

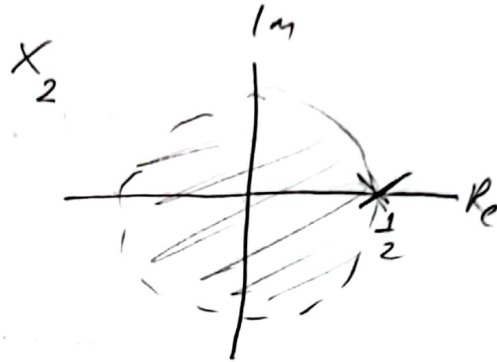
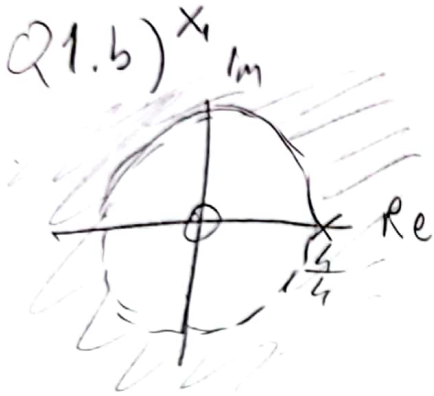
Q1.a)

EEE-391 Homework 2

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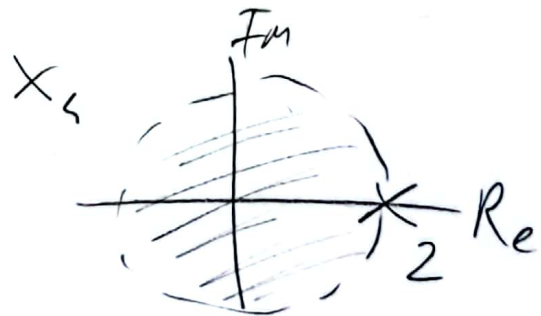
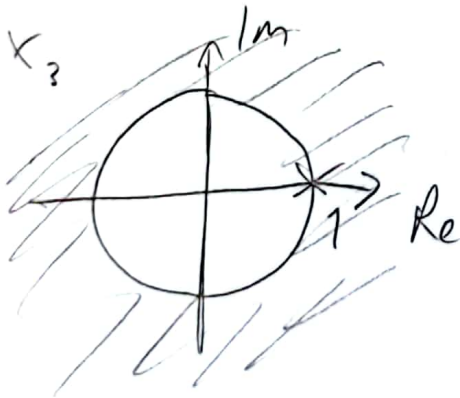
$$i) Z\{x_1[n]\} = \sum_{k=0}^{\infty} z^{-k} z^{-1} = \frac{z}{z-1}, |z| > \frac{1}{2}$$

$$ii) Z\{x_2[n]\} = \sum_{k=-\infty}^{-1} -z^{-k} z^{-1} = -\sum_{k=1}^{\infty} z^k z^{-1} = \frac{z}{2z-1}, |z| < \frac{1}{2}$$



$$Q1.c) x_3(z) = \sum_{k=0}^{\infty} 2z^{-k} = \frac{2z}{z-1}, |z| > 1$$

$$x_4(z) = -\sum_{k=-\infty}^{-1} z^k z^{-1} = -\sum_{k=1}^{\infty} \left(\frac{z}{2}\right)^k = \frac{z}{2-z}, |z| < 2$$



$$\begin{aligned}
 Q2.d) \quad H(e^{j\omega}) &= (1 + e^{-j\omega}) (1 + e^{-j\omega} (e^{-j\frac{2\pi}{3}} - e^{j\frac{2\pi}{3}})) - e^{-2j\omega} \\
 &= (1 + e^{-j\omega}) (1 - \sqrt{3}j e^{-j\omega} - e^{-2j\omega}) \\
 &= 1 - \sqrt{3}j e^{-j\omega} - e^{-2j\omega} + e^{-j\omega} - \sqrt{3}j e^{-2j\omega} - e^{-3j\omega} \\
 &= 1 + (1 - \sqrt{3}j) e^{-j\omega} + (-1 - \sqrt{3}j) e^{-2j\omega} + (-1) e^{-3j\omega}
 \end{aligned}$$

$$y[n] = x[n] + (1 - \sqrt{3}j)x[n-1] + (-1 - \sqrt{3}j)x[n-2] - x[n-3]$$

$$Q2.b) \quad h[n] = \delta[n] + (1 - \sqrt{3}j)\delta[n-1] - (1 + \sqrt{3}j)\delta[n-2] - \delta[n-3]$$

$$Q2.c) \quad 1 + e^{-j\omega} = 0 \rightarrow \omega = -\pi, \pi$$

$$1 - e^{j\frac{2\pi}{3}} e^{-j\omega} = 0 \rightarrow \omega = \frac{2\pi}{3}$$

$$1 + e^{-j\frac{2\pi}{3}} e^{-j\omega} = 0 \rightarrow \omega = \frac{\pi}{3} \quad \text{for } y[n] = 0$$

$$\omega \in \left\{ -\pi, \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\} \quad -\pi \leq \omega \leq \pi$$

$$Q3.a) \quad \frac{1}{1 + \frac{1}{2}z^{-1}} \xrightarrow{z^{-1}} \left(\frac{-1}{2}\right)^n u[n], \quad |z| > \frac{1}{2}$$

$$Q3.b) \quad \frac{1 - \frac{1}{2}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{2}z^{-1})} = \frac{1}{1 + \frac{1}{2}z^{-1}} \Rightarrow X(z) \xrightarrow{z^{-1}} \left(\frac{-1}{2}\right)^n u[n], \quad |z| > \frac{1}{2}$$

$$Q3.c) \quad \frac{1}{z^{-1} - d} - z^{-1} \frac{d}{z^{-1} - d} = -\frac{1}{d} \left( \frac{1}{1 - \frac{z^{-1}}{d}} \right) + z^{-1} \left( \frac{1}{1 - \frac{z^{-1}}{d}} \right)$$

$$X(z) \xrightarrow{z^{-1}} -\left(\frac{1}{d}\right)^{n+1} u[n] + \left(\frac{1}{d}\right)^{n+1} u[n-1], \quad |z| > \left|\frac{1}{d}\right|$$

$$Q4) \quad Z\{y[n]\} = Z\{x_1[n+3]\} Z\{x_2[-n+1]\}$$

$$x_1[n+3] \xrightarrow{Z} z^3 \frac{1}{1 - \frac{1}{2}z^{-1}} \text{ (shift)}$$

$$x_2[-n+1] \xrightarrow{Z} z \frac{1}{1 - \frac{1}{3}z} \text{ (time scale \& shift)}$$

$$Y(z) = \frac{z^4}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z)} = \frac{z^4}{\frac{7}{6} - \frac{1}{2}z^{-1} - \frac{1}{3}z}$$

Q5)

$$Y(z)(1 - \frac{\sqrt{2}}{2}z^{-1} + \frac{1}{4}z^{-2}) = X(z)(1 - z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - \frac{1}{\sqrt{2}}z^{-1} + \frac{1}{4}z^{-2}} = \frac{1 - z^{-1}}{(1 - \frac{1}{2}e^{\frac{\pi}{4}j})(1 - \frac{1}{2}e^{-\frac{\pi}{4}j})}$$

$$= \frac{1 - 2e^{-\frac{\pi}{4}j}}{1 - e^{-\frac{\pi}{2}j}} + \frac{1 - 2e^{\frac{\pi}{4}j}}{1 - e^{\frac{\pi}{2}j}}$$

By Inverse Z-transform:

$$h[n] = \frac{1 - 2e^{-\frac{\pi}{4}j}}{1 - i} \left(\frac{1}{2}e^{\frac{\pi}{4}j}\right)^n u[n] + \frac{1 - 2e^{\frac{\pi}{4}j}}{1 - i} \left(\frac{1}{2}e^{-\frac{\pi}{4}j}\right)^n u[n]$$

$$h[n] = u[n] 2^{-n} \left( \frac{e^{\frac{\pi}{4}jn} - 2e^{\frac{\pi}{4}j(n-1)}}{1+i} + \frac{e^{-\frac{\pi}{4}jn} - 2e^{-\frac{\pi}{4}j(n-1)}}{1-i} \right)$$

$$\downarrow$$

$$h[n] = 2^{-n} \left[ \cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{4}n\right) - 2\cos\left(\frac{\pi}{4}(n-1)\right) - 2\sin\left(\frac{\pi}{4}(n-1)\right) \right] u[n]$$

Q6.a)  $y(t) = x(t) + x(t-1)$  } Time shifted 1 unit Nyquist Rate  $= \omega_0$

Q6.b)  $y(t) = \frac{d}{dt} x(t)$   $y(j\omega) = x(j\omega) \cdot \frac{1}{j\omega}$  (Same  $f_{max}$ ) Nyquist Rate  $= \omega_0$

Q6.c)  $y(t) = x^2(t)$  (Doubled  $f_{max} \Rightarrow$  Nyquist Rate  $= 2\omega_0$ )  
 $= x(t) \cdot x(t)$

Q6.d)  $y(t) = x(t) \cdot \cos(\omega_0 t)$  (Tripled Nyquist Rate  $= 3\omega_0$ )

Q7.a)  $\underbrace{x(t) \xrightarrow{\text{Fourier Transformation}} X(j\omega)}_{x(at+b) \rightarrow \frac{1}{|a|} X\left(\frac{j\omega}{a}\right) e^{\frac{j b \omega}{a}}}$

$x_1(t) = x(1-t) + x(-1-t)$   $X_1(j\omega) = X\left(j\frac{\omega}{-1}\right) e^{-j\omega} + X\left(\frac{j\omega}{-1}\right) \cdot e^{j\omega}$   
 $= X(-j\omega) e^{-j\omega} + X(-j\omega) e^{j\omega}$

Q7.b)  $x_2(t) = x(3t-6)$   $X_2(j\omega) = \frac{1}{3} X\left(\frac{j\omega}{3}\right) e^{-j6\omega}$   
 $= \frac{1}{3} X\left(\frac{j\omega}{3}\right) e^{-2j\omega}$

$$Q8.a) \quad y[n] = \frac{1}{2} y[n-1] + x[n]$$

$$Y(z) - \frac{1}{2} Y(z) z^{-1} = X(z) \rightarrow Y(z) \left[ 1 - \frac{1}{2} z^{-1} \right] = X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2} z^{-1}} \xrightarrow{\text{As } x[n] = u[n]} X(z) = \frac{1}{1 - z^{-1}}$$

$$Y(z) = \left( \frac{1}{1 - \frac{1}{2} z^{-1}} \right) \times \left( \frac{1}{1 - z^{-1}} \right) = \frac{A}{1 - z^{-1}} + \frac{B}{1 - \frac{1}{2} z^{-1}}$$

$$z^{-1} = 1 \Rightarrow A = \frac{1}{1 - \frac{1}{2}} = 2 \quad \text{and} \quad z^{-1} = 2 \Rightarrow B = \frac{1}{1 - 2} = -1$$

$$Y(z) = \frac{2}{1 - z^{-1}} - \frac{1}{1 - \frac{1}{2} z^{-1}} \quad y[n] = 2u[n] - \left(\frac{1}{2}\right)^n u[n]$$

$$Q8.b) \quad x[n] = \left(e^{j\frac{\pi}{4}}\right)^n u[n] \quad Y(z) = \left(\frac{1}{1 - e^{j\frac{\pi}{4}} z^{-1}}\right) \left(\frac{1}{1 - \frac{1}{2} z^{-1}}\right)$$

$$Y(z) = \frac{A}{1 - e^{j\frac{\pi}{4}} z^{-1}} + \frac{B}{1 + \frac{1}{2} z^{-1}}$$

$$z^{-1} = e^{-j\frac{\pi}{4}} \Rightarrow A = \frac{1}{1 - \frac{1}{2} e^{-j\frac{\pi}{4}}} \quad z^{-1} = 2 \Rightarrow B = \frac{1}{1 - 2e^{j\frac{\pi}{4}}}$$

$$Y(z) = \left(\frac{1}{1 - \frac{1}{2} e^{-j\frac{\pi}{4}}}\right) \left(\frac{1}{1 - e^{j\frac{\pi}{4}} z^{-1}}\right) + \left(\frac{1}{1 - 2e^{j\frac{\pi}{4}}}\right) \left(\frac{1}{1 - \frac{1}{2} z^{-1}}\right)$$

$$y[n] = \frac{1}{1 - \frac{1}{2} e^{-j\frac{\pi}{4}}} \cdot \left(e^{j\frac{\pi}{4}}\right)^n u[n] + \frac{1}{1 - 2e^{j\frac{\pi}{4}}} \left(\frac{1}{2}\right)^n u[n]$$