EEE 391: Basics of Signals and Systems

Assignment 2

Due:

1)
   a) Determine the z-transforms of the following two signals. Note that the z-transforms for both have the same algebraic expression and differ only in the ROC.

   i) $x_1[n] = (1/2)^n u[n]$

   ii) $x_2[n] = -(1/2)^n u[-n-1]$

   b) Sketch the pole-zero plot and ROC for each signal in part (a).

   c) Repeat parts (a) and (b) for the following two signals:

      i) $x_3[n] = 2u[n]$

      ii) $x_4[n] = -(2)^n u[-n-1]$

2) The frequency response of a linear time-invariant filter is given by the formula

$$H(e^{jw}) = (1 + e^{-jw})(1 - e^{j2\pi/3}e^{-jw})(1 + e^{-j2\pi/3}e^{-jw})$$

   a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

   b) What is the output if the input is $x[n] = \delta[n]$?

   c) If the input is of the form $Ae^{j\Theta}e^{jwn}$, for what values of $-\pi \leq \mathrm{w} \leq \pi$ will $y[n] = 0$ for all n?

3) For each of the following z-transforms determine the inverse z-transform.

   a) $X(z) = \frac{1}{1+\frac{1}{2}z^{-1}},$ $\quad |z|>\frac{1}{2}$

   b) $X(z) = \frac{1-\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}},$ $\quad |z|>\frac{1}{2}$

   c) $X(z) = \frac{1-az^{-1}}{z^{-1}-a},$ $\quad |z|>|\frac{1}{a}|$

4) Consider a signal $y[n]$ which is related to two signals $x_1[n]$ and $x_2[n]$ by

$$y[n] = x_1[n+3] * x_2[-n+1]$$

where

$$x_1[n] = (1/2)^n u[n] \text{ and } x_2[n] = (1/3)^n u[n]$$

Given that

$$\text{(z-transform) } a^n u[n] \leftrightarrow \frac{1}{1+az^{-1}}, \quad |z|>|a|$$

use properties of z-transform to determine the z-transform $Y(z)$ of $y[n]$.

5) A particular causal LTI system is described by the difference equation

$$y[n] - \frac{\sqrt{2}}{2}y[n-1] + \frac{1}{4}y[n-2] = x[n] - x[n-1]$$

   a) Find the impulse response of this system.

   b) Sketch the log magnitude and the phase of the frequency response of the system.

6) Let $x(t)$ be a signal with Nyquist rate w_0. Determine the Nyquist rate for each of the following signals:

a) $x(t) + x(t-1)$

b) $\frac{dx(t)}{dt}$

c) $x^2(t)$

d) $x(t)cosw_0t$

7) Given that $x(t)$ has the Fourier transform $X(jw)$, express the Fourier transforms of the signals listed below in terms of $X(jw)$.

a) $x_1(t) = x(1-t) + x(-1-t)$

b) $x_2(t) = x(3t-6)$

8) Given an IIR filter defined by the difference equation

$$y[n] = \frac{1}{2}y[n-1] + x[n]$$

a) When the input to the system is a unit step sequence, $u[n]$ determine the functional form of the output signal $y[n]$. Use the inverse z- transform method. Assume that the output signal $y[n]$ is zero for n<0.

b) Find the output when $x[n]$ is a complex exponential that starts at n = 0:

$$x[n] = e^{j\left(\frac{\pi}{4}\right)n}u[n]$$

c) From (b),identify the steady state component of the response, and compare its magnitude and phase to the frequency response at w = $\frac{\pi}{4}$.