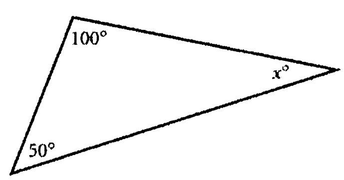
Here, line 1 is parallel to line 2. Angles a, c, e, and g are obtuse, so they are all equal. Angles b, d, f, and h are acute, so they are all equal.

Furthermore, **any of the acute angles is supplementary to any of the obtuse angles.** Angles a and h are supplementary, as are b and e, c and f, and so on.

TRIANGLES—GENERAL

80. INTERIOR ANGLES OF A TRIANGLE

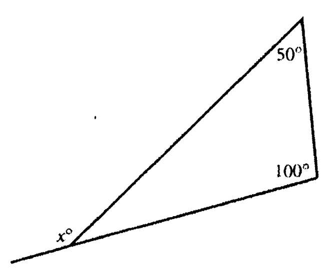
The three angles of any triangle **add up to 180°.**



In the figure above, $x + 50 + 100 = 180$, so $x = 30$.

81. EXTERIOR ANGLES OF A TRIANGLE

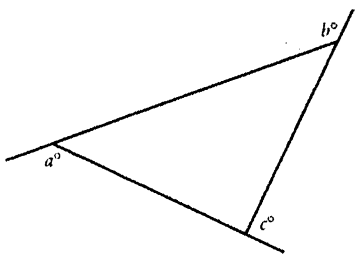
An exterior angle of a triangle is equal to the **sum of the remote interior angles.**



In the figure above, the exterior angle labeled $x°$ is equal to the sum of the remote interior angles:

$$x = 50 + 100 = 150$$

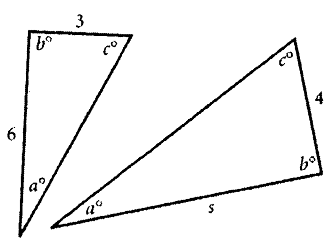
The three exterior angles of any triangle add up to 360°.



In the figure above, $a + b + c = 360$.

82. SIMILAR TRIANGLES

Similar triangles have the same shape: **corresponding angles are equal and corresponding sides are proportional.**



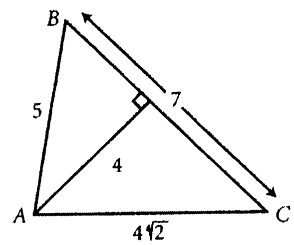
The triangles above are similar because they have the same angles. The 3 corresponds to the 4 and the 6 corresponds to the s.

$$\frac{3}{4} = \frac{6}{s}$$
$$3s = 24$$
$$s = 8$$

83. AREA OF A TRIANGLE

$$\text{Area of Triangle} = \frac{1}{2}(\text{base})(\text{height})$$

The height is the perpendicular distance between the side that's chosen as the base and the opposite vertex.



In the triangle above, 4 is the height when the 7 is chosen as the base.

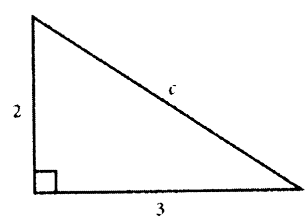
$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(7)(4) = 14$$

RIGHT TRIANGLES

84. PYTHAGOREAN THEOREM

For all right triangles:

$$(\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hypotenuse})^2$$



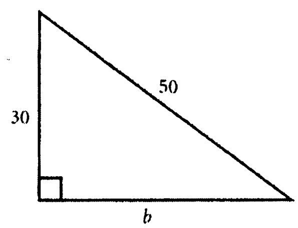
If one leg is 2 and the other leg is 3, then:

$$2^2 + 3^2 = c^2$$
$$c^2 = 4 + 9$$
$$c = \sqrt{13}$$

85. SPECIAL RIGHT TRIANGLES

- ***3-4-5***

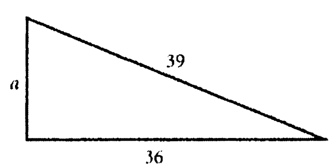
If a right triangle's leg-to-leg ratio is 3:4, or if the leg-to-hypotenuse ratio is 3:5 or 4:5, then it's a 3-4-5 triangle and you don't need to use the Pythagorean theorem to find the third side. Just figure out what multiple of 3-4-5 it is.



In the right triangle above, one leg is 30 and the hypotenuse is 50. This is 10 times 3-4-5. The other leg is 40.

- ***5-12-13***

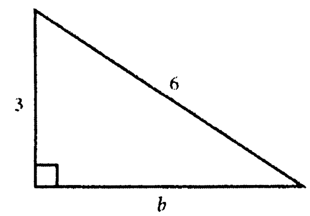
If a right triangle's leg-to-leg ratio is 5:12, or if the leg-to-hypotenuse ratio is 5:13 or 12:13, then it's a 5-12-13 triangle and you don't need to use the Pythagorean theorem to find the third side. Just figure out what multiple of 5-12-13 it is.



Here one leg is 36 and the hypotenuse is 39. This is 3 times 5-12-13. The other leg is 15.

- ***30°-60°-90°***

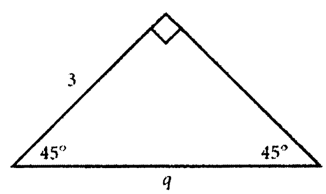
The sides of a 30°-60°-90° triangle are in a ratio of $1:\sqrt{3}:2$. You don't need to use the Pythagorean theorem.



If the hypotenuse is 6, then the shorter leg is half that, or 3; and then the longer leg is equal to the short leg times $\sqrt{3}$, or $3\sqrt{3}$.

- ***45°-45°-90°***

The sides of a 45°-45°-90° triangle are in a ratio of $1:1:\sqrt{2}$.



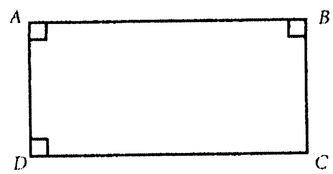
If one leg is 3, then the other leg is also 3, and the hypotenuse is equal to a leg times $\sqrt{2}$, or $3\sqrt{2}$.

OTHER POLYGONS

86. SPECIAL QUADRILATERALS

• ***Rectangle***

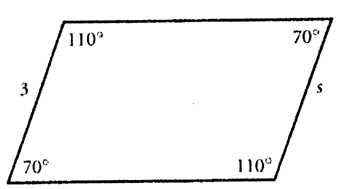
A rectangle is a **four-sided figure with four right angles.** Opposite sides are equal. Diagonals are equal.



Quadrilateral $ABCD$ above is shown to have three right angles. The fourth angle therefore also measures 90°, and $ABCD$ is a rectangle. The perimeter of a rectangle is equal to the sum of the lengths of the four sides, which is equivalent to 2(length + width).

• ***Parallelogram***

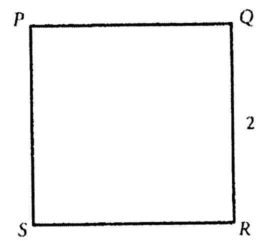
A parallelogram has **two pairs of parallel sides.** Opposite sides are equal. Opposite angles are equal. Consecutive angles add up to 180°.



In the figure above, s is the length of the side opposite the 3, so $s = 3$.

• ***Square***

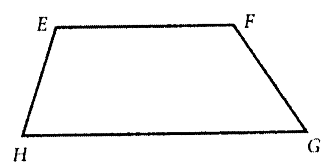
A square is a **rectangle with 4 equal sides.**



If $PQRS$ is a square, all sides are the same length as QR. The perimeter of a square is equal to four times the length of one side.

• **Trapezoid**

A **trapezoid** is a quadrilateral with one pair of parallel sides and one pair of nonparallel sides.

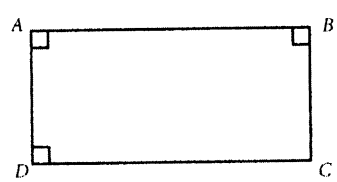


In the quadrilateral above, sides $\overline{EF}$ and $\overline{GF}$ are parallel, while sides $\overline{EH}$ and $\overline{FG}$ are not parallel. $EFGH$ is therefore a trapezoid.

87. AREAS OF SPECIAL QUADRILATERALS

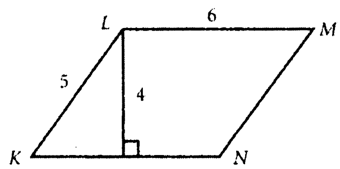
Area of Rectangle = Length × Width

The area of a 7-by-3 rectangle is $7 \times 3 = 21$.



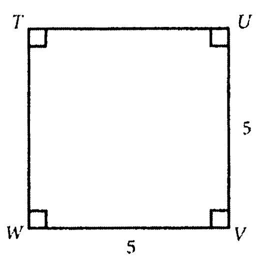
Area of Parallelogram = Base × Height

The area of a parallelogram with a height of 4 and a base of 6 is $4 \times 6 = 24$.



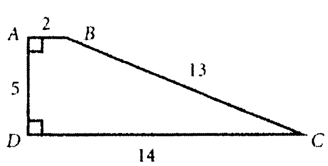
Area of Square = (Side)^2

The area of a square with sides of length 5 is $5^2 = 25$.



Area of Trapezoid $= \left(\dfrac{\text{base}_1 + \text{base}_2}{2}\right) \times$ **height**

Think of it as the average of the bases (the two parallel sides) times the height (the length of the perpendicular altitude).



In the trapezoid $ABCD$ above, you can use side $\overline{AD}$ for the height. The average of the bases is $\dfrac{2 + 14}{2} = 8$, so the area is 5×8, or 40.

88. INTERIOR ANGLES OF A POLYGON

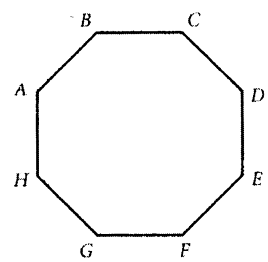
The sum of the measures of the interior angles of a polygon is $(n - 2) \times 180$, where n is the number of sides.

Sum of the Angles = $(n - 2) \times 180$ degrees

The eight angles of an octagon, for example, add up to $(8 - 2) \times 180 = 1{,}080$.

To find **one angle of a regular polygon,** divide the sum of the angles by the number of angles (which is the same as the number of sides). The formula, therefore, is:

$$\textbf{Interior Angle} = \frac{(n-2) \times 180}{n}$$

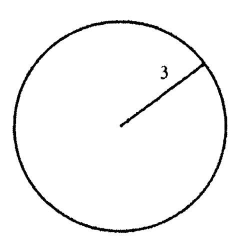


Angle A of the regular octagon above measures $\dfrac{1{,}080}{8} = 135$ degrees.

CIRCLES

89. CIRCUMFERENCE OF A CIRCLE

Circumference of a Circle = $2\pi r$

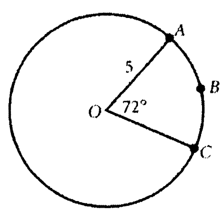


Here, the radius is 3, and so the circumference is $2\pi(3) = 6\pi$.

90. LENGTH OF AN ARC

An **arc** is a piece of the circumference. If n is the measure of the arc's central angle, then the formula is:

$$\textbf{Length of an Arc} = \frac{n}{360}(2\pi r)$$

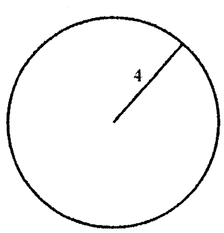


In the figure above, the radius is 5 and the measure of the central angle is 72°. The arc length is $\frac{72}{360}$ or $\frac{1}{5}$ of the circumference:

$$\left(\frac{72}{360}\right) 2\pi (5) = \left(\frac{1}{5}\right) 10\pi = 2\pi$$

91. AREA OF A CIRCLE

$$\textbf{Area of a Circle} = \pi r^2$$

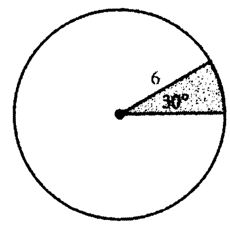


The area of the circle above is $\pi(4)^2 = 16\pi$.

92. AREA OF A SECTOR

A **sector** is a piece of the area of a circle. If n is the measure of the sector's central angle, then the formula is:

$$\textbf{Area of a Sector} = \left(\frac{n}{360}\right)(\pi r^2)$$



In the figure above, the radius is 6 and the measure of the sector's central angle is 30°. The sector has $\frac{30}{360}$ or $\frac{1}{12}$ of the area of the circle:

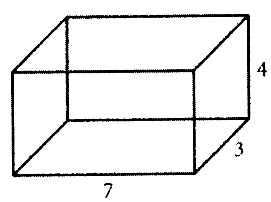
$$\left(\frac{30}{360}\right)(\pi)(6^2) = \left(\frac{1}{12}\right)(36\pi) = 3\pi$$

SOLIDS

93. SURFACE AREA OF A RECTANGULAR SOLID

The surface of a rectangular solid consists of 3 pairs of identical faces. To find the surface area, find the area of each face and add them up. If the length is l, the width is w, and the height is h, the formula is:

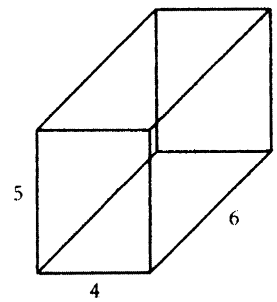
$$\textbf{Surface Area} = 2lw + 2wh + 2lh$$



The surface area of the box above is:
$2 \times 7 \times 3 + 2 \times 3 \times 4 + 2 \times 7 \times 4 =$
$42 + 24 + 56 = 122$

94. VOLUME OF A RECTANGULAR SOLID

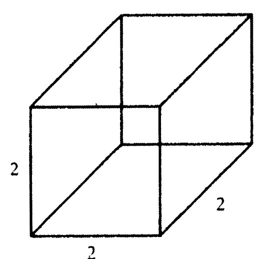
Volume of a Rectangular Solid = lwh



The volume of a 4-by-5-by-6 box is $4 \times 5 \times 6 = 120$.

A cube is a rectangular solid with length, width, and height all equal. The volume formula if e is the length of an edge of the cube is:

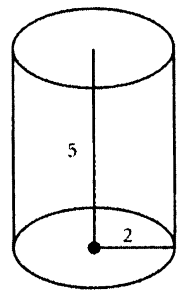
Volume of a Cube = e^3



The volume of the cube above is $2^3 = 8$.

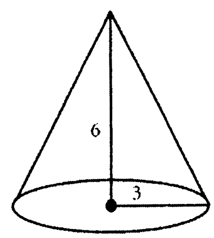
95. VOLUME OF OTHER SOLIDS

Volume of a Cylinder = $\pi r^2 h$



The volume of a cylinder where $r = 2$, and $h = 5$ is $\pi(2^2)(5) = 20\pi$.

Volume of a Cone = $\frac{1}{3}\pi r^2 h$



The volume of a cone where $r = 3$, and $h = 6$ is:

$$\text{Volume} = \frac{1}{3}\pi(3^2)(6) = 18$$

Volume of a Sphere = $\frac{4}{3}\pi r^3$

If the radius of a sphere is 3, then:

$$\text{Volume} = \frac{4}{3}\pi(3^3) = 36\pi$$

TRIGONOMETRY

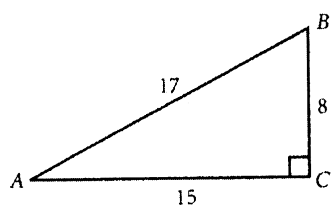
96. SINE, COSINE, AND TANGENT OF ACUTE ANGLES

To find the sine, cosine, or tangent of an acute angle, use SOHCAHTOA, which is an abbreviation for the following definitions:

$$\textit{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\textit{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\textit{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}$$



In the figure above:

$$\sin A = \frac{8}{17}$$

$$\cos A = \frac{15}{17}$$

$$\tan A = \frac{8}{15}$$

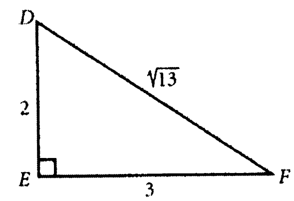
97. COTANGENT, SECANT, AND COSECANT OF ACUTE ANGLES

Think of the cotangent, secant, and cosecant as the reciprocals of the SOHCAHTOA functions:

$$\textit{Cotangent} = \frac{1}{\text{Tangent}} = \frac{\text{Adjacent}}{\text{Opposite}}$$

$$\textit{Secant} = \frac{1}{\text{Cosine}} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$$

$$\textit{Cosecant} = \frac{1}{\text{Sine}} = \frac{\text{Hypotenuse}}{\text{Opposite}}$$



In the preceeding figure:

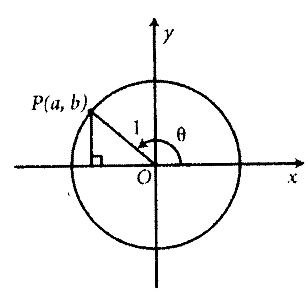
$$\cot D = \frac{2}{3}$$

$$\sec D = \frac{\sqrt{13}}{2}$$

$$\csc D = \frac{\sqrt{13}}{3}$$

98. TRIGONOMETRIC FUNCTIONS OF OTHER ANGLES

To find a trigonometric function of an angle greater than 90°, sketch a circle of radius 1 and centered at the origin of the coordinate grid. Start from the point (1, 0) and rotate the appropriate number of degrees counterclockwise.



In the "unit circle" setup above, the basic trigonometric functions are defined in terms of the coordinates a and b:

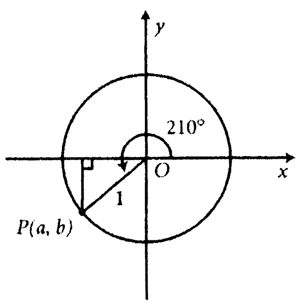
$$\sin \theta = b$$

$$\cos \theta = a$$

$$\tan \theta = \frac{b}{a}$$

Example: $\sin 210° = ?$

Setup: Sketch a 210° angle in the coordinate plane:



Because the triangle shown in the figure above is a 30°-60°-90° right triangle, we can determine that the coordinates of point P are $-\frac{\sqrt{3}}{2}, -\frac{1}{2}$. The sine is therefore $-\frac{1}{2}$.

99. SIMPLIFYING TRIGONOMETRIC EXPRESSIONS

To simplify trigonometric expressions, use the inverse function definitions along with the fundamental trigonometric identity:

$$\sin^2 x + \cos^2 x = 1$$

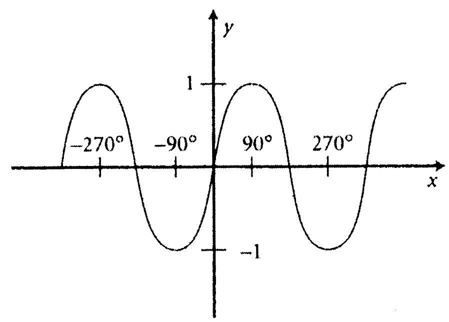
Example: $\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} = ?$

Setup: The numerator equals 1, so:

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$$

100. GRAPHING TRIGONOMETRIC FUNCTIONS

To graph trigonometric functions, use the x-axis for the angle and the y-axis for the value of the trigonometric function. Use special angles—0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180°, etc.—to plot key points.



The figure above shows a portion of the graph of $y = \sin x$.

I. Numbers and Arithmetic Operations

/’nΛmbə(r)/ /ə’riϴmətik/ /opə’ræisyen/

Numbers

Two kinds of activity made our ancestors develop numbers (**cardinal** and **ordinal numbers**). The first for comparing their things (which one has more elements), and the second for creating order.

A. Cardinal Numbers (Counting Numbers) /‘kɑ: dinl ’nΛmbə(r)/ /kaunting ’nΛmbə(r)/

Example:

1	one	/wʌn/
2	two	/tu:/
3	three	/θri:/
4	four	/fɔː/
5	five	/faɪv/
6	six	/sɪks/
7	seven	/'sevən/
8	eight	/eɪt/
9	nine	/naɪn/
10	ten	/ten/
11	eleven	/ɪ'levən/
12	twelve	/twelv/
13	thirteen	/θɜ:'ti:n/
14	fourteen	/fɔː'ti:n/
15	fifteen	/fɪf'ti:n/
16	sixteen	/sɪkst'i:n/
17	seventeen	/seven'ti:n/
18	eighteen	/eɪ'ti:n/
19	nineteen	/naɪn'ti:n/
20	twenty	/'twentɪ/
21	twenty-one	/twentɪ'wʌn/
22	twenty-two	/twentɪ'tu:/
23	twenty-three	/twentɪ'θri:/

24	twenty-four	/twentɪ'fɔː/
25	twenty-five	/twentɪ'faɪv/
26	twenty-six	/twentɪ'sɪks/
30	thirty	/'θɜ:tɪ/
40	forty	/'fɔːtɪ/
50	fifty	/'fɪftɪ/
60	sixty	/'sɪkstɪ/
70	seventy	/'sevəntɪ/
80	eighty	/'eɪtɪ/
90	ninety	/'naɪntɪ/
100	a hundred; one hundred	/ə 'hʌndrəd/ /wʌn 'hʌndrəd/
101	a hundred and one	/ə 'hʌndrəd ən wʌn/
110	a hundred and ten	/ə 'hʌndrəd ən ten/
120	a hundred and twenty	/ə 'hʌndrəd ən 'twentɪ/
200	two hundred	/tu: 'hʌndrəd/
300	three hundred	/θri: 'hʌndrəd/
900	nine hundred	/naɪn 'hʌndrəd/
1 000	a thousand, one thousand	/ə θ'aʊzənd/ /wʌn 'θaʊzənd/
1 001	a thousand and one	/ə 'θaʊzənd ən wʌn/
1 010	a thousand and ten	/ə 'θaʊzənd ən ten/
1 020	a thousand and twenty	/ə 'θaʊzənd ən 'twentɪ/
1 100	one thousand, one hundred	/wʌn 'θaʊzənd wʌn 'hʌndrəd/
1 101	one thousand, one hundred and one	/wʌn 'θaʊzənd wʌn 'hʌndrəd ən wʌn/
9 999	nine thousand, nine hundred and ninety-nine	/naɪn 'θaʊzənd naɪn 'hʌndrəd ən 'naɪntɪ 'naɪn/
10 000	ten thousand	/ten 'θaʊzənd/
15 356	fifteen thousand, three hundred and fifty six	/'fɪfti:n 'θaʊzənd θri: 'hʌndrəd ən 'fɪftɪ sɪks/

100 000	a hundred thousand	/ə 'hʌndrəd 'θaʊzənd/
1 000 000	a million	/ə 'mɪljən/
1 000 000 000	a billion	/ə 'bɪljən/
1 000 000 000 000	a trillion	/ə 'trɪljən/

B. Ordinal Numbers/Place Numbers /’ərdinəl ’nΛmbə(r)/

Example:

1st	first	/fɜ:st/
2nd	second	/'sekənd/
3rd	third	/θɜ:d/
4th	fourth	/fɔ:θ/
5th	fifth	/fɪfθ/
6th	sixth	/sɪksθ/
7th	seventh	/'sevənθ/
8th	eighth	/eɪtθ/
9th	ninth	/naɪnθ/
10th	tenth	/tenθ/
11th	eleventh	/ɪ'levənθ/
12th	twelfth	/'twelfθ/
13th	thirteenth	/θɜ:'ti:nθ/
14th	fourtheenth	/fɔː'ti:nθ/
15th	fidteenth	/fɪf'ti:nθ/
16th	sixteenth	/sɪks'ti:nθ/
17th	seventeenth	/seven'ti:nθ/
18th	eighteenth	/eɪ'ti:nθ/
19th	nineteenth	/naɪn'ti:nθ/
20th	twentieth	/'twentɪəθ/
21st	twenty-first	/twentɪ'fɜ:st/
22nd	twenty-second	/twentɪ'sekənd/
23rd	twenty-third	/twentɪ'θɜ:d/
24th	twenty-fourth	/twentɪ'fɔ:θ/
25th	twenty-fifth	/twentɪ'fɪfθ/

26th	twenty-sixth	/twentɪ'sɪksθ/
27th	twenty-seventh	/twentɪ'sevənθ/
28th	twenty-eighth	/twentɪ'eɪtθ/
29th	twenty-ninth	/twentɪ'naɪnθ/
30th	thirtieth	/'θɜːtɪəθ/
31st	thirty-first	/θɜːtɪ'fɜ:st/
40th	fortieth	/'fɔ:tɪəθ/
50th	fiftieth	/'fɪftɪəθ/
100th	hundredth	/'hʌndrədθ/
1 000th	thousandth	/'θɑʊzəndθ/
1 000 000th	miilionth	/'mɪljənθ/

- **Natural Numbers**

  /’nætʃral ’nΛmbə(r)/

  1,2,3,… *one, two, three, and so forth (without end).*

  1,2,3,…, 10 *one, two, three, and so forth up to ten.*

  Natural numbers can be divided into two sets:

  Odd Numbers /ɒd ’nΛmbə(r)/ and **Even Numbers** /’i:vn ’nΛmbə(r)/

- **Whole Numbers** /həʊl ’nΛmbə(r)/

  Natural Numbers + 0 *zero/o/nought.*

  /’ziərəu/ /nə:t/

- **Integers** /’intəjər/

  ..,--2,1,0,1,.. *.., negative two, negative one, zero, one, ..*

- **Rational numbers** /’ræʃnəl ’nΛmbə(r)/ are numbers that can be expressed as fraction.
- **Irrational Numbers** /i’ræʃnəl ’nΛmbə(r)/ are numbers that cannot be expressed as fraction, such as $\sqrt{2}, \pi$.
- **Real Numbers** /riəl ’nΛmbə(r)/ are made up of rational and irrational numbers.
- **Complex Numbers** /’kompleks ’nΛmbə(r)/

  Complex numbers are numbers that contain **real** and **imaginary** part.

2 + 3i   2 is called the **real part**, 3 is called the **imaginary part**, and i is called **imaginary unit** of the complex number.

- **A Digit** /’dɪdʒɪt/ is any one of the ten numerals 0,1,2,3,4,5,6,7,8,9.

  Example:

  3 is a single-digit number, but 234 is a three-digit number.

  In 234, 4 is the units digit, 3 is the tens digit, and 2 is hundreds digit.

- **Consecutive** /kən'sekjʊtɪv/ **numbers** are counting numbers that differ by 1.

  Examples**:**

  83, 84, 85, 86, and 87 are 5 consecutive numbers.

  84, 85, 86, … are successor /sək-‘ses-ə(r)/ of 83.

  84 is the immediate successor of 83.

  1, 2, …, and 82 are predecessor /’predə-ses-ə(r)/of 83.

  82 is the immediate predecessor of 83.

  36, 38, 40, and 42 are 4 consecutive even numbers.

Operation on Numbers

Addition (+) , Subtraction (-) , Multiplication (×) , Division(:)

/ə’diʃn/ /sab’træksyən/ /’maltəplə’keisyen/ /di’vi3n/

Symbols in Numbers Operation

+	added by/plus/and /ædid bai/ /plΛs/ /ənd/
-	subtracted by/minus/take away /səb’træktid bai/ /’mainəs/ /teik ə’wei/
±	plus or minus /plΛs o:(r) ‘mainəs/
×	multiplied by/times /’mΛltiplaid bai/ /taimz/
:	divided by/over /di’vaidid bai/ /’əuvə(r)/

Symbols for Comparing /kəm'peə(r)ing / **Numbers**

=	is equal to/equals/is /iz "i:kwəl tu:/ /"i:kwəlz/ /iz/
≠	is not equal to/does not equal /iz not "i:kwəl tu:/ /'dΛznt "i:kwəl/
<	is less than/is smaller than /iz les thən/ /iz smōlər thən/
>	is greater than/is more than /iz greitər thən/ /iz mə:(r) thən/
≤	is less than or equal to /iz les thən o:(r) "i:kwəl tu:/
≥	is more/greater than or equal to /iz mə:(r)/ /greitər thən o:(r) "i:kwəl tu:/
≅	is approximately equal to /iz ə'proksimatli "i:kwəl tu:/

The mathematical sentences that use symbols "=" are called **equation**, and the mathematical sentences that use symbols "<", ">", "≤"", or "≥" are called **inequalities**.

Examples

$ax + b = 0$ is a linear equation.

$ax^2 + bx + c = 0$ is a quadratic equation.

$3x^3 - 2x^2 + 3 = 0$ is a cubic equation.

$\frac{a+b}{2} \geq \sqrt{ab}$ is called AM-GM inequality.

Examples

- $2 + 3 = 5$

two	*is added by*	*three*	*is equal to*	*five.*
	plus		*equals*	
	and		*is*	

2 and 3 are called **addends** or **summands**, and 5 is called **sum**. /sΛm/

- $10 – 4 = 6$

Ten	*is substracted by*	*four*	*is equal to*	*six.*
	minus		*equals*	
	take away		*is*	

10 is the **minuend**, 4 is the **subtrahend**, and 6 is the **difference**/'difrəns/

- 7 × 8 = 56

Seven	*is multiplied by* *times*	*eight*	*is equal to* *equals* *is*	*fifty-six*

7 is the **multiplicator**/’mΛltəplə’kətwr/, 8 is the **multiplicand**/’mΛltəplə’kənd/, and 56 is the **product**/’prodəkt/.

- 45 : 5 = 9

forty-five	*is divided by* *over*	*five*	*is equal to* *is*	*nine.*

45 is the **dividend**, 5 is the **divisor** /də’vaizə(r)/, and 9 is the **quotient**/’kwəuʃnt/.

Practice

1. Read out the following operations, and for every operations name each number’s function.
   a. 1,209 + 118 = 1,327
   b. 135 + (-132) = 3
   c. 2 – (-25) = 27
   d. 52 – 65 = -13
   e. 9 x 26 = 234
   f. -111 x 99 = -10,989
   g. 36 : 9 = 4
   h. 1375 : (-25) = -55

2. Fill the blank spaces with the right words.
   a. The ____________ of three and seven is twenty-one.
   b. The operation that uses symbol “:” is called ___________.
   c. 14 is the ________________________of 13, and the predecessor of 13 are _________________.
   d. The result of division is called ______________.
   e. Three multiplied ___________ five equals _____________.
   f. In 123,456,789, the hundred thousands digit is ____, and 9 is the ______________.

g. We select a ________________ number *htu*, as $100h + 10t + u$, where *h* represents the ___________ digit, *t* represents the ______ digit, and *u* represents the units digit.

h. When we __________ two numbers, for example seven plus thirteen, the answer (twenty) is called _______________.

Fractions /frækʃn/

A **common (or simple) fraction** is a fraction of the form ***a/b*** where *a* is an integer and *b* is a counting number

Example: p/q

p is called the **numerator** /nyu:məreitə(r)/of the fraction

q is called the **denominator** /di'nomi'neitə(r)/ of the fraction

If the numerator < the denominator, then (p/q) is a **proper fraction** /propə(r) 'frækʃn/

If the numerator > the denominator, then (p/q) is an **improper fraction** /im'propə(r) 'frækʃn/

3 ¼ is a **mixed numbers** /miksed 'nΛmbə(r)/ because it contains **number part** /'nΛmbə(r) pa:t/ and **fractional part** /'frækʃnəl pa:t/

The fraction ***a/b*** is **simplified** ("in lowest terms") if *a* and *b* have no common factor other than 1

Saying Fraction

$\frac{1}{2}$	A/one half /ə/wΛn ha:f/
$\frac{1}{3}$	A/one third /ə/wΛn θɜ:d/
$\frac{1}{4}$	A/one quarter /ə/wΛn 'kwɔ:tə(r)/
$\frac{5}{6}$	Five sixths/Five over six
$\frac{22+x}{7}$	Twenty-two plus x all over seven
$13\frac{3}{4}$	Thirteen and three quarters

0.3	Nought/zero/o point three
3.056	Three point o five six
273.856	Two hundred and seventy-three point eight five six

Practice

1. Read out the following fractions
    a. $\frac{2}{5}$
    b. $\frac{3}{4}$
    c. $\frac{5}{8} \times \frac{1}{4} = \frac{5}{32}$
    d. $2\frac{1}{2} : \frac{9}{10} = 3\frac{2}{5}$
    e. $\frac{1}{9} - \frac{1}{8} \neq \frac{1}{24}$
    f. 13,945.614
    g. 43.554
    h. 6.9 x 2.2 = 15.18
    i. 72.4 x 61.5 = 4452.6
2. Fill the blank spaces with the right words.
    a. In the fraction seven ninths, __________ is the numerator, and _____________ is the ______________.
    b. The ______________ of two thirds and a half is four over three.
    c. An integer plus a fraction makes a __________________.

Divisibility

4|12

*12 is **divisible** by 4.*
/di'vizəbl/

*12 is a **multiple** of 4.*
/mΛltipl/

*4 **divides** 12.*
/di'vaidz/

*4 **is a factor** of 12*
/'fæktə(r)/

15 is not divisible by 4.

If 15 divided by 4 then the quotient is 3 and **the remainder** is 3.
/thə ri'meɪndə(r)/

0 is divisible by all integers

Prime numbers /praim 'nΛmbə(r)z/

Every numbers is divisible by 1 and itself. These factors (1 and itself) are called **improper divisors**. /im'propə(r) də'vaizə(r)z/

Prime numbers are numbers that have only improper divisors.

Example:

5 is a prime number, but 9 is not a prime number or a **composite number**. /kompəzit 'nΛmbə(r)z/

Common Divisors /'komən də'vaizə(r)z/

Example:

1,2,3,4,6, and 12 are divisors (factors) of 12.

1,3,5, and 15 are divisors of 15.

1 and 3 are ***common divisors*** *of 12 and 15.*

3 is the ***greatest common divisor*** /greitəst 'komən də'vaizə(r)/ *of 12 and 15.*

The ***g.c.d of*** *12 and 15 is 3.*

gcd(12,15) = 3.

Common Multiples /'komən 'mΛltiplz/

Example:

5,10,15,20,25, …are multiples of 5.

4,8,12,16,20,24,… are multiples of 4.

5,10,15,20 are four first multiples of 5.

4,8,12,16,20 are five first multiples of 4.

20,40,60, … are ***common multiples*** *of 4 and 5.*

20 is the ***least common multiple*** /li:st 'komən 'mΛltipl/ *of 4 and 5.*

The ***l.c.m of*** *4 and 5 is 20.*

lcm(4,5) = 20.

Practice

1. Read the following conversation

   A : I have two numbers, 36 and 42. Can you say their factors?

   B : The factors of 36 are 1,2,3,4,6,9,12,24, and 36. 1,2,3,6,7,14,21, and 42 are factors of 42.

   A : So, what are their common factors?

   B : They are 1,2,3, and 6.

   A : And what is the greatest common divisor of 36 and 42?

   B : It's 6.

2. Make a small conversation about gcd or lcm of other numbers.

Exercise

Write down the spelling of these mathematical sentences

- $12 + 1/3 \leq x - 7$
- $3x \times 26 > 20 : y$
- $x (2y + 3) \neq 111.909$
- $(2 + x)/35 < 23/45$

Exercise

Use the right words to complete these sentences.

- 2367 is _______ by nine.
- 3 is _________ of 34.
- The _________ of three and four is twelve.
- Eighteen subtracted __ twenty equals _____.
- 3 is the __________ and 5 is the ________ of three fifths.

Exercise

- Write down five first multiples of 8.
- Write down all divisors of 18.
- Find all common divisors of eighteen and thirty-three.
- Write down the simplest form of 91/234
- Find the sum of the reciprocals of two numbers, given that these numbers have a sum of 50 and a product of 25.

- What is the product of the greatest common divisor of 9633 and 4693 and the least common multiple of the same numbers?
- Let x be the smallest of three positive integers whose products is 720. Find the largest possible value of x.
- If P represents the product of all prime numbers less than 1000, what is the value of the units digit of P?
- Find a positive integer that is eleven times the sum of its digits?
- What is the greatest common divisor of 120 and 49?
- The product of 803 and 907 is divided by the sum of 63 and 37. What is the remainder?
- The average of four consecutive even integers is 17. Find the largest of the four integers.
- When the six-digit number 3456**N**7 is divided by 8, the remainder is 5. List **both** possible values of the digit **N.**

Vocabularies of Chapter I

Words	Pronunciation	Indonesian
Numbers	/’nΛmbə(r)z/	Bilangan
Natural Numbers	/’nætʃral ’nΛmbə(r)z/	Bilangan Asli
Odd Numbers	/ɒd ’nΛmbə(r)z/	Bilangan Ganjil
Even Numbers	/’i:vn ’nΛmbə(r)z/	Bilangan Genap
Whole Numbers	/həʊl ’nΛmbə(r)/	Bilangan Cacah
Integers	/’intəjərz/	Bilangan Bulat
Rational numbers	/’ræʃnəl ’nΛmbə(r)z/	Bilangan Rasional
Irrational Numbers	/i’ræʃnəl ’nΛmbə(r)z/	Bilangan Irrasional
Real Numbers	/riəl ’nΛmbə(r)z/	Bilangan Real
Complex Numbers	/’kompleks ’nΛmbə(r)z/	Bilangan Kompleks
Digit	/’dɪdʒɪt/	Angka
Consecutive numbers	/kən'sekjʊtɪv ’nΛmbə(r)z/	Bilangan berurutan
Prime numbers	/praim ’nΛmbə(r)z/	Bilangan prima
Composite numbers	/kompəzit ’nΛmbə(r)z/	Bilangan komposit
Addition	/ə’diʃn/	Penjumlahan
Subtraction	/sab’træksyən/	Pengurangan
Multiplication	/’maltəplə’keisyen/	Perkalian
Division	/di’vi3n/	Pembagian
Equation	/ɪ’kweɪʃn/	Persamaan
Inequalities	/,ɪnɪ’kwɒləti/	Pertidaksamaan
Difference	/’difrəns/	Selisih
Sum	/sΛm/	Jumlah
Multiplicator	/’mΛltəplə’kətwr/	Pengali
Multiplicand	/’mΛltəplə’kənd/	Yang dikali
Product	/’prodəkt/	Hasilkali
Dividend	/’dɪvɪdend/	Yang dibagi
Divisor	/də’vaizə(r)/	Pembagi

Quotient	/’kwəuʃnt/	Hasilbagi
Fractions	/frækʃn/	Pecahan
Numerator	/nyu:məreitə(r)/	Pembilang
Denominator	/di’nomi’neitə(r)/	Penyebut
Proper fraction	/propə(r) ‘frækʃn/	Pecahan sejati
Improper fraction	/im’propə(r) ‘frækʃn/	Pecahan taksejati
Mixed number	/miksed ’nΛmbə(r)/	Pecahan campuran
Numbert part	/’nΛmbə(r) pa:t/	Bagian bilangan
Fractional part	/‘frækʃnəl pa:t/	Bagian pecahan

II. Powers, Roots, and Logarithm

/'paʊə(r)z/ /ru:tz/ /'lɒgəriðəm/

Powers/Indices /'ɪndɪsi:z/ is used when we want to multiply a number by itself several times.

$$a^b$$

In this term, a is called **base/basis** /beɪs/'beɪsəs/ and b is called **index/exponent** /ɪk'spəʊnənt/. The word *power* sometimes also means the exponent alone rather than the result of an exponential /ɪk'spəʊnənʃl/ expression.

How to Say Powers

x^2	x squared /'skweə(r)d/
x^3	x cubed /kju:bd/
x^n	x to the power of n x to the n-th power x to the n x to the n-th x upper /'ʌpə(r)/ n x raised /reizd/ by n
$(x+y)^2$	x plus y all squared bracket /'brækit/ x plus y bracket closed squared x plus y in bracket squared

Practice

A. Read out the following terms and say their values.

1. 2^6
2. $\left(\frac{2}{3}\right)^3$
3. $x^5 : x^2$
4. $(3ab)^4$

5. $\left(\frac{x}{3y}\right)^3$
6. $(9x)^0$

B. Read these expressions and simplify them.

1. $5^3 \times 5^{13}$
2. $8^{14} : 8^{11}$
3. $\left(2^4\right)^3$
4. $\left(\frac{x^7}{x^3}\right)^2$

LAWS FOR POWERS

for equal exponents

- **First Law for Power:**

$$(ab)^n = a^n b^n$$

A product raised by an exponent is equal to product of factors raised by same exponent

$$(a/b)^n = a^n/b^n$$

For equal basis

- **Second Law for Powers:**

$$a^m \times a^n = a^{m+n}$$

- *The product of two powers with equal basis equals to the basis raised to the sum of the two exponents*
- *When expressions with the same base are multiplied, the indices are added*

How can we say this rule?

$$a^m : a^n = a^{m-n}$$

- **Third Law for Powers:**

$$(a^m)^n = a^{mn}$$

Exponentiating of powers equals to the basis raised to the product of the two exponents

Practice

Try to express in words these another rules of powers:

1. $a^0 = 1$, $a \neq 0$.
2. $a^{-n} = \dfrac{1}{a^n}$, $a \neq 0$
3. $(a/b)^n = a^n/b^n$
4. $a^m : a^n = a^{m-n}$

Roots and Radicals /rædɪklz/

Root is inversion of exponentiation

$$\sqrt[n]{a} = b \leftrightarrow b^n = a$$

$\sqrt[n]{a}$ is called **radical expression** (or **radical form**) because it contains a root.

The radical expression has several parts:

- the **radical sign** /saɪn/ √
- the **radicand** /rædɪkən/: the entire quantity under the radical sign
- the **index**: the number that indicates the root that is being taken

example:

$\sqrt[3]{a+b}$ a + b is the radicand, 3 is the index.

The radical expression can be written in **exponential form** (**powers with fractional exponents**)

example: $\sqrt[n]{x} = x^{1/n}$

So the law of powers can be used in calculating root

Examples:

$$\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}} = \sqrt[n]{a}\sqrt[n]{b}$$

A number is said **perfect square** if its roots are integers.

example:

9, 16, 36, and 100 are perfect squares, but 12 and 20 are not.

How to Say Radicals

$\sqrt{x}$	**(square) root of x**
$\sqrt[3]{y}$	**cube root of y**
$\sqrt[n]{z}$	**n-th root of z**
$\sqrt[5]{x^2y^3}$	**fifth root of (pause) x squared times y cubed** **fifth root of x squared times y cubed in bracket**

Square Root

The square root is in **simplest form** if:

a. the radicand does not contain perfect squares other than 1.
b. no fraction is contained in radicand.
c. no radicals appear in the denominator of a fraction.

Example

$\sqrt{24}$ is not a simplest form because we can write it as $\sqrt{4\times 6}$ where 4 is a perfect square. We can simplify the radical into $2\sqrt{6}$

A radical and a number is called a **binomial** /baɪ'nəʊmɪəl/. The **conjugate** /'kɒndʒʊgeɪt/ of binomial is another binomial with the same number and radical, but the sign of second term is changed.

Example

$2+\sqrt{6}$ is a binomial and its conjugate is $2-\sqrt{6}$

Practice

a. Read out the following radical expressions and say theirs exponential notation.

1. $\sqrt{4x^4}$
2. $\sqrt[4]{m^3n^8}$
3. $\sqrt[5]{a^3}$
4. $\sqrt[3]{8x^6y^9}$
5. $\sqrt{x^2+y^2}$

b. Read out the following terms and say what their values are:

1. $243^{1/5}$
2. -4^{-2}
3. $125^{1/3}$
4. $(-5)^{-1}$
5. 3^{-3}

c. Simplify these radicals

1. $\sqrt{72}$
2. $\sqrt{234}$
3. $\dfrac{5}{2+\sqrt{3}}$
4. $\dfrac{\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

d. Find the conjugate of these binomials

1. $2+\sqrt{5}$
2. $6-\sqrt{4}$

Logarithm

$$x = a^b \Leftrightarrow b = {}^a\log x$$

In this term, a is also called **base.**

How to Say Logarithm

$^n\log x$	**log** /lɒg/**x to the base of n** **log base n of x**
ln 2	**natural log of two** **"L N" of two**
$^5\log^2 25$	**log squared of twenty-five to the base of five** **log base five of twenty-five all squared**

Practice

Read out the following terms:

a. $a\ {}^x\log b$
b. $\log a^2$
c. $^2\log (1/6)$
d. $^5\log (x^2+y)$
e. $(^n\log x)^2$
f. $^6\log^2 22 - {}^6\log x^2 - 1$

Laws for Logarithm

- **First Law for logarithm:**

  The logarithm of a product is equal to the sum of the logarithm of the factors

$$^b\log(xy) = {}^b\log x + {}^b\log y$$

- **Second Law for logarithm:**

  The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor

$$^b\log(x/y) = {}^b\log x - {}^b\log y$$

- **Third Law for logarithm:**

  The logarithm of a power is equal to the exponent times the logarithm of the basis

$$^{b}\log(x^{a}) = a\ ^{b}\log\ x$$

More Examples

$^{2}\log(x+y)+2\ ^{2}\log 4x > 4$	*log base two of x plus y in bracket plus two times log base two of four x's is greater than four*
$x^{2} + \frac{1}{\sqrt{x}} = 1$	*x squared plus (pause) one over root of x equals one*
$3^{x} + 9^{x-1} > 27$	*three upper x plus (pause) nine upper x minus one (pause) is more than twenty-seven*
$9^{x} - 1 < 2$	*nine to the x (pause) minus one is less than two*

Some Algebraic Processes

1. Expand (x-3)(x+2) into x^2-x-6.
2. Simplify (2x+2)/(x+1) into 2
3. Factorize x^3-2x^2+3x-2 into (x-1)(x+1)(x-2)
4. Cancel (x+1) from (2x+2)/(x+1) to get 2
5. Add/subtract/multiply/divide both side

   Examples: multiply both side of equation ½x= 4 with 2 to get x=8
6. Subtitute y=4 into equation 2x+y=12
7. Collect (x+2) from $(x+2)^3-2(x+2)(x+1)$ to get $(x+2)[(x+2)^2-2(x+1)]$

Example

Find x that satisfy equation $3^{x}-3^{x-1}=162$.

Answer

First, we multiply both side with 3 to get $3.3^{x}-3^{x}=486$.

Then, we collect 3^x and we have $3^x(3-1)=486$, which can be simplified into $2.3^x=486$.

Divide both side by 2, we get $3^x=243$.

We know that 243 is 3^5, so we can write $3^x=3^5$.

According to the rule of powers, x must be equal to 5.

Exercise

- How do we say these mathematical terms?

  1. $({}^3\log x)^2 + {}^3\log x^2 = \sqrt[4]{4-x^3}$
  2. $x^{{}^n\log(x+1)} = 0$
  3. $\sqrt{2\sqrt{2}} = \log(\frac{x}{5})$

- Read the complete answers.

  1. 13^2=...
  2. 2^9=...
  3. Every positive real numbers has .... real-numbered square roots.
  4. The cube root of two hundred and sixteen is ....
  5. If the root of eighty-one is raised by three, then we have ....
  6. 7 is the log base ten of ....

- Solve this problem and try to explain it.

  1. Which is greater, $2^{95}+2^{95}$ or 2^{100}?
  2. Which is the larger, $10^{1/10}$ or $2^{1/3}$?
  3. Let A and B are real numbers greater than 1. If there is positive number $C \neq 1$ such that $2({}^A\log C + {}^B\log C) = 9\ {}^{AB}\log C$, then find the largest possible value for ${}^A\log B$.
  4. Given ${}^9\log\ 20 = a$ and ${}^3\log\ n = 4a$. What is the value of n?
  5. In March, the number of students was a perfect square. At the end of the semester, with 100 new students, the number of students became 1 more than a perfect square. At the end of the year with an additional 100 new students, the number of students is a perfect square. How many students were there in September?

Explain the process to find the solution x that satisfies each equation, inequality, or system of equations.

1. $1 - 4x \leq x + 11$
2. $\frac{3}{2}y - \frac{5}{3} = \frac{4-2y}{5}$
3. $\begin{aligned} x + y &= 2 \\ 2x - y &= -5 \end{aligned}$

Vocabularies of Chapter II

Words	Pronunciation	Indonesian
Powers	/'paʊə(r)z/	Bilangan Berpangkat
Indices	/'nætʃral 'nΛmbə(r)z/	Bilangan Asli
Base	/beɪs/	Bilangan Pokok
Basis	/'beɪsəs/	Bilangan Pokok
Exponent	/ɪk'spəʊnənt/	Pangkat
Roots	/ru:tz/	Bentuk Akar
Radicals	/rædɪklz/	Bentuk Akar
Radical sign	/rædɪklz saɪn/	Tanda Akar
Radicand	/rædɪkən/	Bilangan yang diakarkan
Perfect square	/'pɜ:fɪkt skweə(r)/	Kuadrat Sempurna
Binomial	/baɪ'nəʊmɪəl/	Binomial
Conjugate	/'kɒndƷʊgeɪt/	Sekawan
Logarithm	/'lɒgəriðəm/	Logaritma
Expand	/ ɪk'spænd/	Uraikan/Jabarkan
Simplify	/'sɪmplɪfaɪ/	Sederhanakan
Factorize	/'fæktəraɪz/	Faktorkan
Cancel	/'kænsl/	Coret/Hapus/Batalkan
Substitute	/'sΛbstɪtju:t/	Substitusi
Side	/saɪd/	Ruas
Left-hand side	/left hænd saɪd/	Ruas kiri
Right-hand side	/raɪt hænd saɪd/	Ruas kanan
Collect	/kə'ləkt/	Kumpulkan
Eliminate	/ɪ'lɪmɪneɪt/	Eliminasi

III. Sequence, Series, and Trigonometry

/'si:kwəns/ /'sɪəri:z/ /trɪgə'nɒmətrɪ/

Arithmetic /ə'rɪθmətɪk/ **Sequence/Progression** /'prə'greʃn/

An arithmetic sequence is a sequence of the form

$a, a+d, a+2d, \ldots$

The number a is the **first term** /tɜ:m/, and d is the **common difference** /'kɒmən 'dɪfrəns/ of the sequence.

The difference between two **consecutive** /kən'sekjʊtɪv/ **terms** is d

$a_n - a_{n-1} = d$

The *n*th Term of an Arithmetic Sequence

The ***n*th term** of the arithmetic sequence $a, a+d, a+2d, \ldots$ is

$a + (n-1)\,d$

a_1, a_2, a_3 are the **first three terms**.

$a_{(n+1)/2}$ is called **middle** /'mɪdl/ **term**.

a_n and a_{n+1} are **two consecutive terms**

Partial /'pɑ:ʃl/ **sum of Arithmetic Sequence**

For arithmetic sequence $a, a+d, a+2d, \ldots$, the nth **partial sum**

$S_n = (n/2)(2a + (n - 1)d)$

or

$S_n = (n/2)(a + a_n)$

Examples

In arithmetic sequence 1,4,7,10,13,..., the first term is 1, the difference is 3. So, the formula for n-th term is 1+(n-1)3=3n-2. Applying the formula we can say the 100-th term of the sequence is 298. The partial sum of the sequence is $(n/2)(1+3n-2)=(3/2)n^2-(n/2)$.

Geometric /dʒɪə'mətrɪk/ **Sequence**

$a, ar, ar^2, \ldots$ is a geometric sequence

$a_1 = a$ is the **first term**.

$a_n = ar^{n-1}$ is the ***n*-th term** of the geometric sequence.

r is the **common ratio** /'reɪʃɪəʊ/.

Partial Sum of Geometric Sequence

Let Sn be the partial sum of the geometric sequence. Then

$Sn = a + ar + ar^2 + \ldots + ar^{n-1}$

or $Sn = a\dfrac{|r^n - 1|}{|r - 1|}$

Examples

In arithmetic sequence 2,4,8,16,32,..., the first term is 2, the ratio is also 2. So, the formula for n-th term is $2\ 2^{n-1}=2^n$. Applying the formula we can say the 10-th term of the sequence is 1024. The partial sum of the sequence is $2(2^n -1)/(2-1)=2^{n+1}-2$.

The sum of finite /'faɪnaɪt/or infinite /'ɪnfɪnət/ sequence

$$\sum a_n = a_1 + a_2 + \ ..$$

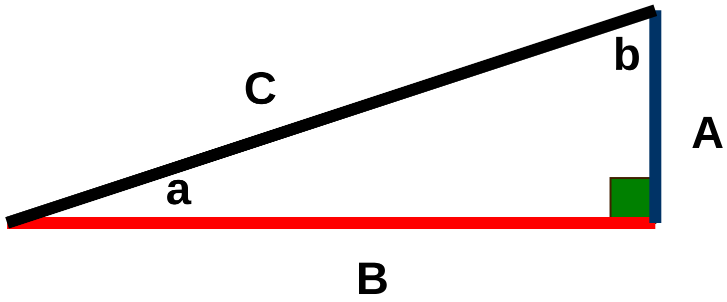
is called **series**.

Practice

1. Find the first 6 terms and the 300th term of the arithmetic sequence 13, 7, ...
2. The 10th term of an arithmetic sequence is 55 and the 2nd term is 7. Find the 1st term.
3. Find the sum of the first 40 terms of the arithmetic sequence 3, 7, 11, 15, ...
4. Find the 8th term of the geometric sequence 5, 15, 45, ...
5. The 3rd term of a geometric sequence is 63/4, and the 6th term is 1701/32. Find the 8th term.
6. Find the sum of the first 5 terms of the geometric sequence 1, 0.7, 0.49, 0.343, ...

Trigonometry

- Trigonometry is the study of angle measurement.
- When you have a right triangle there are 5 things we can know about it: the lengths of the sides (A, B, and C), and the measures of the acute angles (a and b)



Some terminology:

- The **hypotenuse**/haɪ'pɒtənyu:z/ will always be the longest side, and opposite from the right angle.
- The **opposite**/ɒpəzɪt/ **side** is the side directly across from the angle you are considering (angle a).
- The **adjacent**/ə'dʒeɪsnt/ **side** is the side next to angle you are considering.

The Trigonometric Functions

sin	sine	/saɪn/
cos	cos; cosine	/kɒz/;/kɒzaɪn/
tan	tan; tangent	/tæn/;/tændʒənt/
sec	sec;	/sek/
csc	cosec;	/'kəʊsek/
cot	cotangent	/'kəʊtændʒənt /

- Sin /saɪn/ x = Opposite side/Hypotenuse
- Cos /kɒz; kɒzaɪn/ x = Adjacent side/Hypotenuse
- Tan /tæn; tændʒənt/ x = Opposite side/Adjacent side
- Cot /'kəʊtændʒənt /a = adjacent side/opposite side
- Sec /sek/ a = hypotenuse/adjacent side
- Csc /'kəʊsek/ a = hypotenuse/opposite side

Here is a way to remember how to make the 3 basic Trig Ratios

SOHCAHTOA is pronounced "Sew Caw Toe A" and it means

Sin is Opposite over Hypotenuse,

Cos is Adjacent over Hypotenuse,

and Tan is Opposite over Adjacent

We can use the mnemonic

Silly **O**ld **H**en, **C**ackles **A**nd **H**owls, **T**ill **O**ld **A**ge.

- This relationship can be summarized:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Practice

Fill the table below and say the equation, such as sin(0^0)=....

Degrees /dɪ'gri:z/	Radians /'rəɪdɪən/	Sin	Cos	Tan
0	0			
30	π/6			
45	π/4			
60	π/3			
90	π/2			

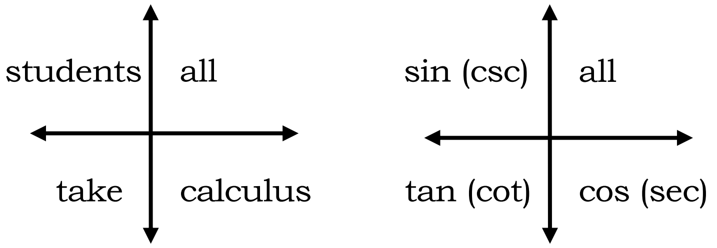
Periodicity

The trigonometric functions are periodic. The sine and cosine functions have the **period** /'pɪərɪəd/ 2π (360^0); the tangent and cotangent functions have the period π (180^0).

Quadrant /'kwɒdrənt/ **relations**.

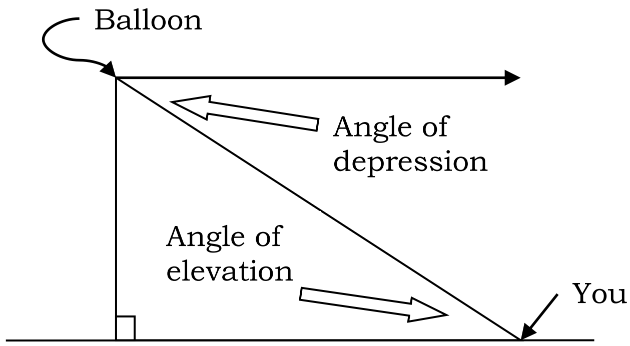
- All functions have positive values for angles in Quadrant I, Sine and Cosecant have positive values for angles in Quadrant II, Tangent and Cotangent have positive values for angles in Quadrant III, Cosine and Secant have positive values for angles in Quadrant IV.
- It will help to memorize by learning these words in Quadrants I - IV: "All students take calculus"

And remembering **reciprocal** /rɪ'siprəkl/identities



Sometimes when we use right triangles to model real-life situations, we use the terms **<u>angle of elevation</u>** and **<u>angle of depression</u>**.

If you are standing on the ground and looking up at a hot air balloon, the angle that you look up from ground level is called the **<u>angle of elevation</u>**. If someone is in the hot air balloon and looks down to the ground to see you, the angle that they have to lower their eyes, from looking straight ahead, is called the **<u>angle of depression</u>**.



<u>Practice</u>

Read the following expressions.

1. (Formulas for Addition and Subtraction)
   sin(A + B) = sin A cos B + cos A sin B
   cos(A - B) = cos A cos B + sin A sin B
   tan(A + B) = tan A + tan B / (1 - tan A tan B)
2. (Phytagorean Identities) $\sin^2\theta + \cos^2\theta = 1$
   $\tan^2\theta + 1 = \sec^2\theta$
3. (Formula for Double Angle) sin 2A = 2 sin A cos A
4. (Formula for half angle) $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$
5. (Cosine Rule) $a^2 = b^2 + c^2 - 2bc \cos A$

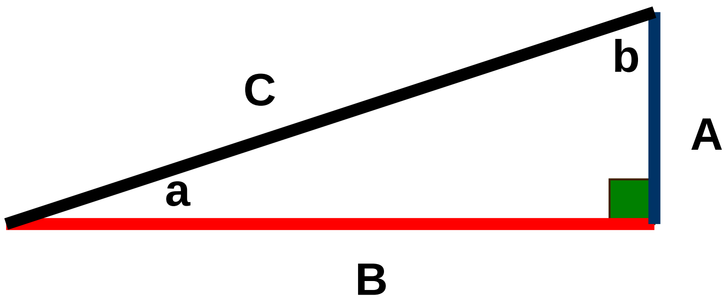
Explaining the solution of these problems

1. Without using calculator, find $\cos(15^0)$.
2. Find the values of x for which $\sin 3x = 0.5$ if it is given that $0<x<90^0$.
3. What is the equal value in degrees for x radians?

Exercise

Complete the sentences or give short answers.

1. Tangent has positive values for angles in _______________, and _______________ has positive values for angles in Quadrant IV.
2. The tangent and cotangent functions have the period __________
3. If $\sin \alpha = 0.8$, then the value of $\sin(180-\alpha)$ is _________ and the value of $\tan \alpha$ is ______________
4. Without using calculator, find $\cos(15^0)$.
5. Find the values of x for which $\sin 3x = 0.5$ if it is given that $0<x<90^0$.
6. What is the equal value in degrees for x radians?
7. Say anything about trigonometry of the picture below



For example:

sin a is A/C.

Greek Alphabets

Big Letters	Small Letters	Words	Pronunciation
Α	α	alpha	/'ælfə/
Β	β	beta	/'bi:tə/
Γ	γ	gamma	/'gæmə/
Δ	δ	delta	/'deltə/
Ε	ε	epsilon	/'epsilən/
Ζ	ζ	zeta	/'ziːtə/
Η	η	eta	/'iːtə/
Θ	θ	theta	/'θiːtə/

Ι	ι	iota	/aɪ'əʊtə/
Κ	κ	kappa	/'kæpə/
Λ	λ	lamda	/'læmdə/
Μ	μ	mu	/'mjuː/
Ν	ν	nu	/'njuː/
Ξ	ξ	xi	/'ksaɪ/
Ο	ο	omicron	/'əʊmɪkrən/
Π	π	pi	/'paɪ/
Ρ	ρ	rho	/'rəʊ/
Σ	σ	sigma	/'sɪgmə/
Τ	τ	tau	/'taʊ/
Υ	υ	upsilon	/'jʊpsɪlən/
Φ	φ	phi	/'faɪ/
Χ	χ	chi	/'kaɪ/
Ψ	ψ	psi	/'psaɪ/
Ω	ω	omega	/'əʊmɪgə/

Vocabularies of Chapter III

Words	Pronunciation	Indonesian
Sequence	/'si:kwəns/	Barisan
Progression	/'prə'greʃn/	Barisan
Arithmetic sequence	/ə'rɪθmətɪk 'si:kwəns/	Barisan Aritmatika
Geometric	/dʒɪə'mətrɪk 'si:kwəns/	Barisan Geometri
Difference	/'dɪfrəns/	Beda
Ratio	/'reɪʃɪəʊ/	Rasio
Series	/'sɪəri:z/	Deret
Finite	/'faɪnaɪt/	Berhingga
Infinite	/'ɪnfɪnət/	Tak Berhingga
Term	/tɜ:m/	Suku
First Term	/fɜ:st tɜ:m/	Suku Pertama
Middle Term	/'mɪdl tɜ:m/	Suku Tengah
Consecutive Terms	/kən'sekjʊtɪv tɜ:mz/	Suku Berurutan
Partial sum	/'pɑ:ʃl sʌm/	Jumlah sebagian (suku)
Trigonometry	/trɪgə'nɒmətrɪ/	Trigonometri
Adjacent side	/ə'dʒeɪsnt saɪd/	Sisi samping
Hypotenuse	/haɪ'pɒtənyu:z/	Sisi miring

Opposite side	/ɒpəzɪt saɪd/	Sisi depan
Angle	/'æŋgl/	Sudut
Degrees	/dɪ'gri:z/	Derajat
Period	/'pɪərɪəd/	Periode
Quadrant	/'kwɒdrənt/	Kuadran
Radian	/'rəɪdɪən/	Radian
Reciprocal	/rɪ'siprəkl/	Kebalikan

IV. Plane Geometry

Objects in plane geometry are **points**, **lines**, and **two-dimensional figures**.

A **point** /pɔɪnt/ is an exact location in space. A point has no size. Points are represented by dots, and named with capital letters.

A **line** /laɪn/ is a straight arrangement of points. Lines extend forever in opposite directions.

Lines are named using a lower case letter. For example: line m

A line could be **straight** or **curvy**. A straight line could be **horizontal**, **vertical**, or **oblique** /ə’bli:k/.

examples:

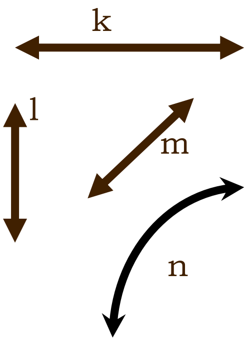
Line k is a **horizontal line**

Line l is a **vertical line**.

Line m is an **oblique line**.

Lines k, l, and m are **straight lines**

Line n is a **curved line**. /kɜ:v/



Position of lines in a plane

Two lines in a plane could be **parallel** /pærəlel/, or **intersecting**.

example:

Line l **is parallel to** line m.

(Lines l and m are parallel).

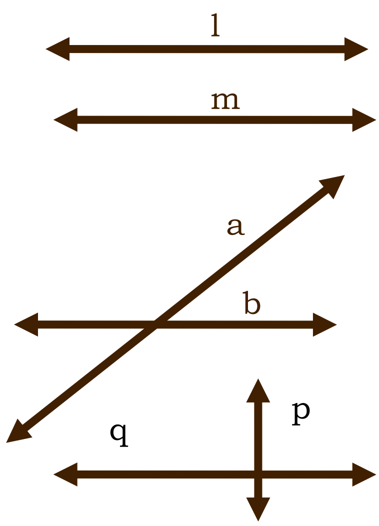
Line a **cuts** line b. Line a **intersects** line b.

Lines a and b are **intersected lines**.

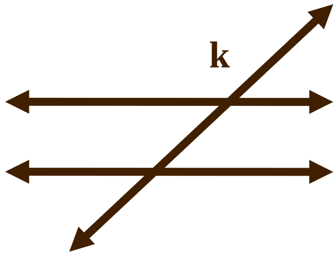
If two lines intersect, their intersection is a point called the **point of intersection**.

Line p is **orthogonal** to line q. (Line p is **perpendicular** /pɜ:pən’dɪkjʊlə(r)/ to line q).

Lines p and q are **orthogonal**.



When we have more than two lines, this situation (see picture below) can happen



Line k is called a **<u>transversal</u>** /trænzvɜ:sl/ line, which is a straight line drawn across a set of two or more parallel lines.

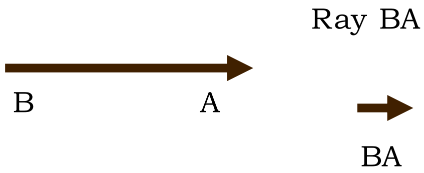
Line Segments

A line segment is part of a line. Line segments consist of 2 endpoints and all the points in between. Line segments are named using their endpoints.

example : line segment AB or AB

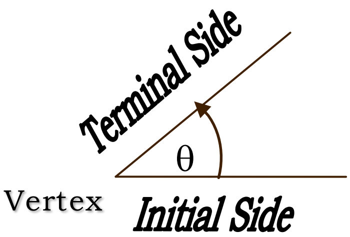
Rays /reɪ/

A ray is a part of a line that has one **endpoint** and extends forever in one direction. Rays are named by writing the endpoint first, then another point on the ray. Example:



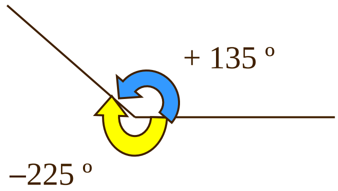
Angle /'æŋgl/

An angle is formed by rotating a ray around its **end point**. One ray is fixed, and is called the **initial side**. The second ray is called the **terminal side**. The common end point is called the **vertex** /vɜ:teks/.



Positive, Negative & Coterminal Angles

- A **positive** angle results from a **counter-clockwise** rotation.
- A **negative** angle results from a **clockwise** rotation.

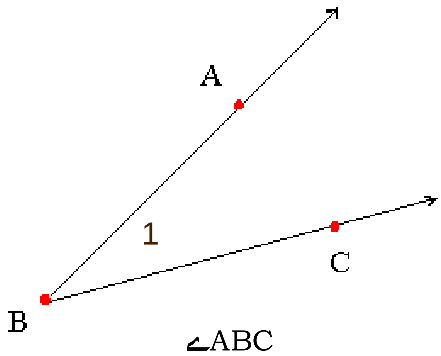


Two angles with the same terminal side are **coterminal**.

So + 135 º and - 225 º are coterminal!

Naming Angle

- Angles can be named using letters or numbers.
- When using letters, the vertex may be named alone, or the vertex may be named between two other points on the angle.



$\angle ABC$
$\angle CBA$
$\angle B$
$\angle 1$

Angles according to their measures.

Measure of Angle θ	Name
$0^0 < \theta < 90^0$	acute /ə'kju:t/angle
$\theta = 90^0$	right /raɪt/angle
$90^0 < \theta < 180^0$	obtuse /əb'tju:s/angle
$\theta = 180^0$	straight angle
$180^0 < \theta < 360^0$	reflex angle
$\theta = 360^0$	full angle

Complementary & Supplementary Angles

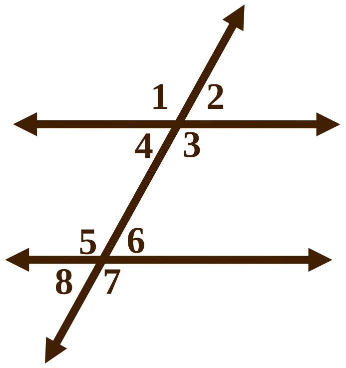
Two angles that have a sum of 90° are **complementary**.

If the measure of Angle A is 40^0 and Angle B is 50^0, then we say Angle A and angle B are **complementary angles.**

Two angles that have a sum of 180° are **supplementary.**

If the measure of Angle A is 120^0 and Angle B is 60^0, then we say Angle A and angle B are **supplementary angles.**

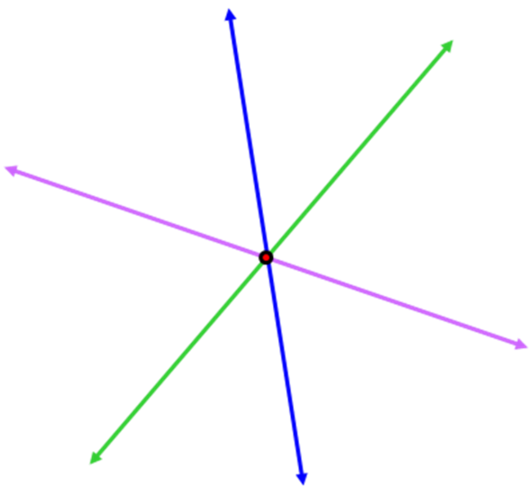
Angles Formed by Transversal Line



- Angle 3 and angle 5 are **alternate interior** angles.
- Angle 1 and angle 7 are **alternate exterior** angles.
- Angle 1 and angle 2 are **adjacent** angles.
- Angle 2 and angle 6 are **corresponding** angles.
- Angle 5 and angle 7 are **opposite** angles.

Could you find another pairs of equal kind of angles?

Concurrent lines are three or more lines that intersect in one point. The point is called **point of concurrency**.



Two Dimensional Figures

There are two groups of two-dimensional figures: **Polygon** and **Circle**.

Polygon

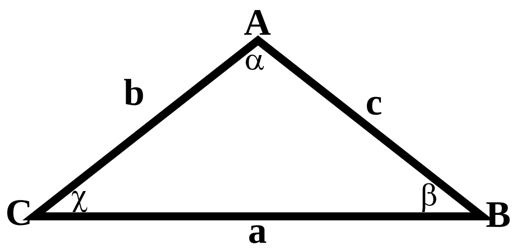
Polygon are two-dimensional figures which consists of n points and nearby points connected by straight lines which are called **sides**. They are named by the number of the sides.

example:

- TRIANGLE – 3 sides
- QUADRILATERAL – 4 sides
- PENTAGON – 5 sides
- HEXAGON – 6 sides
- HEPTAGON – 7 sides
- OCTAGON – 8 sides
- NONAGON – 9 sides
- N-GON – N sides

Triangles

A triangle is a three – sided figure . The three sides of a triangle meet at points called vertices (singular: vertex)



A triangle has three **vertices** (A,B,C), three **sides** (a,b,c), and three interior **angles** (α,β,χ) whose sum is 180 degrees.

Special Kind of Triangles

- **Isosceles triangle** is a triangle that has **two equal sides**.
  The top vertex is called **<u>apex</u>** and the bottom side is called **<u>base</u>**.
- **Equilateral triangle** is a triangle that has **three equal sides**.
- **Right-angle triangle** (usually called **right triangle**) is a triangle that has **one right angle**. The side opposite the right angle is called **<u>hypotenuse</u>**.
- **Scalene triangle** is a triangle that has **three different sides** or all of its sides have different length.

Acute triangle is a triangle which all of its internal angles are **acute** angles.

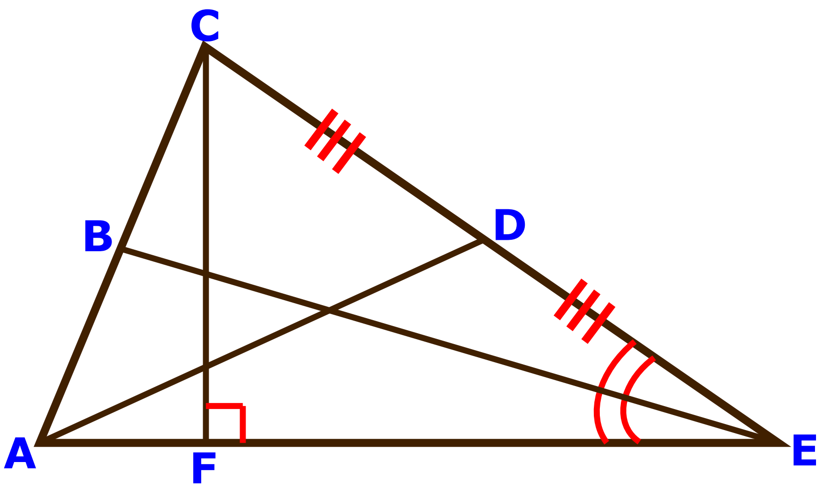
Obtuse triangle is a triangle that has an obtuse angle.

3 Important Lines of Triangles

Lines	Definition
angle bisector	a segment which bisects an angle and connects a vertex and a point on the opposite side.
median (bisector)	a segment that connects a vertex of the triangle and the midpoint of the opposite side
altitude	a segment from the vertex of the triangle perpendicular to the line containing the opposite side

Practice

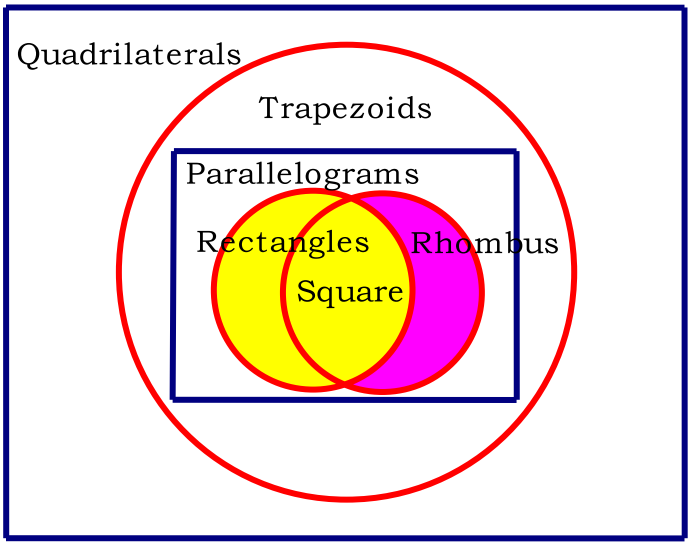
Which one is the angle bisector, median, or altitude of this triangle?



Quadrilateral

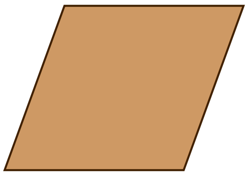
A quadrilateral is any 4 sided shape. The sum of all interior angles of any quadrilateral is 360 degrees.

Classification of Quadrilaterals.



Parallelogram

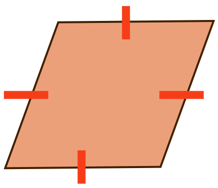
A parallelogram has 2 pairs of parallel sides.



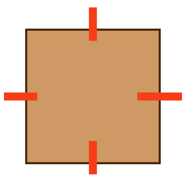
Types of Parallelogram

A rectangle is a parallelogram with 4 right angles.

A rhombus is a parallelogram with 4 sides of equal length



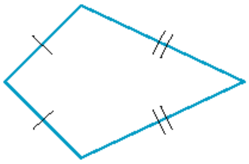
A square is a parallelogram with 4 right angles and 4 sides of equal length.



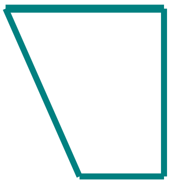
Another Types of Quadrilateral

A trapezoid has exactly one pair of parallel sides.

A kite has exactly two pairs of congruent adjacent sides.

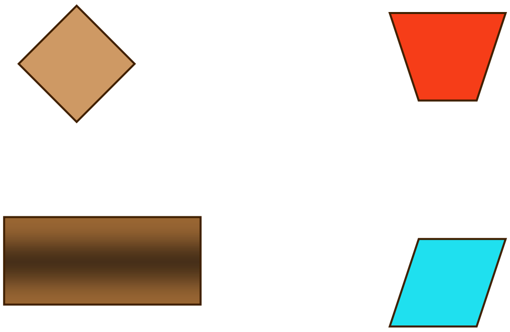


A trapezium has exactly one pair of parallel sides and two right angles.



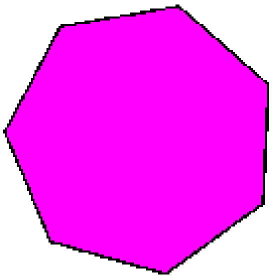
Practice

Classify the quadrilaterals in as many ways as possible.

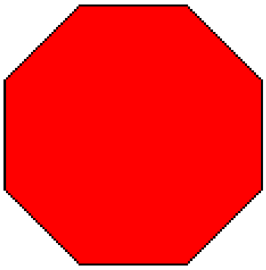


Polygon

Heptagon



Octagon



Area and Perimeter

- The formula of **area** of a triangle is
  ½ length of altitude × length of base
- The **perimeter** of a n-sided figure is the sum of all its side's lengths.

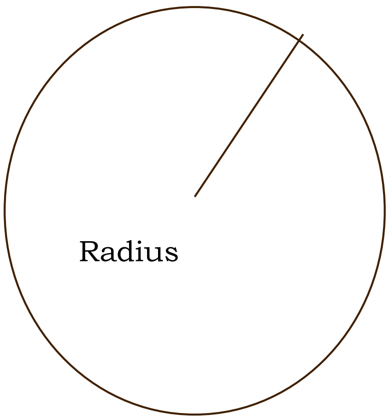
Practice

Find the formula for area of square, rectangle, rhombus, parallelogram, and trapezium.

Circle

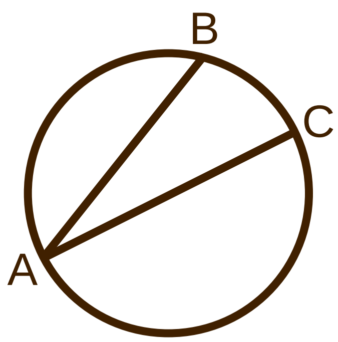
Circle is the set of all points in a plane that are a given distance from the center.

Radius (plural: **radii**) is a segment line that joins the center to a point on the circle.

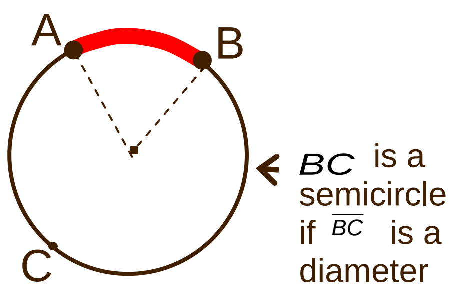


Parts of a Circle

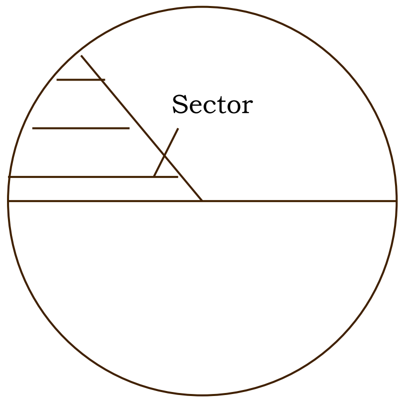
- Chord: a line joining two points on a circle.
  – AC and AB are chords.
- Diameter: a chord that passes through the circles center.
  – AC is a diameter



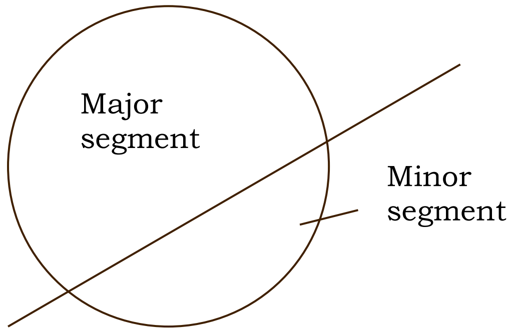
- Arc: two points on a circle and all the points needed to connect them.
  - minor arc AB or
  - major arc ACB or



- Central Angle: an angle whose vertex is at the center of the circle
- Sector: It is a region enclosed by two radii and arc of the circle.



- Segment: It is a region enclosed by a chord and the arc joining the chord. The segment made my minor arc is called minor segment and segment by major arc as major segment.



- Circumference: the perimeter of a circle, the distance around a circle.

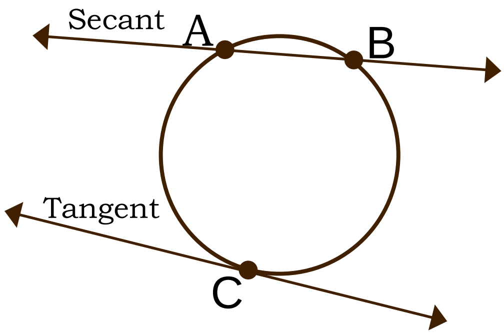
  $C = d\pi$ or $C = 2\pi r$ (d = diameter, r = radius)

- Area of a circle

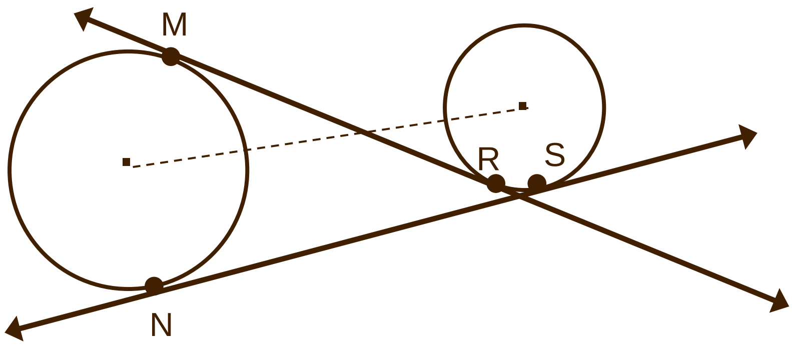
  $A = \pi r^2$

Line that Cuts the Circle

- Secant: a line that intersects a circle at exactly two points
- Tangent: a line that intersects a circle at exactly one point
  - The point of contact is called the **point of tangency** or point of contact.



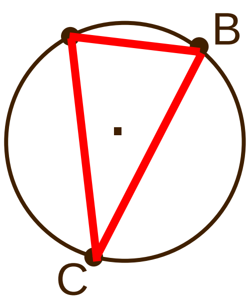
- A **common tangent** is a line tangent to two circles (not necessarily at the same point)



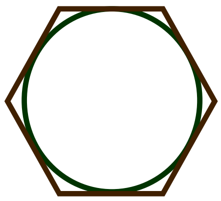
MR is a common internal tangent
NS is a common external tangent

Inscribed and Circumscribed Polygons

- **Inscribed**: A polygon is inscribed in a circle (or another polygon) if all of its vertices lie on the circle (or another polygon).
  - The circle center is the **incenter** of the polygon



- **Circumscribed**: A polygon is circumscribed about a circle if each of its sides is tangent to the circle.
  - The circle center is the **circumcenter** of the polygon



Practice

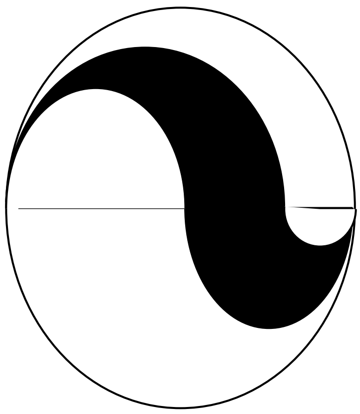
Fill the blank with the right words or answer the questions.

1. ______________ consist of 2 endpoints and all the points in between.
2. If each angle in a triangle is less than 60^0, then the triangle is called ___________.
3. A line which meets another _____ at 90^0 is called a ___________ line.
4. If two angles of a triangle are equal to 45^0, then the triangle is called _____________
5. If we __________ a right angle, we will have two __________ angles of 45^0.
6. If the measure of angle A is 130^0, then the _________________________________ is -230^0.
7. A segment that perpendicular to a side of triangles and through a vertex is called ____________________.
8. Each triangle has 3 points, or ______________
9. ___________ is a rectangle with four congruent sides.
10. An octagon is _______________ in a square if all of its _____________ lie on the square.
11. A _______________ with radius 10 m has _____________________ of 20π m.
12. A quadrilateral which only has one pair of right angle can be called _________________.
13. ______________________ has seven vertices and _____ sides.
14. If the _______________________ of a sector is 60 degrees, then the area of the sector is _________________ of the circle's area.

Answer these questions.

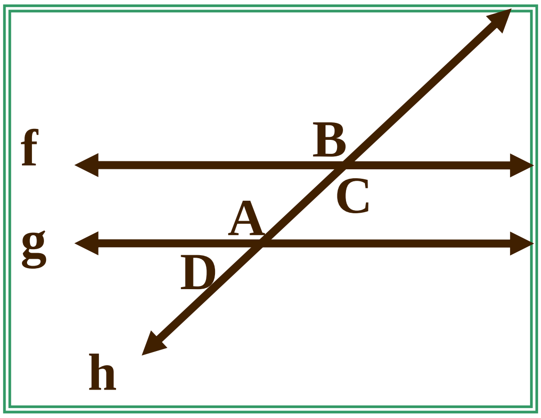
1. A right triangle has a hypotenuse of 6 and a perimeter of 14. Find the area of the triangle.
2. A regular hexagon is inscribed in a circle of radius 4 meters. What is the area of the hexagon?

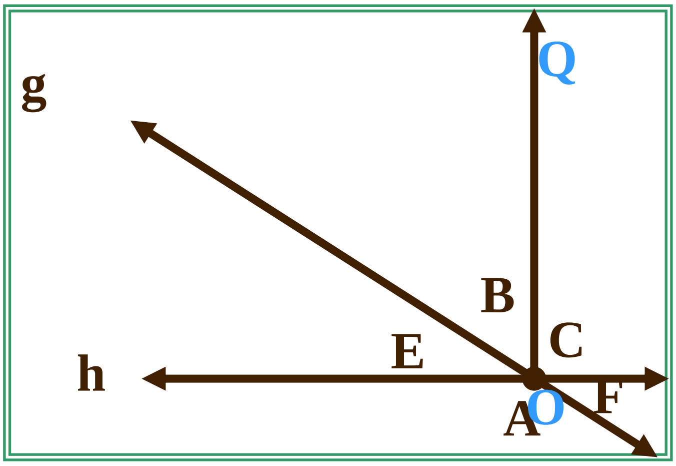
3. The total number of interior angles in two regular polygons is 17, and the total number of diagonals is 53. How many sides does each regular polygon have?
4. A triangle has sides of length 30, 40, and 50 meters. What is the length of the shortest altitude of this triangle?
5. A circle is inscribed in a triangle that has sides of lengths 60, 80, and 100 cm. Find the length of the radius of the circle.
6. We know that the vertices of a quadrilateral are 2, 3, 5, and 6 cm, respectively, from a point P. What is the largest possible area of this quadrilateral?
7. Five of the angles of an octagon have measures whose sum is 845^0. Of the remaining three angles, two are complementary to each other and two are supplementary to each other. Find the measures of these three angles.
8. Gene wants to put a brick border around a tree. The border is to be placed 1.5m from the tree. If the circumference of the tree is 56.52 cm, what is the inner circumference of the brick border?
9. A hexagon is inscribed in a circle, which is inscribed in a square of side 10 cm. What is the length of each side of the hexagon?
10. Find the dimension of a rectangle of maximum area with a given perimeter P.
11. In how many different ways can you divide a square into four congruent shapes?
12. Find the area of the shaded region if the diameter of the circle is 24 cm and it is divided into four equal segments as shown.

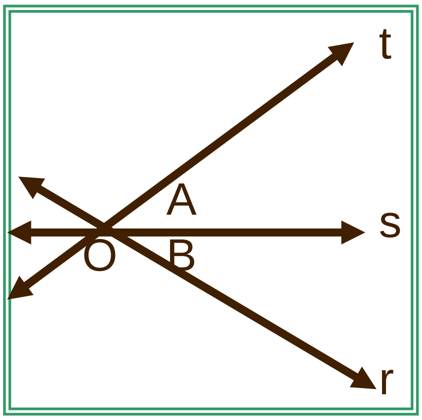


Practice

- Describe points, lines, and angles in these figures







Vocabularies of Chapter IV

Words	**Pronunciation**	**Indonesian**
Point	/pɔɪnt/	Titik
Line	/laɪn/	Garis
Horizontal line	/ˌhɒrɪ'zɒntl laɪn/	Garis datar
Vertical line	/'vɜ:tɪkl laɪn/	Garis tegak
Oblique line	/ə'bli:k laɪn/	Garis miring
Straight line	/streɪt laɪn/	Garis lurus
Curved line	/kɜ:v laɪn/	Garis lengkung
Parallel	/pærəlel/	Sejajar
Parallel line	/pærəlel laɪn/	Garis sejajar
Cut	/kʌt/	Memotong
Intersect	/ˌɪntə'sekt/	Memotong
Intersected line	/ˌɪntə'sektɪd laɪn/	Garis berpotongan
Point of intersection	/pɔɪnt əv ˌɪntə'sekʃn/	Titikpotong
Orthogonal	/ˌɔ:θə'gənəl/	Tegaklurus
Perpendicular	/pɜ:pən'dɪkjʊlə(r)/	Tegaklurus
Line segment	/laɪn 'segmənt/	Ruasgaris
Endpoint	/endpɔɪnt/	Titik ujung
Ray	/reɪ/	Sinar
Angle	/'æŋgl/	Sudut
Vertex	/vɜ:teks/	Titiksudut
Vertices	/vɜ:tɪsi:z/	Titiksudut-titiksudut
Acute angle	/ə'kju:t 'æŋgl/	Sudut lancip
Right angle	/raɪt 'æŋgl/	Sudut siku-siku
Obtuse angle	/əb'tju:s 'æŋgl/	Sudut tumpul
Straight angle	/streɪt 'æŋgl/	Sudut lurus
Full angle	/fʊl 'æŋgl/	Sudut penuh
Complementary angle	/ˌkɒmplɪ'mentrɪ 'æŋgl/	Sudut berpenyiku
Supplementary angle	/'sʌplɪ'mentrɪ 'æŋgl/	Sudut berpelurus
Adjacent angle	/ə'dʒeɪsnt 'æŋgl/	Sudut bersebelahan
Corresponding angle	/ˌkɒrɪ'spɒnding 'æŋgl/	Sudut sehadap
Opposite angle	/ɒpəzɪt 'æŋgl/	Sudut bertolakbelakang
Alternate interior angle	/ɔ:l'tɜ:nət ɪn'tɪərɪə(r) æŋgl/	Sudut dalam berseberangan
Alternate exterior angle	/ɔ:l'tɜ:nət ɪk'stɪərɪə(r) 'æŋgl/	Sudut luar berseberangan
Opposite-interior angle	/ɒpəzɪt ɪn'tɪərɪə(r)	Sudut dalam sepihak

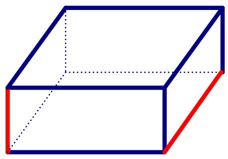
	'æŋgl/	
Opposite-exterior angle	/ɒpəzɪt ɪk'stɪərɪə(r) 'æŋgl/	Sudut luar sepihak
Polygon	/'pɒlɪgən/	Segibanyak
Triangle	/'traɪæŋgl/	Segitiga
Quadrilateral	/ˌkwɒdrɪ'lætərəl/	Segiempat
Pentagon	/'pentəgən/	Segilima
Hexagon	/'heksəgən/	Segienam
Heptagon	/'heptəgən/	Segitujuh
Octagon	/'ɒktəgən/	Segidelapan
Nonagon	/'nəʊnəgən/	Segisembilan
n-gon	/en' gən/	Segi n
Side	/saɪd/	Sisi
Interior angle	/ɪn'tɪərɪə(r) æŋgl/	Sudut dalam
Isosceles triangle	/aɪsɒsəli:z 'traɪæŋgl/	Segitiga samakaki
Equilateral triangle	/ˌikwɪ'lætərəl 'traɪæŋgl/	Segitiga samasisi
Right triangle	/raɪt 'traɪæŋgl/	Segitiga siku-siku
Acute triangle	/ə'kju:t 'traɪæŋgl/	Segitiga lancip
Obtuse triangle	/əb'tju:s 'traɪæŋgl/	Segitiga tumpul
Scalene triangle	/'skeili:n 'traɪæŋgl/	Segitiga sembarang
Apex	/'eɪpeks/	Titikpuncak
Base	/beɪs/	Alas
Height	/haɪt/	Tinggi
Area of triangle	/'eərɪə əv 'traɪæŋgl/	Luas segitiga
Perimeter	/pə'rɪmɪtə/	Keliling
Bisect	/bʌɪ'sɛkt/	Membagi dua sama besar
Median	/'mi:dɪən/	Garis berat
Angle bisector	/æŋglb bʌɪ'sɛktə(r)/	Garis bagi
Altitude	/'altɪtju:d/	Garis tinggi
Parallelogram	/ˌparə'lɛləgram/	Jajargenjang
Square	/'skweə/	Persegi

Rectangle	/ˈrɛktæŋgl/	Persegipanjang
Rhombus	/ˈrɒmbəs/	Belahketupat
Kite	/kʌɪt/	Layang-layang
Trapezium	/trəˈpiːzɪəm/	Trapesium siku-siku
Trapezoid	/ˈtrapɪzɔɪd /	Trapesium
Circle	/ˈsəːk(ə)l/	Lingkaran
Center	/ˈsɛntə/	Titikpusat
Radius	/ˈreɪdɪəs/	Jari-jari
Radii	/ˈreɪdiʌɪ/	Jari-jari (jamak)
Circumference	/səˈkʌmfərəns/	Keliling (lingkaran)
Chord	/kɔːd/	Talibusur
Diameter	/dʌɪˈamɪtə/	Diameter
Arc	/ɒːk/	Busur
Segment	/'segmənt/	Tembereng
Sector	/ˈsɛktə/	Juring
Secant	/ˈsiːkənt/	Garis memotong lingkaran
Tangent	/ˈtæŋʒənt/	Garissinggung lingkaran
Internal tangent	/ɪnˈtəːnəl ˈtæŋʒənt/	Garissinggung dalam
External tangent	/ɪkˈstəːnəl ˈtæŋʒənt/	Garisinggung luar
Inscribed	/ɪnˈskrʌɪb/	(segibanyak) Termuat (dalam lingkaran atau segibanyak)
Circumscribed	/ˈsəːkəmskrʌɪb/	(lingkaran) Memuat (segibanyak)

V. Solid/Space Geometry

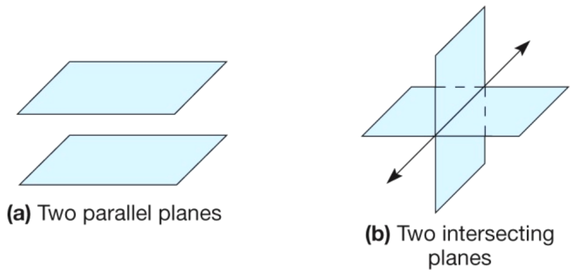
Position of Lines in Space

In plane geometry, two lines can be parallel or intersect. But in space, there is another choice. Two lines can be skew. Skew Lines are nonintersecting lines that are not parallel.



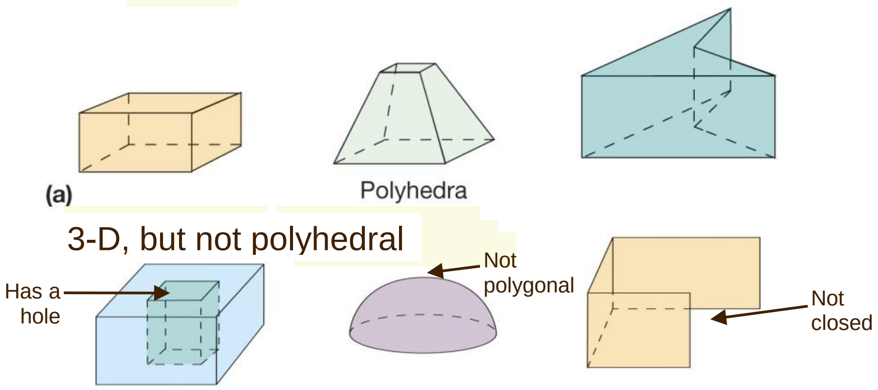
Position of Planes in a Space

In three dimensions, planes are similar to lines in two dimensions. They can be parallel, or intersect.



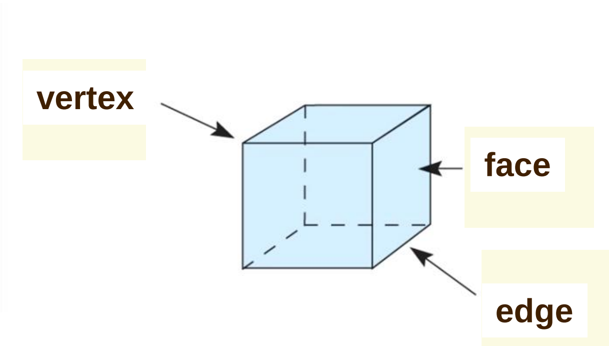
Polyhedra (Bidang Banyak)

A ***polyhedron*** is the union of polygonal regions such that a finite region of space is enclosed without any holes in the interior.



Parts of a Polyhedra

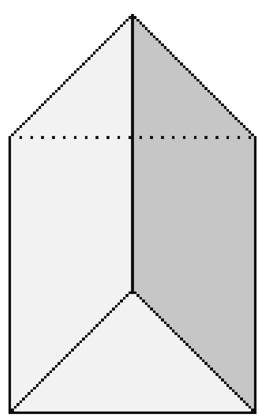
- The polygonal shapes that form the polyhedron are called its ***faces/sides***.
- The line segments where two faces meet are its ***edges***.
- A point where three or more edges meet is a ***vertex***



Types of a Polyhedra

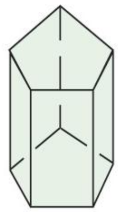
A. Prisms

- If a polyhedron has identical polygonal faces that are opposite each other, then it is a **prism**
- The segments that connect base side and top side is called **side-edges**. The others is called **based-edges**.
- A n-sided prism has n side-edges and 2n base-edges

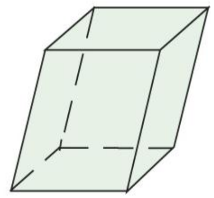


- The opposite, identical side of a prism are called its ***bases***. The other sides are parallelograms and are called the ***lateral sides***.

- If the lateral sides of a prism are rectangles, it is a ***right prism.*** If not, it is called an ***oblique prism***.



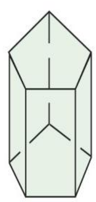
right prism



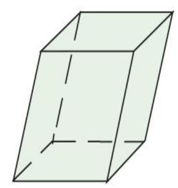
oblique prism

Naming Prism

When naming a prism we use two main descriptors. First, we say whether it is right or oblique, then we say what type of polygon form the prism's bases or the number of its lateral sides.



right pentagonal prism
right 5-sided prism



oblique rectangular prism
oblique 4-sided prism

Formula for Volume and Surface Area of a Prism

Generally, the formula for volume of prism is

V = Area of its base × its height

The height in this formula is the real height not the slant height.

General formula for its surface area is

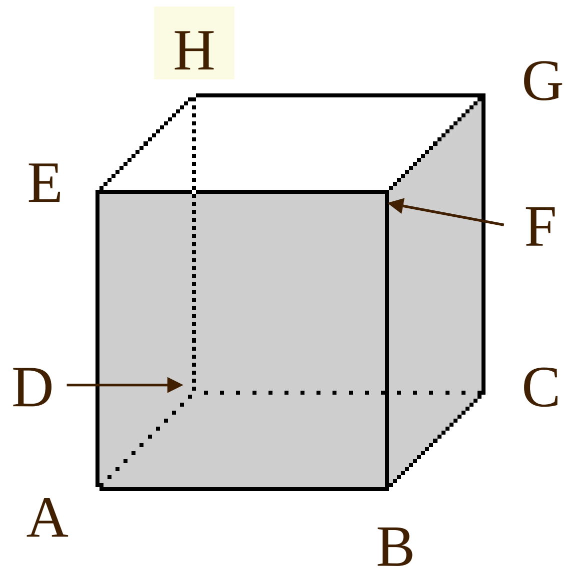
A = sum of all its sides area

Special Kinds of (Rectangular) Prism

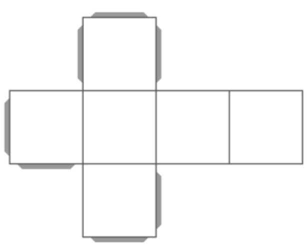
1. Cube

Parts of a Cube

- 6 flat sides of equal squares
  (ex: side ABCD)
- 12 edges of equal lengths
  (ex: edge AB)
- 8 vertices (ex: point A)
- 12 face-diagonals
  (ex: segment line AF)
- 4 space-diagonals
  (ex: segment line AG)
- 6 diagonal planes
  (ex: side ABGH)



Net of a Cube



Formula for Volume and Surface Area of a Cube

The formula for volume of cube is

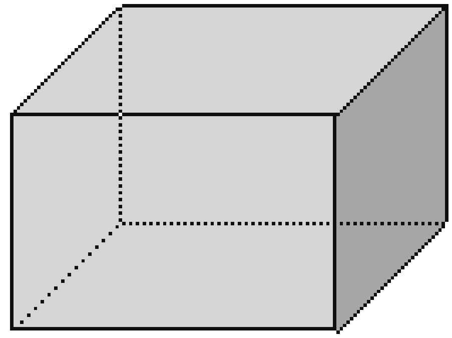
$$V = (\text{length of edge})^3$$

And for surface area is

$$A = 6\times(\text{length of edge})^2$$

2. Cuboid (Rectangular Prism)

- A rectangular prism has 8 rectangular solid angles, 12 edges, equal and parallel in fours.
- It is bounded by three pairs of congruent rectangles lying in parallel planes.



Formula for Volume and Surface Area of a Cuboid

Suppose that we can name the three different edges of cuboid as length, width, and height. Then, the formula for volume of cuboid is

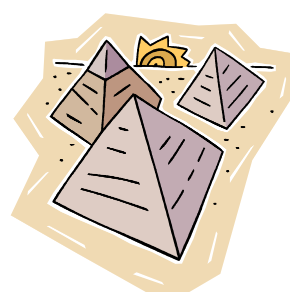
$V = \text{length} \times \text{width} \times \text{height}$

And for surface area is

$A = 2\times[(\text{length} \times \text{width}) + (\text{length} \times \text{height}) + (\text{width} \times \text{height})]$

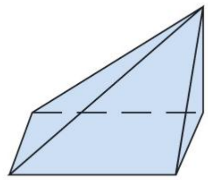
Pyramid

- A **pyramid** is a three-dimensional solid with one polygonal base and with line segments connecting the vertices of the base to a single point somewhere above the base.

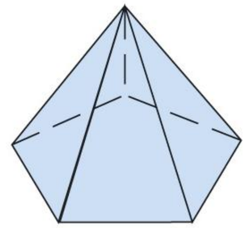


Naming Pyramid

- The lateral sides of a pyramid are triangles. If they are isosceles triangles, then it is a ***right pyramid***, otherwise it is an ***oblique pyramid***.



Oblique square pyramid



Right pentagonal pyramid

Formula for Volume and Surface Area of a Pyramid

Generally, the formula for volume of prism is

$$V = (1/3) \times \text{Area of its base} \times \text{its height}$$

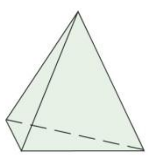
The height in this formula also means the real height not the slant height.

General formula for its surface area also like prism which is

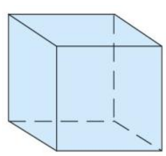
$$A = \text{sum of all its sides area}$$

The Platonic Solids

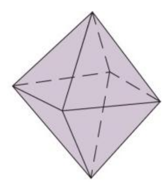
Only five possible regular polyhedra exist. These five solids are called PLATONIC SOLIDS



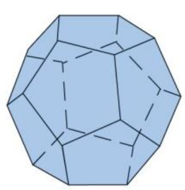
The tetrahedron
--4 triangular



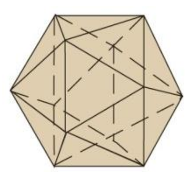
The cube
--6 square



The octahedron
--8 triangular



The dodecahedron
--12 hexagonal



The icosahedron
--20 triangular

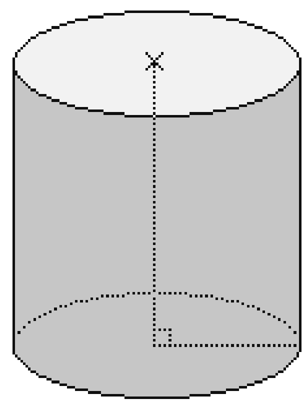
Regular Polyhedron	Number of Sides	Each Side is a	Number of Polygons at a vertex
Tetrahedron	4	Triangle	3
Octahedron	8	Triangle	4
Icosahedron	20	Triangle	5
Hexahedron	6	Square	3
Dodecahedron	12	Pentagon	3

Cylinder

- A **cylinder** is a prism in which the bases are circles or ellips.
- The volume of a cylinder is the area of its base times its height $V = \pi r^2 h$
- The surface area of a cylinder is $A = 2\pi r^2 + 2\pi rh$

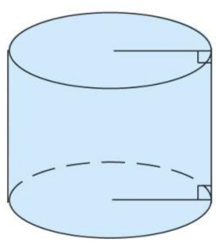
r = length of base's radius

h = height of cylinder



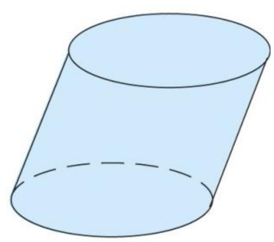
Types of Cylinder

- A cylinder can be right or oblique, and cylinders are named in the same way as prisms and pyramids



A Right Circular Cylinder

The base is a circle



An Oblique Elliptical Cylinder

The base is an ellips

Cone

- A **cone** is like a pyramid but with a circular base instead of a polygonal base.
- The volume of a cone is one-third the area of its base times its height:

  $V = \frac{1}{3}\pi r^2 h$

- The surface area of a cone is base surface area + curved surface area:

  $A = \pi r^2 + \pi rs$

  or

  $A = \pi r^2 + \pi r\sqrt{r^2 + h^2}$

  r = length of base's radius

  h = heigth of cone

  s = slant height of cone

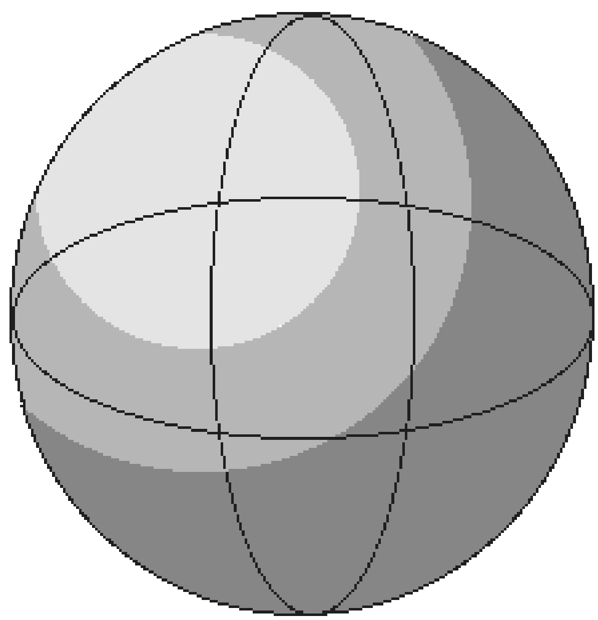
Sphere

- **Sphere** is the mathematical word for "ball." It is the set of all points in space a fixed distance from a given point called the center of the sphere.
- A sphere has a radius and diameter, just like a circle does.
- The volume of a sphere is:

  $V = \frac{4}{3}\pi r^3$

- The surface area of a sphere is:

  $A = 4\pi r^2$



Exercises

A. Answers these questions

1. Intersection of the wall in a class can be described as____________.
2. Two planes in a space can _______________ or _________________, but can't ___________each other.
3. A _____________________ have six vertices, six sides, and __________ edges.
4. If a cuboid has edges whose lengths are 8, 6, and 5 cm, then its surface area is _______________.
5. A cylindrical oil tank can be filled with 7,700 liter gasoline. If its base's radial is 70 cm, then find its height.
6. Draw a net of regular square pyramid and a cone.
7. A prism is inscribed in a cube. The top of the pyramid is at the center of the top side of cube, while its base is the base side of cube. If the cube has edges of 9 cm, then volume of the pyramid is _____________.
8. A three dimensional object that has 8 sides, 12 vertices, and 18 _______ is called ____________________.
9. A cone is inscribed in a cylinder. Their base side is equal, while the top of the cone is placed in the center of the top of cylinder. If the cylinder has radii of 7 cm and height of 24 cm, find the curved surface area of the cone.
10. What is the diameter of the sphere inscribed in a cube that is inscribed in a sphere of diameter 10?
11. The total surface area of a cube, expressed in square centimeters, is equal to the volume of the cube, expressed in cubic centimeters. Compute the length, in centimeters, of a side of the square.
12. Two cylindrical water tank stand side by side. One has radius of 4 meters and contains water to a depth of 12.5 meters. The other has a radius of 3 meters and is empty. Water is pumped from the first tank to the second tank at a rate of 10 cubic meters per minute. How long must the pump run before the depth of the water is the same in both tanks?
13. The pyramid ABCDE has a square base and all four triangular faces are equilateral. The measure of the angle ABD is ___________.

B. *Plan a presentation about one of three-dimensional objects that contains a description of the object, its main parts, the formula for its volume and surface area, and a problem with solution related to the object.*

Vocabularies of Chapter V

Words	**Pronunciation**	**Indonesian**
Skew Line	/skju: laɪn/	Garis Bersilangan
Polyhedron	/ˌpɒlɪˈhɛdrən/	Bidang Banyak
Polyhedra	/ˌpɒlɪˈhɛdrə/	Bidang Banyak (jamak)
Face	/feɪs/	Sisi
Side	/saɪd/	Sisi
Edge	/ɛdʒ/	Rusuk
Vertex	/vɜ:teks/	Titiksudut
Vertices	/vɜ:tɪsi:z/	Titiksudut-titiksudut
Side-edge	/saɪd ɛdʒ/	Rusuk tegak
Based-edge	/beɪsd ɛdʒ/	Rusuk alas
Base	/beɪs/	Alas
Lateral Side	/ˈlatərəl saɪd/	Sisi tegak
Prism	/ˈprɪz(ə)m/	Prisma
Right Prism	/raɪt ˈprɪz(ə)m/	Prisma tegak
Oblique Prism	/əˈbli:k ˈprɪz(ə)m/	Prisma miring
Volume	/ˈvɒlju:m/	Isi
Surface Area	/ˈsə:fɪs ˈɛ:rɪə/	Luas Permukaan
Curved Surface	/kə:vd ˈsə:fɪs/	Selimut (tabung/kerucut)
Height	/haɪt/	Tinggi
Slant Height	/slɑ:nt haɪt/	Panjang ruas garis Pelukis
Cube	/kju:b/	Kubus
Cuboid	/ˈkju:bɔɪd/	Balok
Face-diagonal	/feɪs dʌɪˈag(ə)n(ə)l/	Diagonal bidang
Space-diagonal	/speɪs dʌɪˈag(ə)n(ə)l/	Diagonal ruang
Diagonal Plane	/dʌɪˈag(ə)n(ə)l pleɪn/	Bidang diagonal
Net	/nɛt/	Jaring-jaring
Pyramid	/ˈpɪrəmɪd/	Limas
Platonic Solid	/pləˈtɒnɪk ˈsɒlɪd/	Bangun padat Plato
Tetrahedron	/ˌtɛtrəˈ hɛdrən/	Tetrahedron
Hexahedron	/ˌhɛksəˈhɛdrən/	Heksahedron
Dodecahedron	/ˌdəʊdɛkəˈhɛdrən/	Dodekahedron
Icosahedron	/ˌʌɪkɒsəˈhɛdrən/	Ikosahedron
Cylinder	/ˈsɪlɪndə/	Tabung
Cone	/kəʊn/	Kerucut
Sphere	/sfɪə/	Bola

VI. Logic and Set

The Logic Vocabularies

Pernyataan	statement/proposition
nilai kebenaran	truth value
Benar	true
Salah	false
tabel kebenaran	truth table
Negasi	negation
pernyataan majemuk	compound statement
Konjungsi	conjunction
Disjungsi	disjunction
Implikasi	impication
Anteseden	antecedent
Konsekuen	consequent
Biimplikasi	biimplication
Deduksi	deduction
Argumen	argument
Premis	premise
konklusi/kesimpulan	conclusion

Practice
Fill the blank with the right words or answer the questions.

1. ____________ is a sentence that is either __________ or false.
2. If a statement is true, then its ____________ is false.
3. A compound statement that use the word "or" is called _____________
4. A compound statement that is true only when statements that make it are true is called ________________________
5. If the antecedent is true, and the _________________ is false, then an _________ is false.
6. What is the name of compound statement which is true if the two statements that combined have the same truth value?
7. Given two statements P and Q. P is true. What can you say about the truth value of their disjunction and implication?
8. An _______________ contains ____________ and a conclusion.

Symbols in Logic

$\forall$	*For all/for every*
$\exists$	*There exists/For some*
$\ni$	*Such that*
$\neg$ A	*The negation of A/not A*
A $\wedge$ B	*The conjunction of A and B/A and B*
A $\vee$ B	*The disjunction of A and B/A or B*
P $\Rightarrow$	*If P, then Q/P implies Q*

Q	*P is a sufficient condition for Q* *Q is a necessary condition for P*
$A \Leftrightarrow B$	*A if and only if B* *A is equivalent to B*

Practice

Read out the following sentences:

1. $S \Rightarrow (H \wedge U)$
2. $(S \Rightarrow H) \wedge U$
3. $((N \vee G) \wedge (\neg N) \Rightarrow (G \Rightarrow N)$
4. $(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Leftrightarrow (P \Rightarrow R)$
5. $\forall x,\ Ax \Rightarrow Mx$
6. $\exists x,\ \neg Cx \vee Dx$

Set

Set is collection of objects which have same or equal characteristic.

The objects in a set are called elements or members of the set

$t \in A$	*t is an element of A* *t is a member of A* *t belongs to A*
$u \notin A$	*u is not an element of A* *u is not a member of A* *u does not belong to A*

Special Kind of Set

- **Empty set** is a set which has no member.
- **Power set** of set S is collection of all subsets of the set S.

Cardinality of Set

- The number of distinct objects in a set is called the **cardinality** of the set.
- The cardinality of empty set is zero.
- If a set has finite member, we called the set **finite set**. Otherwise, we called the set **infinite set**.

Comparing two sets

- $A = B$ *A is equal to B*
- $A \subset B$ *A is subset of B*
  A is contained in B
- $A \subseteq B$ *A is subset of or equal to B*
- $A \supset B$ *A is superset of B*
  A contains B

Set operation

- $A \cup B$ *The union of A and B*

		A cup/joint B
➢	$A \cap B$	*The intersection of A and B*
		A cap/meet B
➢	$A \setminus B$	*The difference between A and B*
		A minus B
➢	$A \times B$	*The Cartesian product of A and B*
		A cross B

Exercise

Let $A = \{a,b,c\}$. What can you tell about:

1. a and c
2. f
3. $\{b,c\}$
4. $\{\,\}$
5. 8

How do we say these mathematical terms?

1. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$
2. $A \subseteq (B \setminus C) \cap E$
3. $A = A \setminus (A \setminus B)$
4. $A = (A \setminus B) \cup (A \cap B)$
5. $(D \setminus E) \cap (D \cap E) = \varnothing$

VII. Probability and Statistics

Experiment

A situation whose results depend on chance.

Examples:

- rolling a dice,
- flipping a coin,
- choosing a card from a deck,
- selecting a domino piece.

Events and Sample Space

- Event is the result of an experiment that can, but need not occur.
- Sample space is the set of all events

  Example:

  In one flipping coin experiment, the sample space is {head, tail}

Measures of Central Tendency

- **Mode**

  the value that occurs most often in the data
- **Mean or the average**

  is the sum of the observations divided by the number of observations.
- **Median**

  is the middle number (when data is odd) or average of two middle numbers (when data is even) when data are arranged from the smallest to the largest.

Measure of Dispersion

- ranges: difference between greatest and smallest data
- variance and standard of deviation
- quartiles, deciles, and percentiles
- interquartile: difference between third and first quartiles.

Grouped Data

Range of grouped data = Midpoint of highest class - Midpoint of lowest class

$$\text{Lower boundary of a class interval} = \frac{\text{Upper limit of the class before it + Lower limit of the class}}{2}$$

$$\text{Upper boundary of a class interval} = \frac{\text{Upper limit of the class + Lower limit of the class after it}}{2}$$

Size of class interval = Upper boundary - Lower boundary

$$\text{Size of class interval} = \frac{\text{Range of data}}{\text{Number of classses}}$$

$$\text{Midpoint of a class} = \frac{\text{Lower Limit + Upper Limit}}{2}$$

$$\text{Mean} = \frac{\text{Sum of (Midpoint} \times \text{Frequency)}}{\text{Sum of frequencies}}$$

Modal class = Class interval with the highest frequency

Example of a grouped data

Marks	Frequency	Cumulative Frequency	Lower Limit	Upper Limit	Lower Boundary	Upper Boundary	Midpoint
41-50	6	6	41	50	40,5	50,5	45,5
51-60	14	20	51	60	50,5	60,5	55,5
61-70	42	62	61	70	60,5	70,5	65,5
71-80	28	90	71	80	70,5	80,5	75,5
81-90	8	98	81	90	80,5	90,5	85,5
91-100	2	100	91	100	90,5	100,5	95,5

There are many types of graph that can be used to show information. It is important that every graph used is appropriate, accurate, and has a title, labels and key.

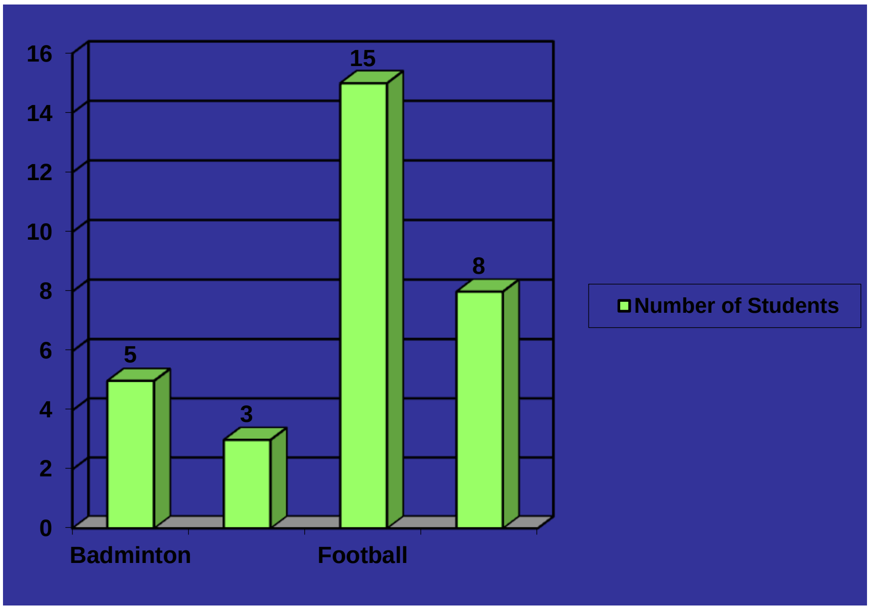
To represent data effectively, we can choose several form of diagrams like: tables, bar and compound bar graphs, histograms, frequency polygons, pie charts, dot plots, line and broken line graphs, or stem and leaf plots.

- Table

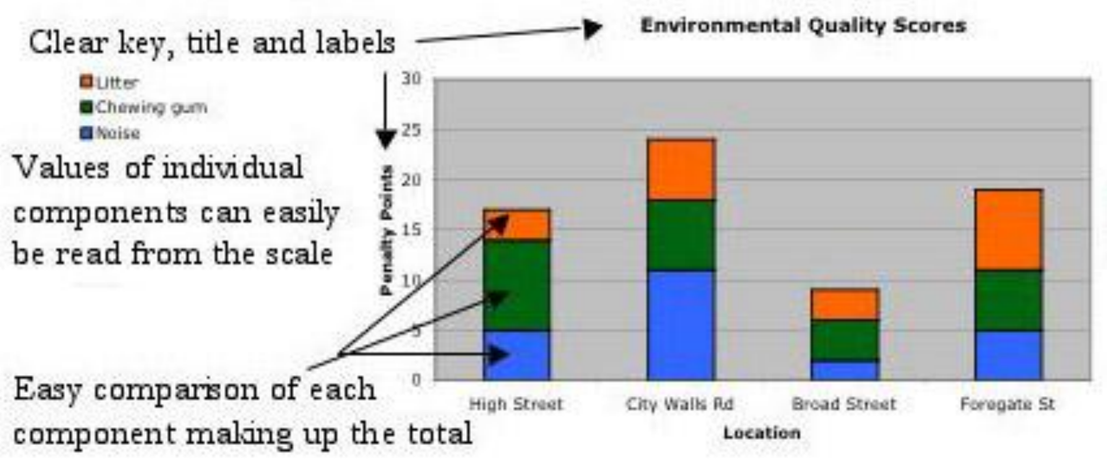
Food stamp participants, people in poverty, and unemployed people, 1980-2002

Year	Food stamp participants[1]	People in poverty[2]	Unemployed people[3]
		Millions	
1980	21.1	29.3	7.6
1981	22.4	31.8	8.3
1982	21.7	34.4	10.7
1983	21.6	35.3	10.7
1984	20.9	33.7	8.5
1985	19.9	33.1	8.3
1986	19.4	32.4	8.2
1987	19.1	32.2	7.4
1988	18.6	31.7	6.7
1989	18.8	31.5	6.5
1990	20.1	33.6	7.0
1991	22.6	35.7	8.6
1992	25.4	38.0	9.6
1993	27.0	39.3	8.9
1994	27.5	38.1	8.0
1995	26.6	36.4	7.4
1996	25.5	36.5	7.2
1997	22.9	35.6	6.7
1998	19.8	34.5	6.2
1999	18.2	32.3	5.9
2000	17.2	31.1	5.7
2001	17.3	32.9	6.7
2002	19.1	--	8.4

Note: Fiscal 2002 data for FSP participation are preliminary. Poverty measures are based on questions about income in the previous year, so they are not yet available for 2002.
1/ Source: Food and Nutrition Service, USDA. Data as of April 24, 2003. Data refer to the average monthly number of participants during the fiscal year.
2/ Source: U.S. Bureau of Commerce, Census Bureau. Data refer to the number of people in poverty during the calendar year.
3/ Source: U.S. Department of Labor, Bureau of Labor Statistics. Data refer to the average monthly number of unemployed people during the calendar year.

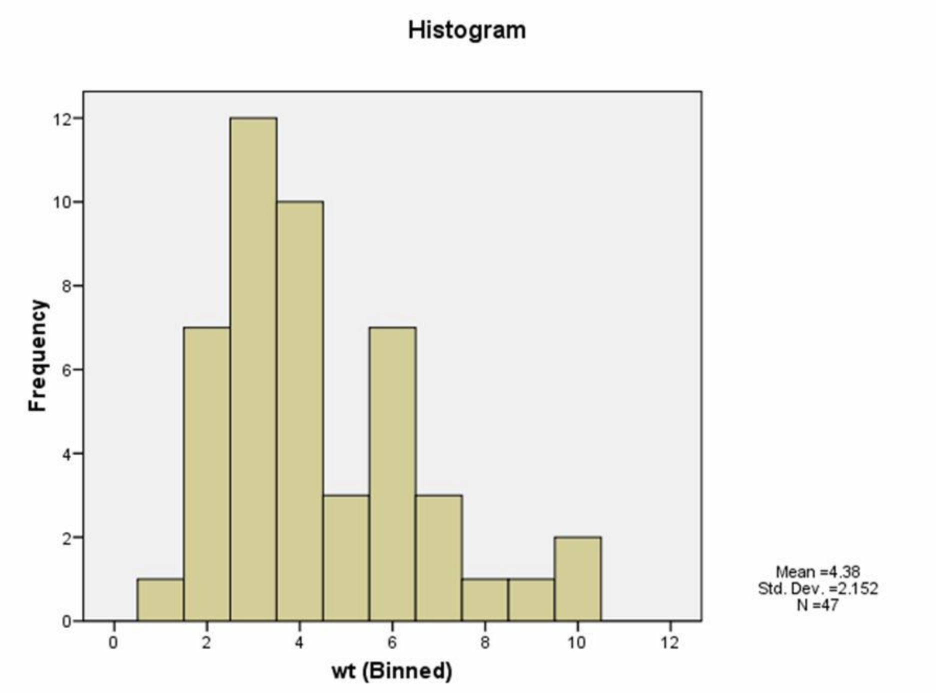


- Simple Bar Graphs
- Compound Bar Graphs



➢ Histograms

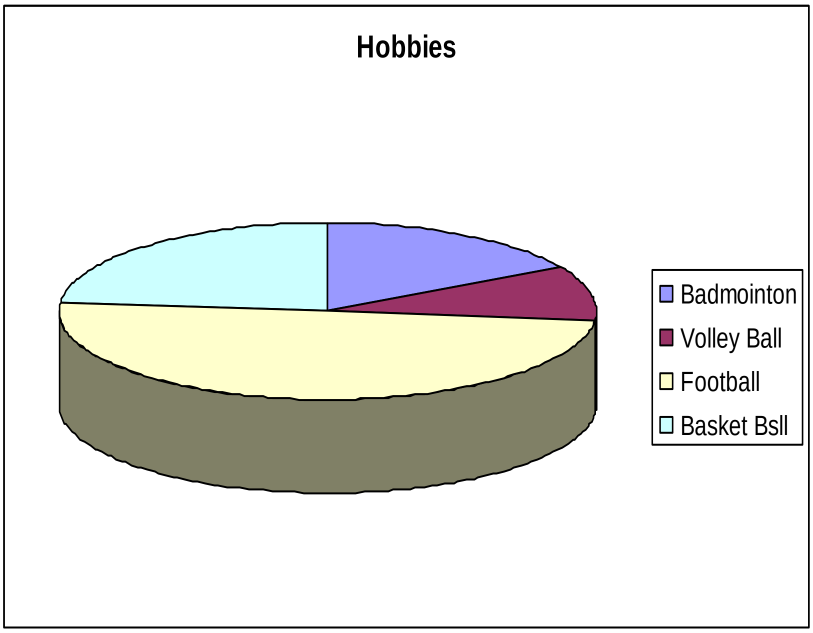
Histograms are similar to bar charts but a **continuous** scale divided into groups or "**classes**" is used (x axis). Vertical axis shows the frequency of each class



➢ Pie Charts

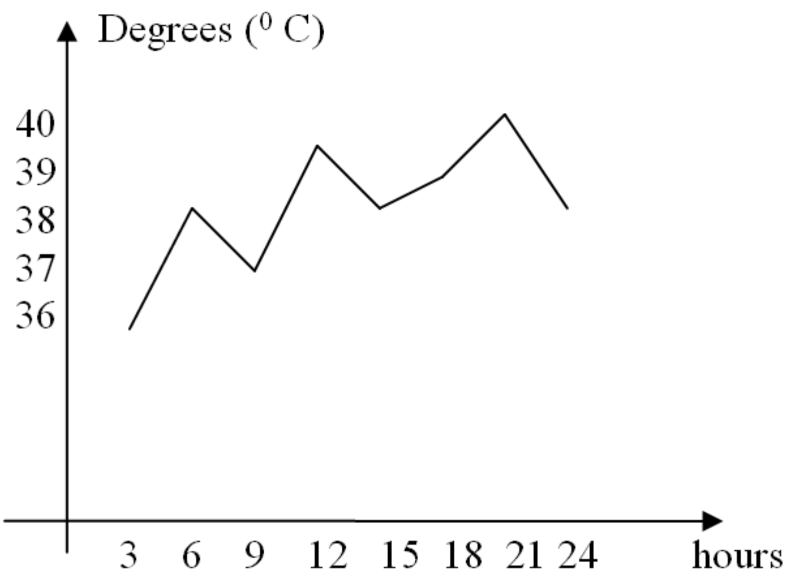
The pie chart is useful to show the total data divided into proportions. It often has good visual impact but can it is difficult to read the data accurately, particularly if there are several categories.

The segments should be drawn from the largest first and the smallest last unless there is an "others" category in which case that should be last regardless of its size. Segments should be shaded in different colours and a suitable key or labels added. The raw data and percentage figures can be added to the key if appropriate.



- Line Graphs

Line graphs show **changes** over time. All the points are joined up and the axes should normally begin at zero. Rates of change are shown well, although careful thought to the scale should be given. Unsuitable if there are only a few data points.



- Stem-and-leaf Plot

  The heights of 11 badminton players are (in inches): 56, 57, 58, 58, 59, 59, 60, 61, 61, 61, 63

HEIGHT	IN INCHES
Stem	Leaves
5 6	6, 7, 8, 8, 9, 9 0, 1, 1, 1, 3

Probability

- Probability of an Event is number of outcomes in an event divide by number of outcomes in sample space

$$P(E) = \frac{n(E)}{n(S)}$$

Some properties of probability

- **P(E)=1** means the probability of an event that is certain to happen.

  So, P(S) = 1
- **P(E)=0** means the probability of an event that is certain not to happen
- **$0 \le P(E) \le 1$**
- **$P(A^C)=1-P(A)$**
- **$P(A \cup B) = P(A) + P(B) - P(A \cap B)$**

  Probability of A or B equals probability of A plus probability of B minus probability of A and B.
- Two events are said to be **independent** of the happening of one event has <u>no effect</u> on the happening of the other.

  Example: Choosing an apple and an egg. Let C be the event that a rotten apple is chosen.Let D be the event that a rotten egg is chosen. Then C and D are independent events.
- If there are 2 *independent events* A and B, we can calculate <u>the probability of A and B</u> by applying to Multiplication/Product Law:

  $P(A \cap B)= P(A)\ P(B)$

- If there are 2 *independent events* A and B, we can calculate the probability of A or B by applying to Addition/Additive Law:

$$P(A \cup B)= P(A) + P(B)$$

Permutations

The product of the n first natural numbers is called n factorial and denoted by n!

$$n! = 1.2.3.4..(n-1).n$$

Example

How many three-digit numbers that can be formed using the digits 1,2, and 4 if no digit is repeated.

Answer

3 digits can be placed for the unit digit.

2 digit left for the tens digit.

And there has been 1 remaining digit for the hundreds digit.

Thus, 1.2.3 = 6 numbers that can be formed

Permutation is number of arrangements of a number of discrete objects when order matters.

There are two kinds of permutation

a. of r different items

$$r!$$

b. of r items selected from n

$$_nP_r = P(n,r) = \frac{n!}{(n-r)!}$$

Example

Find the number of permutation of the letters in "mathematics".

Combination is number of possible arrangements when order is not important.

Formula for combination of r items from n where all are different or distinguishable.

$$_nC_r = C(n,r) = \frac{n!}{r!(n-r)!}$$

Example

List all of the three digit combinations that can be formed from the number 2,3,4,5,6

Exercise

1. Give an example of an occasion when you would use a histogram rather than a bar graph, and say how you would construct it.
2. Two mathematics classes took the same test. The mean of the first class of twenty students was 90 percent. The mean of the second class was 83 percent. If the mean grade for the combined classes was 85 percent, how many students were in the second class?
3. In a football match, team A has a penalty kick. The coach is deciding which player to take that place. It is known that the goalkeeper will defend the left, the central and the right parts with probabilities of 0.3, 0.2 and 0.5 respectively. Find the probability that Ronaldo has a goal if the probability of kicking the ball to the left, central and right part is 0.2, 0.5 and 0.3.
4. After the ace of spades is removed from an ordinary deck of cards, the cards are shuffled and the top card is discarded without its face value known. What is the probability that the next card is an ace?
5. Both the mean and the median of a set of five distinct natural numbers are 7, and the range is 6. What numbers are included in the set?
6. An apple, an orange, a banana, and a pear are laid out in a straight line. The orange is not at either end and is somewhere to the right of the banana. In how many ways can the fruit be laid out ?
7. If the chances of rain are 40 percent and 20 percent for the two days of a weekend, what is the chance that it will rain on at least one of the two days?
8. A social club contains seven woman and four men. The club wants to select a committee of three members the represent it at state convention. How many of the possible committees that could be chosen contain at least one man?

9. A student has three mathematics books, two English books, and four science books. The number of ways that the books can be arranged on a shelf, if all books of the same subject are kept together, is:
10. Six peoples want to get in line in front of teller, and two people don't want to stand near each other. How much queuing patterns can be formed?

Vocabularies of Chapter VII

Words	**Pronunciations**	**Indonesians**
bar	/ba:(r)/	Batang
boundary	/'baʊndrɪ/	Batas
chart	/tʃɑ:t/	Tabel/diagram
coin	/kɔɪn/	Koin
combination	/ˌkɒmbɪ'neɪʃn/	Kombinasi
deviation	/ˌdi:vɪ'eɪʃn/	Simpangan/Deviasi
diagram	/'daɪəgræm/	Diagram
dice	/daɪs/	Dadu
event	/ɪ'vənt/	Kejadian
experiment	/ɪk'sperɪmənt/	Percobaan/Eksperimen
factorial	/fæk'tɔːrɪəl/	Faktorial
frequency	/'fri:kwənsɪ/	Frekuensi
graph	/'gra:f/	Grafik
histogram	/'hɪstəgræm/	Histogram
leaf	/li:f/	Daun
mean	/mi:n/	Rata-rata
median	/'mɪ:dɪən/	Median/nilai tengah
mode	/məʊd/	Modus
mutually exclusive	/'mju:tʃʊəlli ɪk'sklu:sɪv/	Saling lepas
outcome	/'aʊtkʌm/	Hasil percobaan
permutation	/ˌpɜmju:'teɪʃn/	Permutasi
pie	/paɪ/	Lingkaran
plot	/plɒt/	Plot
probability	/prɒbə'bɪlətɪ/	Peluang
quartile	/kwɔ:taɪl/	Kuartil

range	/reɪndʒ/	Jangkauan
sample	/'sa:mpl/	Sampel
standard	/'stændəd/	Standar/Baku
statistics	/stə'tɪstɪks/	Stastik
stem	/sti:m/	Dahan/batang
variance	/'veərɪəns/	Variansi

VIII. Calculus

Three Big Calculus Topics

- Limits
- Derivatives
- Integrals

Limit

Given real numbers c and L. If the values *f*(x) of a function approach or equal L as the values of x approach (but do not equal) c, then *f* has a **limit** L as x approaches c.

How to say Limits

$\lim_{x\to\infty} f(x)$	The limit of f x as x approaches infinity The limit as x goes/tends to infinity of f x
$\lim_{x\to a^+} f(x)$	The limit of f x as x approaches a from above/right The right-hand limit of f x
$\lim_{x\to a^-} f(x)$	The limit of f x as x approaches a from below The left-hand limit of f x

Examples:

Determine $\lim_{x\to 10} \frac{x^2-100}{x-10}$

Answer

Step 1 : Simplify the expression.

The numerator can be factorised.

$$\frac{x^2-100}{x-10} = \frac{(x-10)(x+10)}{x-10}$$

Step 2 : Cancel all common terms

x − 10 can be cancelled from the numerator and denominator.

$$\frac{(x-10)(x+10)}{x-10} = x+10$$

Step 3 : Let x → 10 and write final answer

$$\lim_{x\to 10}\frac{x^2-100}{x-10} = \lim_{x\to 10} x+10 = 20$$

Exercise

Find the limit of the following

$$\lim_{x \to 3} \frac{x^2 - 9}{x + 3}$$

$$\lim_{x \to 3} \frac{x + 3}{x^2 + 3x}$$

$$\lim_{x \to 2} \frac{3x^2 - 4x}{3 - x}$$

Derivatives

The derivative of f at point a with respect to x is the limit of changing rate of f near point a.

The derivative of a function f at a, denoted by f`(a), is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

A function has a derivative at a point if and only if the function's right-hand and left-hand derivatives exist and are equal. In this case, we say the function is **differentiable**.

If the right-hand limit does not equal to the left-hand limit, then the limit does not exist and we said that f **is not differentiable** at a.

There are much rules about derivative (**rules of differentiation**)

- Factor rule
- Sum rule
- Product rule
- Quotient rule
- Chain rule (for composite function)
- Rule for trigonometry function

Exercise

1. Find the derivative of the function $f(x) = x^2-8x+9$ at a using definition of derivative.
2. If $f(x) = 3x-2x^2$, find $f`(x)$ from definition and hence evaluate $f`(4)$.

How to say Derivatives

$f`(x)$	f prime x/f dash x The (first) derivative of f with respect to x
$f``(x)$	f double-prime x/f double-dash x The second derivative of f with respect to x
$f```(x)$	f triple-prime x/f triple-dash x The third derivative of f with respect to x
$f^{(IV)}x$	f four x/f four prime x the fourth derivative of f with respect to x
$\frac{df}{dx}$	"D F D X" The derivative of f with respect to x
$\frac{d^2f}{dx}$	"D" squared "F D X" squared the second derivative of f with respect to x
$\frac{\partial y}{\partial x}$	"D Y partially by X" The (first) partial derivative of y with respect to x Delta y by delta x
$\frac{\partial^2 y}{\partial x^2}$	"D squared Y partially by X squared" The second partial derivative of y with respect to x Delta two y by delta x squared

There are much rules about derivative

- Factor rule
- Sum rule
- Product rule
- Quotient rule
- Chain rule (for composite function)
- Rule for trigonometry function

Maxima and minima

- Find f'(x) and solve f'(x) = 0. This value of x (say x*) is the **stationary/extreme/critical point**; probably maxima or minima will occur at this point.
- Find f''(x). If f''(x) > 0 then x is a **local minima**. If f''(x) < 0 then x is a **local maxima**. We can say that x is a **maximum/minimum point**.

- If f”(x)=0, then x may be a **point of inflection**.

Example

Michael wants to start a vegetable garden, which he decides to fence off in the shape of a rectangle from the rest of the garden. Michael only has 160 m of fencing, so he decides to use a wall as one border of the vegetable garden. Calculate the width and length of the garden that corresponds to largest possible area that Michael can fence off.

Answer

Step 1 : Examine the problem and formulate the equations that are required

The important pieces of information given are related to the area and modified perimeter of the garden. We know that the area of the garden is:

$$A = W \times L \qquad \text{(Equation 1)}$$

We are also told that the fence covers only 3 sides and the three sides should add up to 160 m. This can be written as:

$$160 = W + L + L$$

However, we can use the last equation to write W in terms of L:

$$W = 160 - 2L \qquad \text{(Equation 2)}$$

Substitute Equation 2 into Equation 1 to get:

$$A = (160 - 2L)L = 160L - 2L^2 \qquad \text{(Equation 3)}$$

Step 2 : Differentiate

Since we are interested in maximizing the area, we differentiate Equation 3 to get:

$$A'(L) = 160 - 4L$$

Step 3 : Find the stationary point

To find the stationary point, we set $A'(L) = 0$ and solve for the value of L that

maximizes the area.

$$A'(L) = 160 - 4L$$
$$0 = 160 - 4L$$
$$4L = 160$$
$$L = 40 \text{ metres}$$

Substitute into Equation 2 to solve for the width.

$$W = 160 - 2L = 160 - 2(40) = 160 - 80 = 80m$$

Step 4 : Write the final answer

A width of 80 m and a length of 40 m will yield the maximal area fenced off.

Exercises

1. The sum of two positive numbers is 20. One of the numbers is multiplied by the square of the other. Find the numbers that make this products a maximum
2. After doing some research, a transport company has determined that the rate at which petrol is consumed by one of its large carriers, travelling at an average speed of x km per hour, is given by P(x) = (55/2x) + (x/200) litres per kilometre.
   i. Assume that the petrol costs Rp.4,000 per litre and the driver earns Rp.18,000 per hour (travelling time). Now deduce that the total cost, C, in Rupiahs, for a 2,000 km trip is given by:

   $$C(x) = (256000)/x + 40x$$

   ii. Hence determine the average speed to be maintained to effect a minimum cost for a 2,000 km trip.

Integral

Integral is could be described as the set of all antiderivatives, because if the derivative of F is f, then the integral of f is F plus a constant.

$$\int f \, dx = F + c$$

The process of finding F is called **integration**, the function f is called **integrand**, and the differential dx indicates that x is **the variable of integration.**

If the **bounds or limits** of integral is given, we said the integral as **definite integral**, but when the integral has no bounds nor limits, we said the integral as **indefinite integral**.

How to Say Integrals

$\int_a^b f(x)\,dx$	The integral of f x from a to b The integral from a to b of f x
$\int_0^\infty$	integral from zero to infinity
$\int f(x)\,dx$	The indefinite integral of f x

There are some technique for integration, such as

- Integration by subtitution
- Integration by parts or partial integration

Tasks

- Write 2 problems and complete solution about calculus. At least, one of the problem is applied problem.
- Next week, the results will be presented by selected students.

Vocabularies of Chapter VIII

Words	Pronunciations	Indonesians
antiderivative	/ˌæntɪdɪ'rɪvətɪv/	
approach	/ə'prəʊtʃ/	
dash	/dæʃ/	
definite	/'defɪnət/	
derivative	/dɪ'rɪvətɪv/	
differential	/ˌdɪfə'renʃl/	
extreme	/ɪk'stri:m/	
function	/'fʌŋkʃn/	
infinity	/ɪn'fɪnətɪ/	
inflection	/ɪn'flekʃn/	
integral	/'ɪntɪgrəl/	
interval	/'ɪntəvl/	
limit	/'lɪmɪt/	
maximize	/'mæksɪmaɪz/	
maximum	/'mæksɪməm/	
monotone	/'mɒnətəʊn/	
prime	/praɪm/	
respect	/'rɪspekt/	
stationary	/'steɪʃənrɪ/	
tend	/tend/	

IX. Sentence Analysis

PART I: Jenis-jenis Kalimat (Sentence Types)

Kalimat dibentuk dari **Klausa** (Clause), yaitu kelompok kata yang memiliki **Subject** (S) dan **Predicate** (P)

Example:

1. **We** entered the room. (**S**,P)
2. **Arif** and **Alvin** chased the ball. (**CS**, P)
3. **Beni** caught the ball and threw it to Ilham. (**S**, CP)
4. The **dean** and the **students** stood and sang the national anthem (**CS**,CP)

Ada dua macam klausa: **Klausa Bebas** (Independent Clause/IC) dan **Klausa Terikat** (Dependent Clause/DC)

IC adalah klausa yang dapat berdiri sendiri (tetap memiliki makna)

Examples:

1. **I** nearly **score a goal**. (with adverb modifier)
2. **The building** near the river **caught fire**. (with phrase modifier)
3. **They bought the book** that sold over a million copy around the world. (with clause modifiers)

DC adalah klausa yang tidak dapat berdiri sendiri (kehilangan makna) dan biasanya mengikuti Kata Penghubung (Conjunction: as, since, because, etc) atau (Relative Pronoun: that, which, who, etc)

Examples:

1. Adjective Clause: menerangkan noun/pronoun
   - This is the **car** that broke the speed record.
   - **Anybody** who is tired may leave.
   - Malaysia is the **nation** (that) we made the treaty with.
2. Adverb Clause: menerangkan verb/adjective/adverb
   - The child **cried** when the dentist appeared.
   - I am **sorry** (that) he is sick.
   - He thinks more **quickly** than you do.

3. Noun clause: berfungsi sebagai noun (subjek, predikat, objek dari verb/kt. depan)
    - <u>What John wants</u> is a better job
    - This is <u>where we came in.</u>
    - Please **tell** him (that) <u>I will be late.</u>
    - He has no interest **in** <u>what he is reading.</u>

Jenis-jenis Kalimat	**IC**	**DC**
Kalimat Sederhana (Simple Sentence)	1	-
Kalimat Majemuk (Compound Sentence)	≥ 2	-
Kalimat Kompleks (Complex Sentence)	1	≥ 1
Kalimat Majemuk Kompleks (Compound-complex Sentence)	≥ 2	≥ 1

<u>Examples</u>:

1. The wind **blew**. (Simple)
2. The wind **blew** <u>and</u> the leaves **fell**. (Compound)
3. <u>When</u> the wind **blew**, the leaves **fell**. (Complex)
4. <u>When</u> the sky **darkened**, the wind **blew** <u>and</u> the leaves **fell**. (Compound-complex)

<u>Catatan</u>:

A. Simple sentence may have two or more subjects and predicates.

1. <u>Addition</u> and <u>scalar multiplication</u> **are** two well-defined operations in vector space.
2. <u>We</u> **left** the class and **went** to library.
3. <u>Bambang</u> and <u>Susilo</u> **watched** the movie and **ate** popcorns.

B. IC joined with a **coordinating conjunction** (and, but, for, or, nor, so, etc) by a **comma** or an **adverb** (then) by **semicolon**.

1. I **like** geometry, <u>but</u> I **like** algebra better.
2. Jarwo **ran** into the house, <u>and</u> he **locked** the door.
3. You **first** read the problem; <u>then</u> you **write** the solution.

Exercise

1. This is the student who got the highest grade in calculus.
2. Algebra is the subject (that) we thought very difficult.
3. Four scholarships have been offered, and I hope that you will apply for one.
4. My cousin has applied for three colleges, and the application that she sent to UNP has just been accepted
5. What Riski wants is a better grade.
6. This is where we came in.
7. He has no interest in what he is reading.

PART II: Pola-pola Kalimat (Sentence Patterns)

1. Subject + Intransitive Verb
2. Subject + Transitive Verb + Direct Object
3. Subject + Linking Verb + Subject Complement
4. Subject + Transitive Verb + Indirect Object + Direct Object
5. Subject + Transitive Verb + Direct Object + Object Complement

Examples

Subject	Intransitive Verb	
The Students	argued	(clearly)
Ani	walks	(to the campus)
Bambang	sat	(quietly in mosque)

Subject	Transitive Verb	Direct Object
The Students	began	final examination
Ani	underlined	the keywords
We	shall discuss	the property of natural numbers

Subject	Linking Verb	Subject Complement	
		Predicate Substantive	Predicate Adjective
Ani	may become	a president	
The winner	is	me	
The perfume	smells		good
The book	seem		obscene
His face	went		white
Bambang	became		very angry

Subject	**Transitive Verb**	**Indirect Object**	**Direct Object**
The lecturer	gave	him	an A
He	showed	us	the proof
The suspect	told	the interrogator	the truth

Subject	**Transitive Verb**	**Direct Object**	**Object Complement**
The senate	elected	him	chairman
We	thought	algebra	difficult
They	made	the lecturer	angry

Dua perubahan dari Pola Dasar

Passive

Subject	**Passive Verb**	**Original Subject**
The solution	is presented	(by Budi)
He	was called dummy	(by Jono)
Victory	was bought to Indonesia	(by Andik)

Expletive

Expletive	**Verb**	**Complement**	**Subject**
It	is		time to leave
It	is	doubtful	that they will arrive
There	are		forty students
There	was		rain yesterday

Appositive

Appositive adalah keterangan tambahan (additional explanation) yang biasanya ditulis diantara dua koma atau dua strip (-). Appositive ini dapat diabaikan karena kehilangannya tidak mempengaruhi arti.

Examples

1. This chapter, <u>an excellent summary of the real number properties</u>, presents some basic concepts in real analysis.
2. The notion of absolute value, <u>which is based on order properties</u>, is discussed later.

PART III: Empat Langkah dalam Menganalisa Kalimat

1. **Menemukan Penghubung (Connector)** jika ada. (sekaligus mengetahui jenis kalimat)
   - Diantara dua IC (sometimes between two DC)
     and, but, for, or, nor, so (following a comma).
     then (following a semicolon).
   - Di depan DC
     as, since, because, when, etc.
     that, which, who, etc.
2. **Pisahkan Klausanya**
   Example:
   - Two sets are said to be disjoint **if** their intersection is empty.

   The connector : if
   IC : Two sets are said to be disjoint.
   DC : Their intersection is empty.
3. **Temukan Predicate**
   Predicate ada dua macam, yaitu: **verbs** dan **linking verb**
   Verbs dapat dikenali melalui:
   - Kemampuannya berubah dari bentuk infinitive ke bentuk:
     berakhiran ***–s***: *I **run** → he **runs***
     berakhiran ***–ed*** *he **walks** → he **walked***
     berakhiran ***–ing*** *she **talks** → she is **talking***
   - Ditambah ***–er*** bisa menjadi pelaku: ***walk**er*, ***receive**r*, ***admire**r*, ***love**r*, ***hang**er*.
   - Ditambah awalan ***dis–*** menjadi negatif: *dis**agree***, *dis**approve***, *dis**miss***, *dis**like***.
   - Karakteristik posisinya setelah kata-kata seperti ***let's***, ***please***, dan ***simple noun***.
     *let's **play**, please **go**, Joko **hit** Kadir*
4. **Temukan Subject**
   Subject biasanya ada di depan Predicate, kecuali dalam bentuk Expletive.

Examples

(1) Four scholarship have been offered, and I hope that you will apply for one

- Connectors : and, that
- Clauses:
  1. Four scholarship have been offered. (IC)
  2. I hope (something) (IC)
  3. You will apply for one. (DC)
- Verbs
  1. have been offered
  2. hope
  3. will apply
- Subjects
  1. Four scholarship
  2. I
  3. You

(2) An angle at the circumference has a unique arc, but an arc subtends infinitely many angles at circumference.

- Connectors : and, that
- Clauses:
  1. an angle at the circumference has a unique arc. (IC)
  2. an arc subtends infinitely many angles at circumference. (IC)
- Verbs
  1. has
  2. subtends
- Subjects
  1. an angle at the circumference
  2. an arc

Exercise

Find the connectors, clauses, subjects, and verbs in these sentences

- Any finite field is of finite characteristic.
- Cardinal and ordinal numbers have developed in close interconection and form the two aspects of natural numbers.

- Points, lines, and planes are the foundation stones of elementary geometry in three-dimensional space.
- The matrices operation is discussed in the last chapter.
- We shall use the notation (a,b) for the greatest common divisor of a and b.
- Sequences and functions are the last concept that we will be studying.
- Every natural number has exactly one immediate successor.
- Our goal is to gain an understanding of the theory and to be able to solve applied problems
- An event is the result of a trial that can, but need not occur.
- Prime numbers are numbers that have only improper divisors.

X. Paragraph Analysis and Mathematical Writing

PART I: Analisis Kalimat (Paragraph Analysis)

Dalam menganalisis paragraf, perlu diingat bahwa

- Suatu paragraf memuat:

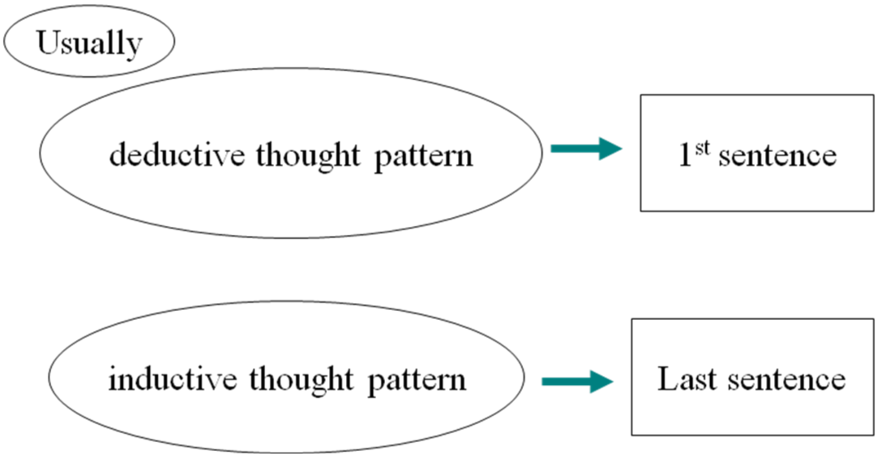
  1 topik (**topic**)

  1 kalimat utama (**main idea/sentence**),

  Dan beberapa kalimat penjelas/pendukung (**subordinate sentences**).

- Kalimat utama biasanya terletak di awal paragraf (**first sentence**), tetapi kadang-kadang juga terletak pada akhir paragraf (**last sentence**) atau di tengah paragraf.

Teks matematika sebagian besar menggunakan pola berfikir deduktif sehingga pada umumnya kalimat utama ada di awal paragraf.



Jadi, langkah dalam analisis kalimat pada teks matematika adalah:

1. Temukan kalimat utama (sebagian besar adalah kalimat pertama).
2. Temukan Subjek (adakalanya Objek) dari kalimat tersebut. Ini adalah topik dari paragraf.

Example

In arithmetic, it is never possible to add <u>unlike quantities</u>. For example, we should not add inches and gallons and expect to obtain a sensible answer. Neither should we attempt to add volts, amperes, farads, ohms, etc. So it goes through algebra – we can never add quantities unless they are expressed in the same units.

Topic: Adding (unlike) quantities

Main Sentence: In arithmetic, it is never possible to add unlike quantities

Example

<u>The study of mathematics</u> may be likened to the study of a language. In fact, mathematics is a language of number and size. Just as the rules of the grammar must be studied in order to master English, so must certain concepts, definitions, rules, terms, and words be learned in the pursuit of mathematical knowledge. These form the vocabulary or structure of the language. The more language is studied and used, the greater becomes the vocabulary; the more mathematics is studied and applied, the greater becomes its usefulness.

Topic: Study of mathematic

Main sentence: The study of mathematics may be likened to the study of a language

<u>Exercise</u>

Find the topic and main sentence of paragraphs below

1. So far, we have described the properties of the real number system relative to addition, multiplication, and order. A system which satisfies all the postulates that we have stated so far is called an ordered field. We repeat the definition of an ordered field as follows
2. Applied mathematics should read like good mystery, with an intriguing beginning, a clever but systematic middle, and a satisfying resolution at the end. Often the resolution of one mystery opens up a whole new problem, and the process starts all over. For the applied scientist, there

is the goal to explain or predict the behavior of some physical situation. One begins by constructing a mathematical model which captures the essential features of the problem without masking its content with overwhelming detail. Then comes the analysis of the model where every possible tool is tried, and some new tools developed. Finally, one must interpret and compare these results with real world facts. Sometimes this comparison is quite satisfactory, but most often one discovers that important features of the problem are not adequately accounted for, and the process begins again.

PART II: Kelompok Pernyataan Matematika (Mathematical Structure)

1. Definitions

Definition is used to state/prescribe the meaning of a word, phrase or expression.

In math, definitions usually use term : is/are, if, if and only if

Example:

- A matrix is a rectangular array of numbers.
- A is contractive with respect to the norm || || if ||A|| < 1.
- An integer n is called even if n is divisible by two.

2. Theorems

a. A theorem is a mathematical statement (major result) that has been proved.
b. An auxiliary result is called lemma.
c. A minor result is called a proposition.
d. A immediate consequences of a previous result is called corollary.
e. A result which has been accepted to be true and need not a proof is called axiom.

Example:

- For all real numbers ***a*, *b*,** the following identity holds

  $$(a+b)(a-b) = a^2 - b^2$$

- A natural number n is divisible by 9 if and only if the sum of it digits is divisible by 9.

3. Proofs

A proof of a mathematical statement is an argument that shows that the statement is correct

Example:

If S=T, then, by definition, S and T have precisely the same elements. In particular, this means that $x \in S$ implies that $x \in T$ and also $x \in T$ implies that $x \in S$. That is, $S \subset T$ and $T \subset S$.

PART III: Menulis Teks Matematika (Mathematical Writing)

Aturan Kecil (Small Rules):

Digunakan untuk kalimat:

Pemisahan, Gaya Penulisan, Aturan Simbol, Penggunaan Singkatan, dan Pemilihan “that” atau “which”.

A. Pemisahan (Breaking up)

Aturan 2-3-4 (2-3-4 rule)

Pertimbangkan unttuk memisahkan

1. Setiap kalimat yang melebihi 2 baris
2. Setiap kalimat yang memiliki lebih dari 3 verb
3. Setiap paragraf yang memiliki lebih dari 4 kalimat “panjang”

Example

- If $\Delta = b^2 - 4ac \geq 0$, then the roots are real. (bad)
- Set $\Delta = b^2 - 4ac$. If $\Delta \geq 0$, then the roots are real. (good)

B. Gaya Penulisan (Narrative Style)

1. Lebih baik menulis kalimat aktif.

   example:

   - Gaussian elimination gave the answer to sixteen decimal places. (active)
   - The answer was given to sixteen decimal places by Gaussian elimination. (passive)

2. Kata “we” (“penulis dan pembaca bersama-sama”) berguna untuk menghindari penggunaan kalimat pasif. Hindari penggunaan “I”.

   example:

   - We solve the equation to get x = 0. (good)
   - The equation was solved to get x = 0. (bad)
   - I solved the equation and got x = 0. (bad)

3. Penggunaan Present Tense selalu OK

   Kalau ragu, pilihlah present tense.

   example:

   - This is proved in Theorem 3 below. (present)

     This will be proved in Theorem 3 below. (future)
   - Banach shows that the conclussion is correct. (present)

     Banach showed that the conclussion is correct. (past)
   - We have proved that f(x) was equal to g(x). (bad)

     We have proved that f(x) is equal to g(x). (good)

C. Aturan Penulisan Kalimat yang Memuat Simbol Matematika.

1. Pisahkan simbol-simbol dari rumus berbeda dengan kata-kata.

   example:

   - Consider x_k, k = 1, .., n. (bad)
   - Consider x_k for k = 1, .., n. (good)

2. Jangan memulai kalimat dengan simbol.

   example:

   - $x^n - a$ has n distinct zeroes. (bad)

     The polynomial $x^n - a$ has n distinct zeroes. (good)
   - f is a continuous function. (bad)

     The function f is continuous. (good)
   - m is the negative reciprocal of -3/4. (bad)

     The slope m is the negative reciprocal of -3/4. (good)

3. Jangan gunakan simbol-simbol $\therefore, \Rightarrow, \Leftrightarrow, \forall, \exists, \ni, \in$ kecuali dalam rumus logika. Gantilah dengan kata yang sesuai (that is/therefore, then/implies, if and only if, for all, there exist, such that, elemen/in, etc).

example:

- ◉ $\therefore x \in \mathbf{R} \Rightarrow \exists n \in \mathbf{N} \ni n > x.$ (bad)
- ◉ Therefore, if x is a real number, then there is a natural number n such that n > x. (good)

4. Gunakan kalimat lengkap sebelum definisi atau teorema.

example:

- ◉ We now have the following.
  Theorem 1. H(x) is continuous. (bad)
- ◉ We can now prove the following result.
  Theorem 1. The function H(x) defined by formula (2.1) is continuous. (good)

5. Bilangan yang kecil ditulis secaa lengkap, kecuali sebagai nama.

example:

- ◉ Method 2 recquires <u>2</u> steps. (bad)
- ◉ Method 2 recquires <u>two</u> steps. (good)

6. Nama ditulis dalam huruf besar.

example:

Theorem 1, Lemma 2, Definition 3, Equation 4.

D. Penggunaan Singkatan secara Tepat

1. Yang diterima

i.e	id est	that is (to say) in other words
e.g.	exempli gratia	for example
etc.	et cetera	and so forth and the rest

Example:

- ◉ However, the converse is not true; i.e., bounded sequences are not necessarily convergent.
- ◉ If a function is Riemann-integrable, e.g. if it is continuous on a closed finite interval, then the integral is the limit of the Riemann sums.

2. Yang tidak diterima

B

Singkatan berikut tidak formal, hanya bisa digunakan pada penulisan di kertas "buram".

- iff — if and only if
- s.t. — such that
- w.r.t. — with respect to
- w.l.o.g. — without loss of generality

E. Penggunaan "that" atau "which"

1. Kata "assume" dan "suppose" sebaiknya diikuti oleh kata "that"

<u>example</u>:

- Assume A is a group. (bad)
- Assume that A is a group. (good)

But, never write

- We have that A = B. (bad)
- We have A = B. (good)

2. Gunakan "which" jika didahului oleh koma atau kata depan, atau secara interogatif.
3. Bereksperimenlah dengan bunyi untuk mendapatkan yang lebih enak didengar.

<u>Example</u>:

- Don't use symbols which aren't necessary.
- Don't use symbols that aren't necessary.

The last is better.

<u>Exercise</u>

Write a math problem with solution that follows the rules of mathematical writing.

<u>Exercise</u>

- Find two definitions and two theorems, and try to analyze the sentence; then try to translate it.
- Translate the next two texts using paragraph ad sentence analysis.

First Text.

Egyptian fractions

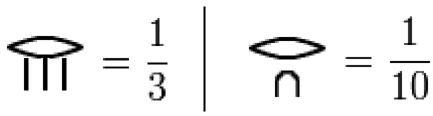
Fractions and the notation to represent them were developed in the Middle Kingdom of Egypt, (between 2055 BC and 1650 BC). Five early texts in which fractions appear were: the Egyptian Mathematical Leather Roll, the Moscow Mathematical Papyrus, the Reisner Papyrus, the Kahun Papyrus and the Akhmim Wooden Tablet. A later text, the Rhind Mathematical Papyrus, introduced improved ways of writing Egyptian fractions.

The Rhind papyrus was written by A'h'mes and dates from the Second Intermediate Period (1650–1550 BC) ; it includes a table of Egyptian fraction expansions for rational numbers 2/*n*, as well as 84 word problems.

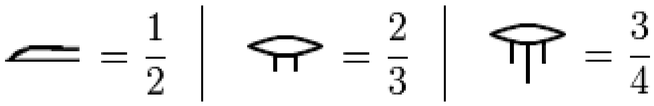
1. Notation

To write the unit fractions used in their Egyptian fraction notation, in hieroglyph script, the Egyptians placed the hieroglyph

(translation: " one among" or possibly "re", mouth) above a number to represent the reciprocal of that number. Similarly in hieratic script they drew a line over the letter representing the number. For example:



The Egyptians had special symbols for 1/2, 2/3, and ¾ that were used to reduce fractions larger that ½.



After subtracting one of these special fractions, the remaining number was written using as a sum of distinct unit fractions.

The reason of this representation is not clear; certainly it's not because it makes the calculation with fractions simpler.

But it is easier to compare two Egyptian fractions than it is to compare two regular fractions because unit fractions compare easily.

Egyptian fraction notation continued to be used in Greek times and into the Middle Ages despite complaints by Ptolemy (AD 90 – AD 168)) about the clumsiness of the notation compared to alternatives such as the Babylonian base-60 notation.

2. Egyptian fraction

In number theory, an **Egyptian fraction** is the sum of **distinct** unit fractions, such as $\frac{1}{2}+\frac{1}{3}+\frac{1}{16}$. The value of an expression of this type is a positive rational number *a*/*b*; for instance the Egyptian fraction above sums to 43/48.

Sums of this type, and similar sums also including 2/3 and 3/4 as summands, were used as rational numbers by the ancient Egyptians, and continued to be used by other civilizations into medieval times. In modern mathematical notation, Egyptian fractions have been superseded by regular fractions and decimal notation. However, Egyptian fractions continue to be an object of study in modern number theory and recreational mathematics, as well as in modern historical studies of ancient mathematics.

3. Fibonacci's greedy algorithm for Egyptian fractions

An important text of medieval mathematics, the Liber Abaci (1202) of Leonardo of Pisa (more commonly known as Fibonacci), provides some insight into the uses of Egyptian fractions in the Middle Ages, and introduces topics that continue to be important in the number theory. He also provides *a greedy algorithm* to represent every proper reduced fraction (i.e., fractions that are <1 and are such that the denominator and numerator do not have common factors) into an Egyptian fraction. *A greedy algorithm* is an algorithm that makes a locally optimal choice at each stage of the problem solving process with the hope of finding a global optimum. Fibonacci's algorithm for transforming rational numbers into Egyptian fractions represents the first published systematic method for constructing such expansions, and it is described in the Liber Abaci (1202).

The general principle behind the algorithm it is the following. At each step we fix d, and we chose the expansion

$$\frac{x}{y}=\frac{1}{d}+\frac{xd-y}{yd},$$

We use this algorithm to prove the following

Theorem*: Every proper reduced fraction with numerator n can be written as an Egyptian fraction with <= n terms.*

The proof is by induction**. Suppose first that n=2.** We consider the fraction 2/m, with m>2 odd (if m is even the fraction can be simplified).

We can chose d so that $\boxed{2/m = 1/d + 1/(md);}$ after computing the common denominator, we have the equation 2d=m+1, which gives $\boxed{d= (m+1)/2}$. m is odd, and so m+1 is even.

When n=3, we shall consider two cases:

a) **if the remainder of the division of m by 3 is 2**, then we can chose d so that $\boxed{3/m = 1/d + 1/(md);}$ after computing the common denominator, we have the equation

3d=m+1, which gives $\boxed{d= (m+1)/3}$. This is an integer because by assumption m=3p+2, and so m+1=3p+3 is divisible by 3. In this case, we only need 2 fractions to represent 3/m.

b) **if the remainder of the division of m by 3 is1**, then we can chose d so that $\boxed{3/m = 1/d + 2/(md);}$ after computing the common denominator, we have the equation 3d=m+2, which gives $\boxed{d= (m+2)/3}$. In this case, 3/m= 1/d+2/(md). If either d or m are even, we can simplify the fraction 2/(md). and we are done. If that is not the case, we have shown how to write this fraction as sum of 2 unit fractions. So, 3/m can be represented as the sum of 3 fractions.

The proof for n=3 applies well to prove this proposition for general n's.

We assume that when the denominator is <= n, the proper reduced fractions of the form of n/m can be written as a sum of < =n fractions.

Suppose that k >=1 is the remainder of the division of m by n. Then we chose d so that $\boxed{n/m= 1/d+ (n-k)/(md).}$

That implies nd= m+ (n-k), and d= (m+ n-k)/n, which is an integer because m= np+k.

By assumption, the fraction (n-k)/(md) can be written as an Egyptian fraction with n-k terms, and so n/m can be written as an Egyptian fraction with n-k+1 terms or less. Since k>=1, we can write n/m as an Egyptian fraction with n terms or less. This concludes the proof.

Example. Write 5/13 as an Egyptian fraction

Solution. The remainder of the division of 13 by 5 is 3, so we find d such that

5/13= 1/d+ 2/(13d). So, 5d=13+2=15, which gives d=3.

We have

5/13= 1/3+ 2/39. We need to decompose 2/39 in Egyptian fraction. We chose d so that

2/39= 1/d+ 1/(39 d)

That gives 2d= 39+1=40, and d=20.

So, 2/39= 1/20+ 1/780, and

5/13= 1/3+ 1/20+ 1/780

This method is effective and easy to apply, but when the numerator is large, it can produce fractions with a very large denominator. Other methods are able to produce Egyptian fractions with smaller denominators, although they may have a large number of terms. In other cases, we get less terms and smaller denominators with a "smart" choice of d. For example,

$$\frac{5}{121} = \frac{1}{25} + \frac{1}{757} + \frac{1}{763309} + \frac{1}{873960180913} + \frac{1}{1527612795642093418846225},$$

But also

$$\frac{5}{121} = \frac{1}{33} + \frac{1}{121} + \frac{1}{363}.$$

Natural questions that arise are: what is the minimum number of (distinct) unit fraction that are necessary to represent a given rational number? And what is the smallest denominator of one such representations?

D. The Erdős–Straus conjecture

The **Erdős–Straus conjecture** states that for all integers $n \geq 2$, the rational number $4/n$ can be expressed as the sum of **three** unit fractions. Paul Erdős and Ernst G. Straus formulated the conjecture in 1948; It is one of many conjectures by Erdős.

More formally, the conjecture states that, for every integer $n \geq 2$, there exist positive integers x, y, and z such that

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}.$$

These unit fractions form an Egyptian fraction representation of the number $4/n$. For instance, for $n = 5$, there are two solutions:

$$\frac{4}{5} = \frac{1}{2} + \frac{1}{4} + \frac{1}{20} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10}.$$

The restriction that x, y, and z be positive is essential to the difficulty of the problem, for if negative values were allowed the problem could be solved trivially. Also, if n is a composite number, $n = pq$, then an expansion for $4/n$ could be found immediately from an expansion for $4/p$ or $4/q$. Therefore, if a counterexample to the Erdős–Straus conjecture exists, the smallest n forming a counterexample would have to be a prime number. Computer searches have verified the truth of the conjecture up to $n \leq 10^{14}$, but proving it for all n remains an open problem.

Second Text

Al-Biruni's measurement of the earth

For showing how history may help us to find a suitable example relating to the real world, here we bring the method that Al-Biruni measured the earth circumstance; actually it is an elegant application of elementary trigonometry in real world.[2] Al- Biruni introduces his method,"Here is another method for the determination of the circumference of the earth. It does not require walking in deserts." Since the method assumes one knows how to determine the height of the mountain, al-Biruni first explains how to do that. The problem is nontrivial since a mountain is not a pole and therefore we cannot easily measure the distance from us to the point within the mountain where the perpendicular from its summit hits ground level.

To measure the height of a mountain al-Biruni first requires that we prepare a square board ABGD whose side AB is ruled into equal divisions and which has pegs at the corner B,G. Then ,at D, we must set a ruler, ruled with the same divisions as the edge AB and free to rotate around D. It should be as long as the diagonal of the square. Set the apparatus as in Fig.2 so that the board is perpendicular to the ground and the line of sight from G to B just touches the summit of the mountain. Fix the board there and let H be the foot of the summit of the perpendicular from D. Also, rotate the ruler around D until, looking along it, the mountain peak is sighted along the ruler's edge DT. Now AD is parallel to EG, $\angle ADT = \angle DEG$, and therefore the right triangles ADT and GED are similar. Thus $TA : AD = DG : GE$, and since of the four quantities in this proportion only GE is unknown we may solve for $GE = AD.DG / TA$. However, since both $\angle EGZ + \angle DGH$ and $\angle EGZ + \angle GEZ$ are equal to right angles it follows that $\angle DGH = \angle GEZ$, and thus the two right triangles DGH and GEZ are similar, so that $GE : EZ = DG : GH$. This means that we may solve for the single unknown $EZ = GE.GH / DG$, which is desired height.

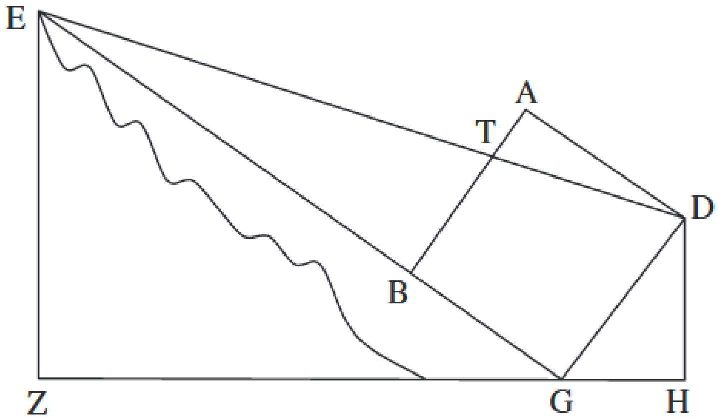


Figure 1

Then al-Biruni uses this method of measuring the height of a mountain to determine the circumference of the earth as following, illustrated in Fig.2:

Let KL be the radius of the earth and EL the height of the mountain. Let $ABGD$ be a large ring whose edge is graduated in degrees and minutes, and let ZEH be a rotatable ruler, along which one can sight, which runs through E, the center of the ring. An astrolabe, would be perfectly suitable for this, using the ruler and scale of degree on the back of this instrument.

Now move the ruler from a horizontal position BED until you can see the horizon, at T, along it. The angle BEZ is called the dip angle, d. From L on the earth, imagine LO drawn so LO is tangent to the earth at L. By the law of sines applied to (ELO)

$EL : LO = Sin(O) : Sin(E) = Sin\triangle : Sin(90^\circ - d)$

Since the two angles, as well as EL, the height of the mountain, are known, we may determine LO; but, TO=LO, since both are tangets to a circle from a point o outside it. Also, since EL and LO are known, it follows from the Pythagorean Theorem that $EO = \sqrt{EL^2 + LO^2}$ is known, and hence ET=EO+OT is known. Again, by the law of Sines, $ET : KT = Sin(d) : Sin(90^\circ - d)$. Since KT, the radius of the earth, is the only unknown quantity in this proportion, we may solve for KT and so find the radius of the earth.

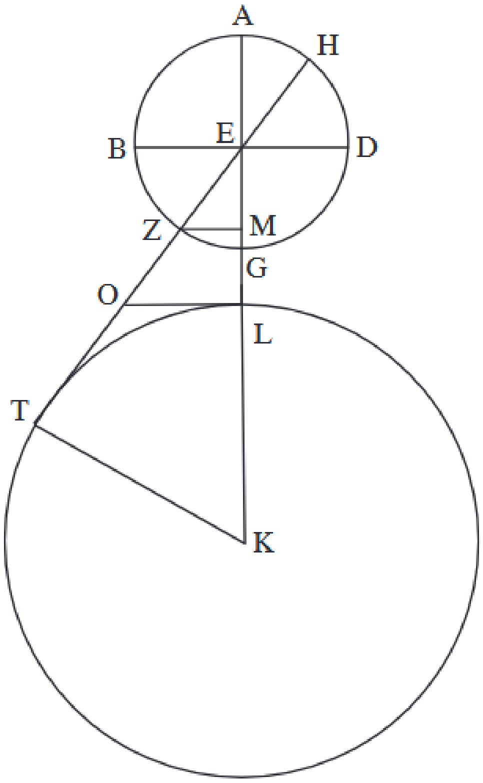
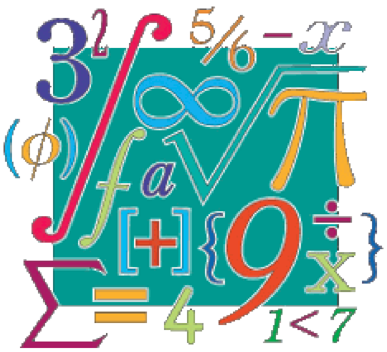


Figure 2

As al-Biruni said, he tried the method on a mountain near Nandana in India where the height, EL, was 652;3,18 cubits and the dip angle was 34'.(Note the very small angles and the rather optimistically accurate height.)These give for the radius of the earth 12,803,337;2,9 cubits. Al-Biruni takes the value $3\frac{1}{7}$ for π and arrives at the value 80,478,118;30,39 cubits for the circumference of the earth, which, upon division by 360 yields the value of 55;53,15 miles/degree on a meridian of the earth.



Math Vocabulary

Mathematics (math) is the study of numbers, quantities, shapes, and space using mathematical processes, rules, and symbols. There are many branches of mathematics and a large vocabulary associated with this subject.

Here are some math words and terms you will likely come across but may not know their precise meanings. Any word or term shown in **bold** is defined in the following alphabetical list of math words and terms.

Algorithm A step-by-step mathematical procedure used to find an answer.

Coefficient A number that multiplies a **variable**. For example, 9 is the coefficient in 9x.

Denominator The bottom number in a fraction. The denominator represents the number of parts into which the whole is divided. For example, 6 is the denominator in the fraction $\frac{5}{6}$.

Equation A mathematical statement used to show that two expressions are equal. It contains an equals sign. For example, 16 - 9 = 7 (the expression 16 - 9 and the expression 7 are equal).

Greatest Common Factor (*greatest common divisor*) The largest number that will divide two or more other numbers equally. For example, the greatest common factor of 32 and 48 is 16.

Improper Fraction (*mixed fractions*) A fraction that has a larger **numerator** than **denominator**. For example, $\frac{9}{5}$ is an improper fraction.

Inverse Operations Opposite or reverse operations. Addition and subtraction are inverse operations, as are multiplication and division.

Negative Number A number that is less than zero. A minus sign is used to show that a number is negative. For example, -12 is a negative number.

Numerator The top number in a fraction. The numerator represents the number of parts of the whole. For example, 5 is the numerator in the fraction $\frac{5}{6}$.

Ordinal Number A number that shows place or position, as in 2nd place.

Positive Number A number that is greater than zero. While a minus sign is used to signify a **negative number**, the absence of a minus sign signifies a positive number.

Prime Number A number that can be divided evenly only by itself and 1. For example, 7 is a prime number.

Square Number A number that results from multiplying another number by itself. For example, 49 is the square of 7 (7 x 7 = 49).

Square Root of a Number A number that is multiplied by itself to produce a **square number**. For example, 7 is the square root of 49. It is designated by the symbol √.

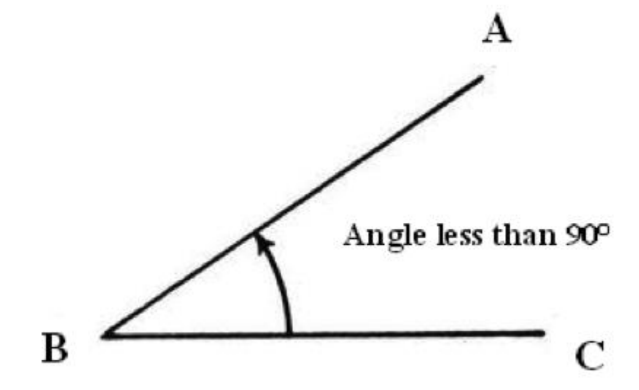
Variable A quantity that can change or vary, taking on different values. It is typically represented by a letter of the alphabet. For example x is the variable in 9x (x can be any number that is being multiplied by 9).

These are just some of the many words and terms found in mathematics. It is important to know the meanings of words and terms you will encounter as you progress through the study of math.

Absolute Value:

1. A number's distance from zero.
2. For any x, is defined as follows: $|x| = -x$, if $x < 0$; x, if $x \geq 0$.

Acute Angle: An angle whose measure is greater than 0 and less than 90 degrees.



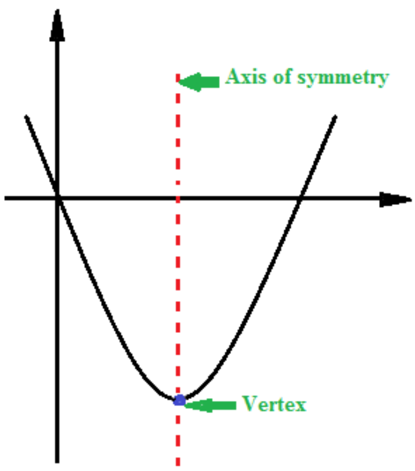
Example of an acute angle

Acute Triangle: A triangle in which all three angles are acute angles.

Algebraic Expression: An expression that includes one or more variables and may also include symbols indicating an operation or a relationship.

Area Model: A mathematical model based on the area of a rectangle, used to represent multiplication or to represent fractional parts of a whole.

Axis of Symmetry: The vertical line $x = -b/(2a)$ for the parabola given by $f(x) = ax^2 + bx + c$ or the vertical line $x = h$ when written $f(x) = a(x - h)^2 + k$.



Example of axis of symmetry for a parabola

Base:

1. For any number x raised to the nth power, written as x^n, x is called the base of the expression.
2. In geometry any side of a triangle may be called the base.

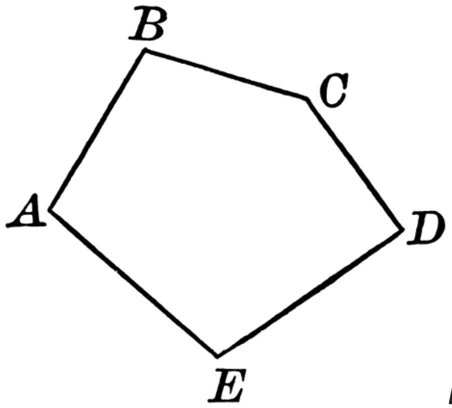
Cartesian Plane: See: Coordinate Plane
Coefficient: In the product of a constant and a variable the constant is the numerical coefficient of the variable and is frequently referred to simply as the coefficient.

Common Factor: A factor that appears in two or more terms.

Compound Interest: Interest paid on previous interest which was added to the principal.

Consistent system: See: System of Equations

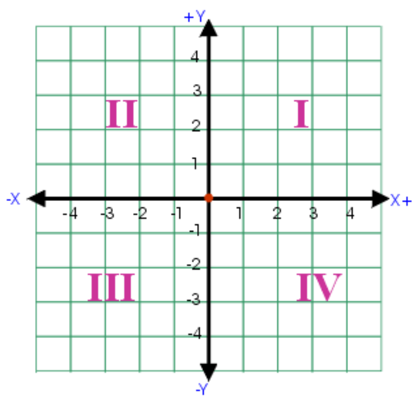
Convex Polygon: A plane, closed, figure formed by three or more line segments intersecting only at end points and each interior angle being less than 180 degrees.



Example of a convex polygon

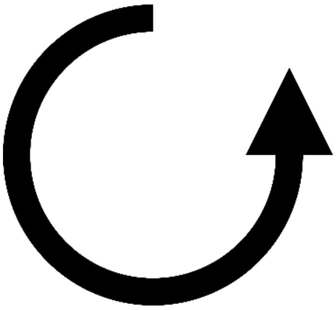
Coordinate(s): A number assigned to each point on the number line which shows its position or location on the line. In a coordinate plane the ordered pair, (x, y), assigned to each point of the plane showing its position in relation to the x-axis and y-axis.

Coordinate Plane: A plane that consists of a horizontal and vertical number line, intersecting at right angles at their origins. The number lines, called axes, divide the plane into four quadrants. The quadrants are numbered I, II, III, and IV beginning in the upper right quadrant and moving counterclockwise.



Coordinate Plane

Counterclockwise: A circular movement opposite to the direction of the movement of the hands of a clock.
Counting numbers: The counting numbers are the numbers in the following never-ending sequence: 1, 2, 3, 4, 5, 6, 7, . . . We can also write this as +1,+2,+3,+4,+5,+6,+7, ... These numbers are also called the positive integers or natural numbers.



Counterclockwise direction

Cube: The third power of a number.

Cube Root: For real number x and y, y is the cube root of x, (written $\sqrt[3]{x}$), if $y^3 = x$.

Data Set: A collection of information, frequently in the form of numbers.
Degree: The degree of a term is the sum of the exponents of the variables. The degree of a polynomial is the highest degree of any of its terms. If it contains no variables its degree is 0 if it is non-zero, and undefined if the polynomial is zero.

Dependent System: See: System of Equations

Dependent Variable: The variable in a function representing the elements of the range; the output values.

Direct Variation: For real variables x and y, y varies directly with x if $y = Kx$ or $yx = K$ for a constant K, $K \neq 0$. K is called the constant of proportionality.

Discriminant: The expression $b^2 - 4ac$ that appears under the radical sign in the Quadratic Formula.

Domain: The set of input values in a function.

Elements: Members of a set.
Equation: A math sentence using the equal sign to state that two expressions represent the same number.

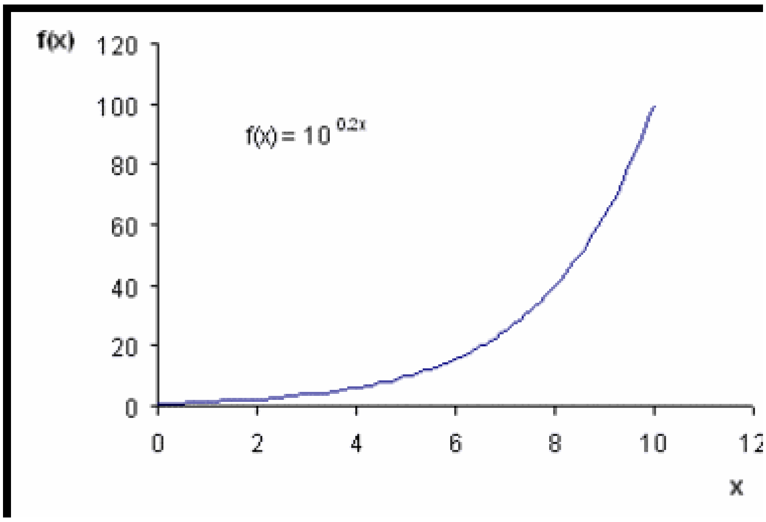
Equivalent Equations: Two equations are equivalent if they have the same solution or solution set.

Equivalent Inequalities: Two inequalities are equivalent if they have the same solution set.

Equivalent Expression: Expressions that have the same numerical value for given values of the variables.

Exponent: Suppose that n is a whole number. Then, for any number x, the nth power of x, or x to the nth power, is the product of n factors of the number x. This number is usually written x^n. The number x is usually called the base of the expression x^n, and n is called the exponent.

Exponential Function: For numbers a, k and b ≠ 0, the function f(t) = (ab)^ t/k is called an exponential function. The number a = f(0) is the initial value, b is the base and k is a constant related to growth rate or period.



Example of an exponential function

Exponential Growth/Decay: Also see Exponential Function.
For a > 0 and b > 1 the function denotes growth; for a > 0 and 0 < b < 1 the function denotes decay.

Exponential Notation: A notation that expresses a number in terms of a base and an exponent.

Extraneous Solutions: Apparent solutions which do not satisfy the given equation; usually introduced by raising to a power or multiplying by the variable in obtaining the solution.

Factor:

1. For integers a, b and c, a and b are factors of c if c = ab. Similarly f(x) and g(x) are factors of p(x) if p(x) = f(x) · g(x).
2. Factor is also used as an instruction or command to express a given integer or polynomial as a product.

Formula: An equation showing the relationship between two or more quantities represented by variables.

Function: A function is a rule which assigns to each member of a set of inputs, called the domain, a member of a set of outputs, called the range.

Function Notation: f(x), read ”f of x”, forming one side of an equation and used to indicate the value of the function when the input is x. Also see Function.

Graph of a Function: The pictorial representation of a function by plotting all of its input-output pairs on a coordinate system.
Height: In a triangle it is the segment from a vertex perpendicular to the selected base. Also used to refer to the length of that segment.

Examples of triangle heights, labeled h

Horizontal Axis: See: Coordinate Plane

Hypotenuse: The side opposite the right angle in a right triangle.

Inconsistent system: See: System of Equations
Independent system: See: System of Equations

Independent Variable: The variable in a function representing the elements of the domain; the input values.

Inequality: A statement that two expressions represent different values. There are various forms.

Strict Inequalities: Statements such as “x is less than y”, ($x < y$), and “x is greater than y”, ($x > y$).

Weak inequalities: Statements such as “x is less than or equal to y”, ($x \leq y$), and “x is greater than or equal to
y”, ($x \geq y$).

General inequality: The statement ”x is not equal to y”, ($x \neq y$).

Input Values: The values of the domain of a function.

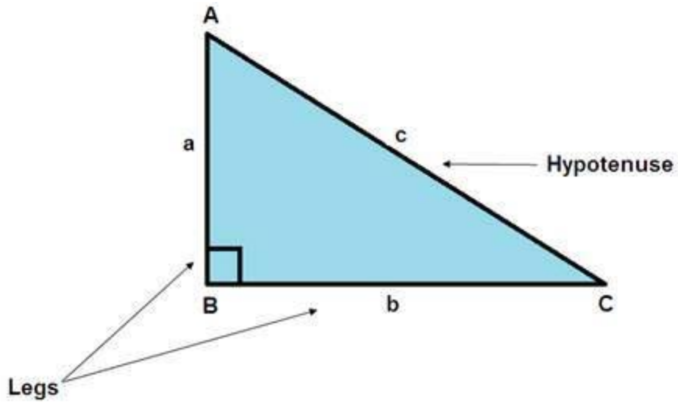
Integers: The collection of integers is composed of the negative integers, zero and the positive integers: . . . ,−4,−3,−2,−1, 0, 1, 2, 3, 4, ...

Intersection of Sets: A set whose elements are all the elements that the given sets have in common.

Inverse Variation: For real variables x and y, y varies inversely with x if yx = K or y = Kx and K is a non-zero constant.

Irrational Number: A decimal number that neither repeats nor terminates. A number that can not be expressed as an integer divided by an integer.

Joint Variation: For variables x, y and z, z varies jointly with x and y if z = Kxy and K is a non-zero constant.
Legs: The two sides of a right triangle that form the right angle.



Triangle legs

Less Than, Greater Than: The statement that the number a is less than the number b, written a < b means that there is a positive number x such that b = a + x. The number x must be b− a. If a is less than b, then b is greater than a, written b > a.

Like Terms: Algebraic terms that contain the same variables and for each variable the power is the same.

Linear Model for Multiplication: Skip counting on a number line.

Multiple: An integer or polynomial is said to be a multiple of any of its factors.
Multiplicative Identity: For each n, n · 1 = n and 1 is called the identity element for multiplication.

Multiplicative Inverse: The number x is called the multiplicative inverse or reciprocal of n, n ≠ 0 , if n · x = 1. This may also be written as n · 1/n = 1.

Natural Numbers: See: Counting Numbers
Negative 1 Power: If x is non-zero, x^−1 is the number 1/x

Non-negative Numbers: Numbers greater than or equal to zero.

Obtuse Angle: An angle whose measure is greater than 90 and less than 180 degrees.

Example of an obtuse angle

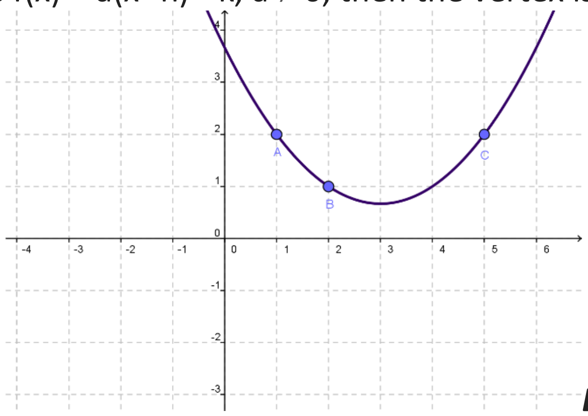
Obtuse Triangle: A triangle that has one obtuse angle.

Ordered pair: A pair of numbers that represent the coordinates of a point in the coordinate plane with the first number measured along the horizontal scale and the second along the vertical scale.

Origin: The point with coordinate 0 on a number line; the point with coordinates (0, 0) in the coordinate plane.

Output Values: The set of results obtained by applying a function rule to a set of input values.

Parabola: The shape of the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$. If the function is written as $f(x) = a(x-h)^2+k$, $a \neq 0$, then the vertex is (h, k).



Example of a parabola

Parallel Lines: Two lines in a plane that never intersect.

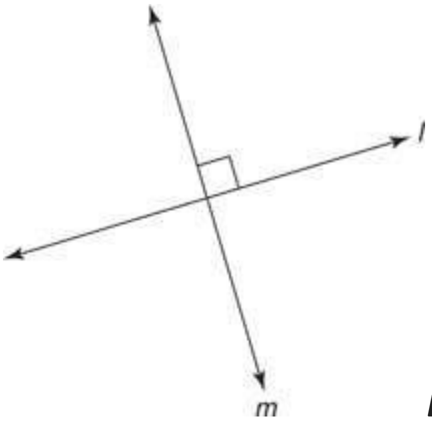
Parent function: The simplest example of a family of functions.

Perfect Cube: An integer n that can be written in the form $n = k^3$, where k is an integer.

Perfect Square:

1. An integer n that can be written in the form $n = k^2$, where k is an integer.
2. Also a trinomial which can be written in the form $(ax \pm b)^2$.

Perpendicular: Lines are perpendicular if they intersect to form a right angle. Segments are perpendicular if the lines containing the segments are perpendicular.



Example of two perpendicular lines

Point-slope form: A form of a linear equation written as (y − y1) = m(x−x1) where m is the slope and the line passes through the point (x1, y1).

Positive Integers: See: Counting Numbers

Power: See: Exponent

Polynomial: A polynomial is an algebraic expression obtained by adding, subtracting and /or multiplying real numbers and variables.

Quadrant: See: Coordinate Plane
Quadratic Equation: An equation with a second degree term as its highest degree term.

Quadratic Expression: A polynomial containing a second degree term as its highest degree term.

Radical: An indicated root of a number or polynomial denoted by n√ so that r = n√x implies r^n = x. The index n is generally omitted when n = 2 and we write √x to mean the square root of x.
Radical Equation: An equation in which the variable appears in the radicand.

Radical Function: A function which is the square root of a variable expression. More generally, a function of the nth root of a variable expression.

Radicand: The number or expression that appears in the radical sign; the number or expression whose root is to be found.

Range: See: Function

Rate:

1. A rate is a division comparison between two quantities with different units of measure. Also see Unit Rate.

2. The amount of interest charged on an annual basis.

Rational Equation: An equation involving one or more rational expressions.

Rational Expression: An expression in the form a/b with a and b being polynomial expressions, b of at least degree one and $b \neq 0$.

Rational Number: A number that can be written as a/b where a is an integer and b is a natural number.

Reciprocal: See: Multiplicative Inverse

Right Angle: An angle formed by the intersection perpendicular lines; an angle with a measure of 90 deg.

Right Triangle: A triangle that has a right angle.

Roots of a Quadratic: The solutions of $ax^2+bx+c = 0$. The same values are zeros of the quadratic expression or the function $f(x) = ax^2 + bx + c$. They are also the x-intercepts of the intersection of the graph of f(x) with the x-axis.

Scale Factor: For the parabola $f(x) = a(x-h)^2 + k$, a is the scale factor.
Scientific Notation: Base ten numbers written in the form a × 10^n where $1 \leq a \leq 10$ and n is an integer.

Sequence: A list of terms ordered by the natural numbers. The outputs of a function whose domain is the natural numbers.

Set: A collection of objects or elements.

Set Notation: A symbolic description of the elements of a set. "A is the set of all x's such that x is an element of the natural numbers with x greater than 2 and less than 11" would be written $A = \{x | x \text{ is a natural number}, 2 < x < 11\}$.

Simplifying: Combining like terms of a polynomial by carrying out the indicated additions or subtractions.

Simple Interest: Interest paid a single time on a principal invested or borrowed. Computed using I = Prt.

Simultaneously: See: System of Equations in two Variables

Slope of a Line: If (x1, y1) and (x2, y2) are two points on a line, then the slope of the line is the ratio m = rise/run = y/x = (y2−y1)/(x2−x1) provided $x2 \neq x1$

Slope-intercept form: The equation y = mx + b is the slope-intercept form of a line, where m is the slope and b is the y-intercept of the line.

Solution of an Equation: A solution to an equation with variable x is a number that, when substituted for x, makes the two sides of the equation equal. If the equation has more than one solution, then the collection of solutions is called the solution set.

Solution of an Inequality: The values that may be substituted for the variable in an inequality to form a true statement.

Solution of a System of Linear Equations: See System of Linear Equations.

Square Root: For non-negative x and y, y is the square root of x if $y^2 = x$. For x a real number, $\sqrt{x^2} = |x|$ because the square root symbol denotes the non-negative root.

Square Root Function: See Radical Function.

Standard Form: A form of a linear equation written as $Ax + By = C$.

Subset: Set B is a subset of set A if every element of set B is also an element of set A.

System of Linear Equations: Two equations that both impose conditions on the variables. An ordered pair is a solution of the system if and only if it is a solution of each of the given equations.

Systems of equations may be classified as follows:

1. A system with one or more solutions is called Consistent.
2. A system with no solution is called Inconsistent.
3. A Consistent system with exactly one solution is called Independent.
4. A Consistent system with more than one solution is called Dependent.

System of inequalities in two variables: Two inequalities that both impose conditions on the variables. If the inequalities form an ”and” statement the solution is all ordered pairs that satisfy both inequalities. If the inequalities form an ”or” statement the solution is any ordered pair that satisfies either inequality.

Term: 1. Each member of a sequence. 2. Each expression in a polynomial separated by addition or subtraction signs.
Translation: A transformation that moves a figure along a line in a plane but does not alter its size or shape. For a parabola, a horizontal or vertical shift in the position of the parent function.

Unit Rate: A ratio of two unlike quantities that has a denominator of 1 unit.
Variable: A letter or symbol that represents an unknown quantity.
Vertex:

1. The common endpoint of two rays forming an angle.
2. The highest or lowest point of the graph of a parabola.

Vertex form of a Parabola: A quadratic function written as $f(x) = a(x - h)^2 + k$ is in vertex form. The vertex is (h, k) and the coefficient a is the scale factor.

Vertical Axis: See: Coordinate Plane

Whole Numbers: The whole numbers are the numbers in the following never-ending sequence: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . .
x-axis: The horizontal axis of a coordinate plane.
x-intercept: The point of intersection of a graph with the x-axis.

y-axis: The vertical axis of a coordinate plane.
y-intercept: The point of intersection of a graph with the y-axis.

Zero power: For any number x, $x \neq 0$, x^0 = 1.
Zeros of a Quadratic: See Roots of a Quadratic.