EEE 391

Basics of Signals and Systems

Fall 2023-2024

Homework 1

Due: 5 November 23:55 on Moodle

1) Solve the following equation for θ:

Find all possible answers. Make sure to give the final answer in radians.

$Re\left[(1-j)e^{j\theta}\right]$ = 1 - $Re\left[(1-j)e^{-j\theta}\right]$

2) A periodic signal $x(t)$ with a period of $T = 4$ is described over one period 0 < t < 4 by the equation:

$$x(t) = \begin{cases} (t-2)^2 & 2 \le t \le 4 \\ 0 & 0 \le t < 2 \end{cases}$$

This signal can be represented by the Fourier series which is valid for all time.

a) Sketch the periodic function $x(t)$
b) Find the a_k coefficient.
c) Compare the area of curve in one period and DC coefficient a_0 values.
d) If $y(t) = \frac{d}{dt}x(t)$, and b_k is the Fourier series of $y(t)$, find b_2

3) Figure 1 shows an ideal C to D and D to C converter.
   a) Suppose that the discrete-time signal $x[n]$ is:

$$x[n] = 5\cos\left(0.26\pi n + 30^\circ\right)$$

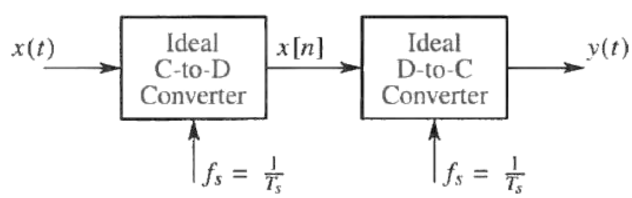


Figure 1

If the sampling rate is $f_s = 500$ samples per second, determine two different continuous-time signals that could have been inputs to the system.

b) $x(t)$ is shown in Figure 2. Determine a formula for $y(t)$ when $f_s = 1000$ for both converters.

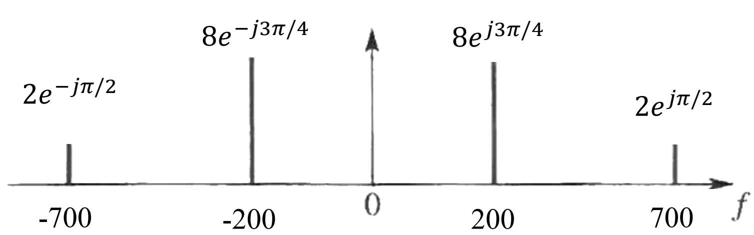


Figure 2

4) A signal is given as below:

$$x(t) = [3 + \cos(3\pi(500)t)]\cos(3\pi(1000)t)$$

a) Is this signal periodic? If so, what is the period of the signal.
b) Plot the two-sided spectrum of this signal.
c) What relation should the sampling rate satisfy so that $y(t) = x(t)$ in figure 1?

5) Filter coefficients of an FIR system are $\{b_k\}$ = {-2 ,2 ,4 ,6}. Determine the $y[n]$ if $x[n]$ is:

$$x[n] = \begin{cases} -1 & n = 3k \\ 0 & n = 3k + 1 \\ 1 & n = 3k + 2 \end{cases} \qquad \mathrm{k} \in Z$$

6) For each of the systems, determine whether or not the system is (1) linear (2) time-invariant and (3) casual.

(a) $y[n] = 2x[n]\cos(\pi n)$

(b) $y[n] = x[n] - x[2n + 1]$

(c) $y[n] = -x[n]^2$

(d) $y[n] = x[n] - \mathrm{u[n]}$

(e) $y[n] = 2^{x[n]}$

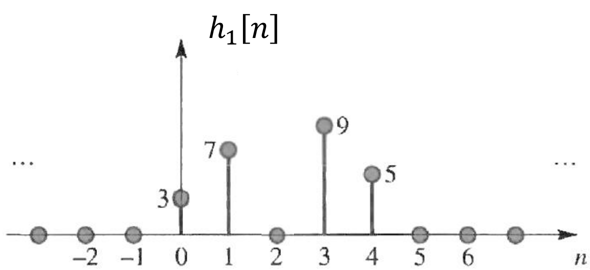
(f) $y[n] = 5 + x[n]$

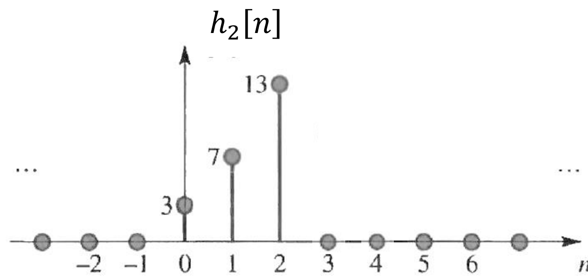
7) A linear time-invariant system is described by the difference equation.

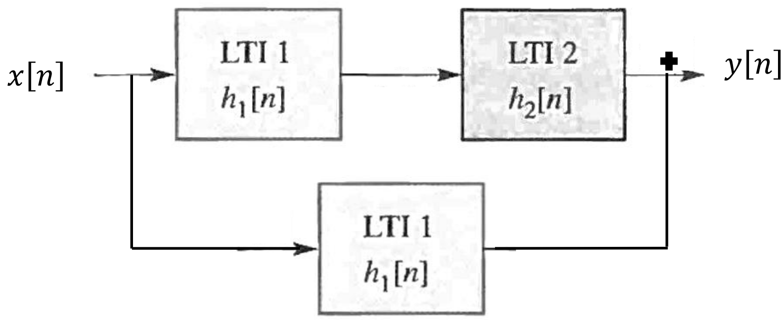
$$y[n] = x[n] + 5x[n-1] + 3x[n-3]$$

a) Draw the implementation of this system as a block diagram in direct form.
b) Write the impulse response for this system and plot it.

8) For two linear time-invariant systems, $h_1[n], h_2[n]$ are given in the figures below. Find $y[n]$, when $x[n] = 2u[n]$


$h_1[n]$
3
7
9
5
...
...
−2 −1 0 1 2 3 4 5 6
n


$h_2[n]$
13
7
3
...
...
−2 −1 0 1 2 3 4 5 6
n


$x[n]$
LTI 1
$h_1[n]$
LTI 2
$h_2[n]$
+
$y[n]$
LTI 1
$h_1[n]$