Arda Göktaş

26708 ~~gdd~~

Answer for question 1

A)
$$\begin{array}{cccc} x_1 & x_2 & x_3 & rhs \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 18 & 19 \end{array}\right] = [\underbrace{A}_{A} : \underbrace{b}_{b}]$$

$[A:b]$ Augmented matrix of a system

pivot

First we try to makes zeros in column 1 except the row 1 column 1

B)
$$\begin{array}{c} R_1 \to \\ R_2 \to \\ R_3 \to \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 18 & 19 \end{array}\right] \begin{array}{l} R_2 = R_2 - 5R_1 \\ R_3 = R_3 - 9R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -9 & -17 \end{array}\right] R_3 = R_3 - R_2$$

Pivot

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 7 & 7 \end{array}\right] \to R_2 = \frac{R_2}{-4} \to \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 7 & 7 \end{array}\right] \to R_1 = R_1 - 2R_2 \to \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 7 & 7 \end{array}\right] \to R_3 = \frac{R_3}{7}$$

Pivot

Now, makes zeros in column 2 except row 2 column 2

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right] \to \begin{array}{l} R_1 = R_1 + R_3 \\ R_2 = R_2 - 2R_3 \end{array} \underbrace{\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]}_{r.r.e.f}$$

reduced row echelon form

C) $x_1 = -1, \; x_2 = 1, \; x_3 = 1$

$$\left[\begin{array}{ccc|c} x_1 & 0 & 0 & -1 \\ 0 & x_2 & 0 & 1 \\ 0 & 0 & x_3 & 1 \end{array}\right]$$

$(-1, 1, 1)$ all solutions of the linear system

Arda Göktaş
26708 ~~ab~~

Answer of Question 2:

A) Firstly, linearly dependent means that one is a scalar multiple of other

$$v \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = k \overset{w}{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}} \quad \begin{cases} k = 2 \\ -1k = -1 \\ 2k = 1 \end{cases} \Rightarrow \text{no such } k \text{ exists}$$

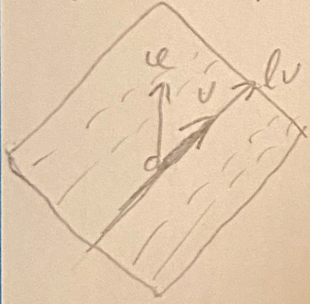
$\{v, w\}$ is linearly independent

b) $S(v,w) = \{av + bw : a,b \in \mathbb{R}\} = \{a \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + b \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \boxed{x,y \in \mathbb{R}}$

if $a = 1$ and $b = 1$ and consider $v + w = \underbrace{\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}}_{w_1} \in S(v,w)$.

, $w_1 \neq v$, $w_1 \neq w$ and $w_1 \in S(v,w)$

C) $v = (1,-1,2)$. determines a line in $\mathbb{R}^3$ $l_v = \{a(1,-1,2)\} : a \in \mathbb{R}\}$ passing through the origin, by part(a), v and w linearly independent



$\Rightarrow$ all linear combinations of v and w will fill a plane in $\mathbb{R}^3$, that is $S(v,w) = \{av + bw : a,b \in \mathbb{R}\}$ is a plane in $\mathbb{R}_3$ passing through the origin

by part(b) $w_1 \in S(v,w)$ w_1 lies on the plane determined by $(v$ and $w)$

$S(v,w,w_1) = S(v,w)$ since $w_1 \in S(v,w)$

$S(v,w,w_1)$ is a plane in $\mathbb{R}^3$

D) $w_2 \in S(v,w) \iff av + bw = w_2$ for $a,b \in \mathbb{R}^3$

$$\begin{bmatrix} 1 & 2 \\ -1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} a + 2b \\ -a - b \\ 2a + b \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix} \Rightarrow \begin{matrix} a + 2b = 3 \\ -a - b = 2 \\ 2a + b = -2 \end{matrix} \rightarrow \begin{matrix} a + 2b = 3 \\ b = 5 \\ -b = 2 \end{matrix} \text{ contradict each other}$$

this system has no solution, that is no such a and b exists

$w_2 \notin S(v,w)$

w_2 can't be written as a L.C of v and w

E) by part (c) $S(v,w)$ is a plane in $\mathbb{R}^3$ pass through the origin, by part(d), $w_2 \notin S(v,w)$ w_2 does not lie on this plane

$S(v,w,w_2) = \mathbb{R}^3$