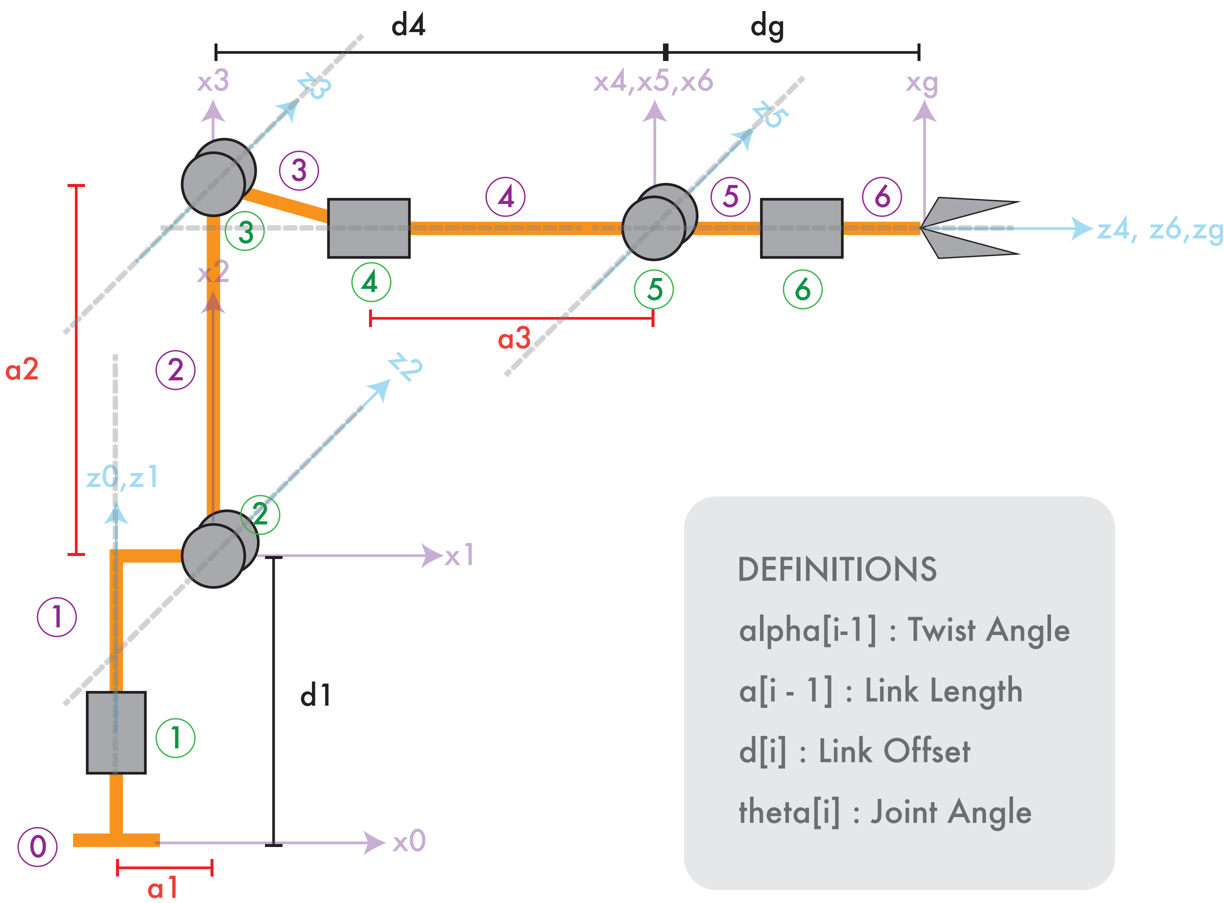
# Project - 2 : Pick and Place


*Arda Kaya* *Udacity Robotics Nanodegree* *ardakayaa@gmail.com*


## Modified DH-Parameters

The Kuka KR210's modified DH-Parameters are the following:

| i | alpha [ i - 1 ] | a [ i - 1 ] | d [ i ] | theta [ i ] |
|---|---|---|---|---|
| **1** | 0 | 0 | 0.75 | q1 |
| **2** | - pi / 2 | 0,35 | 0 | q2 - pi / 2 |
| **3** | 0 | 1,25 | 0 | q3 |
| **4** | - pi / 2 | -0,54 | 1,50 | q4 |
| **5** | pi / 2 | 0 | 0 | q5 |
| **6** | - pi / 2 | 0 | 0 | q6 |
| **g** | 0 | 0 | 0,303 | 0 |




DEFINITIONS

alpha[i-1] : Twist Angle

a[i - 1] : Link Length

d[i] : Link Offset

theta[i] : Joint Angle

# TRANSFORMATION MATRICES ABOUT EACH JOINT

T01

| | | | |
|---|---|---|---|
| cos(q1) | - sin(q1) | 0 | 0 |
| sin(q1) | cos(q1) | 0 | 0 |
| 0 | 0 | 1 | 0.75 |
| 0 | 0 | 0 | 1 |

T12

| | | | |
|---|---|---|---|
| sin(q2) | cos(q2) | 0 | 0.35 |
| 0 | 0 | 1 | 0 |
| cos(q2) | - sin(q2) | 0 | 0 |
| 0 | 0 | 0 | 1 |

T23

| | | | |
|---|---|---|---|
| cos(q3) | - sin(q3) | 0.0 | 1.25 |
| sin(q3) | cos(q3) | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

T34

| | | | |
|---|---|---|---|
| cos(q4) | - sin(q4) | 0 | - 0.054 |
| 0 | 0 | 1 | 1.5 |
| -sin(q4) | - cos(q4) | 0 | 0 |
| 0 | 0 | 0 | 1 |

T45

| | | | |
|---|---|---|---|
| cos(q5) | - sin(q5) | 0 | 0 |
| 0 | 0 | - 1 | 0 |
| sin(q5) | cos(q5) | 0 | 0 |
| 0 | 0 | 0 | 1 |

T56

| | | | |
|---|---|---|---|
| cos(q6) | - sin(q6) | 0 | 0 |
| 0 | 0 | 1 | 0 |
| -sin(q6) | - cos(q6) | 0 | 0 |
| 0 | 0 | 0 | 1 |

T6g

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0.303 |
| 0 | 0 | 0 | 1 |

Note: I have used following code to get these matrices.

```
DH_Table = {alpha0:    0.,   a0:      0.,   d1:  0.75,   q1: q1,
                 alpha1:-pi/2.,   a1:   0.35,    d2:     0.,  q2:-pi/2. + q2,
                 alpha2:    0.,   a2:   1.25,    d3:     0.,  q3: q3,
                 alpha3:-pi/2.,   a3: -0.054,    d4:  1.50,   q4: q4,
                 alpha4: pi/2.,   a4:      0.,   d5:     0.,  q5: q5,
                 alpha5:-pi/2.,   a5:      0.,   d6:     0.,  q6: q6,
                 alpha6:    0.,   a6:      0.,   d7: 0.303,   q7: 0.}

# Define Transformation Matrix
    def TF_Matrix(alpha, a, d ,q):
        TF = Matrix([[            cos(q),          -sin(q),          0,            a],
                    [sin(q)*cos(alpha),cos(q)*cos(alpha),-sin(alpha),-sin(alpha)*d],
                    [sin(q)*sin(alpha),cos(q)*sin(alpha), cos(alpha), cos(alpha)*d],
                    [                0,                0,          0,            1]])
        return TF

    # Create individual transformations Matrixis
    T0_1 = TF_Matrix(alpha0, a0, d1, q1).subs(DH_Table)
    T1_2 = TF_Matrix(alpha1, a1, d2, q2).subs(DH_Table)
    T2_3 = TF_Matrix(alpha2, a2, d3, q3).subs(DH_Table)
    T3_4 = TF_Matrix(alpha3, a3, d4, q4).subs(DH_Table)
    T4_5 = TF_Matrix(alpha4, a4, d5, q5).subs(DH_Table)
    T5_6 = TF_Matrix(alpha5, a5, d6, q6).subs(DH_Table)
    T6_EE = TF_Matrix(alpha6, a6, d7, q7).subs(DH_Table)

    T0_EE = T0_1 * T1_2 * T2_3 * T3_4 * T4_5 * T5_6 * T6_EE
```

I have multiplied individual matrices to create transformation matrix from base link to gripper .

# THE TRANSFORMATION MATRIX BETWEEN THE GRIPPER FRAME AND THE BASE FRAME GIVEN THE POSITION AND ORIENTATION OF THE GRIPPER

Rotation X

```
Matrix([[1      ,     0,      0 ],
        [0      , cos(r), -sin(r)],
        [0      , sin(r),  cos(r)]])
```

Rotation Y

```
Matrix([[cos(p) ,     0,  sin(p)],
        [0      ,     1,     0  ],
        [-sin(p),     0,  cos(p) ]])
```

Rotation Z

```
Matrix([[cos(y) , -sin(y),     0],
        [sin(y) ,  cos(y),     0],
        [     0,      0,       1]])
```

Note: I have used following code to get these matrices.

```
# Extract end-effector position and orientation from request px,py,pz = end-effector
        # position, roll, pitch, yaw = end-effector orientation
        px = req.poses[x].position.x
        py = req.poses[x].position.y
        pz = req.poses[x].position.z

        (roll, pitch, yaw) = tf.transformations.euler_from_quaternion(
        [req.poses[x].orientation.x, req.poses[x].orientation.y, req.poses[x].orientation.z,
        req.poses[x].orientation.w ])

        # Find EE rotation Matrix
        # Define RPY rotation matrices
        # http://planning.cs.uiuc.edu/node102.html

        r, p, y  = symbols('r p y')

        ROT_x = Matrix([[1      ,      0,      0],
                        [0      , cos(r), -sin(r)],
                        [0      , sin(r),  cos(r)]]) #roll

        ROT_y = Matrix([[cos(p) ,      0,  sin(p)],
                        [0      ,      1,      0],
                        [-sin(p),      0,  cos(p)]]) #pitch

        ROT_z = Matrix([[cos(y) , -sin(y),      0],
                        [sin(y) ,  cos(y),      0],
                        [     0,       0,       1]]) #yaw

        ROT_EE = ROT_z * ROT_y * ROT_x

        # More Information can be found in KR210 FK section
        Rot_Error = ROT_z.subs(y, radians(180)) * ROT_y.subs(p, radians(-90))

        ROT_EE = ROT_EE * Rot_Error
        ROT_EE = ROT_EE.subs({'r' : roll, 'p' : pitch, 'y' : yaw})

        EE = Matrix([[px],
                     [py],
                     [pz]])
        WC = EE - (0.303) * ROT_EE[:,2]
```

I have multiplied gripper's roll, pitch and yaw matrices to get its rotation matrix.

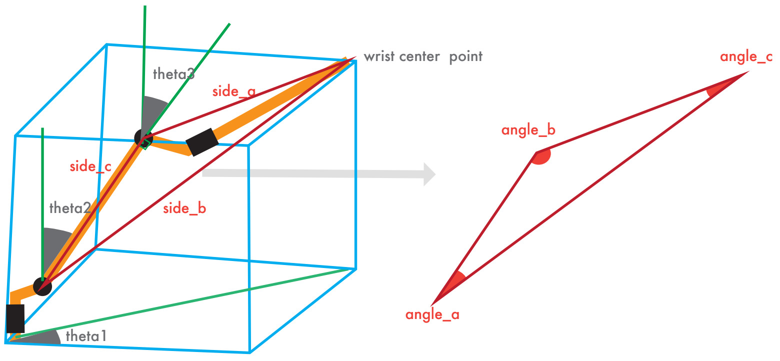
To get the wrist center I have used this equation.

$$^0r_{WC/0} = {^0r_{EE/0}} - d \cdot {^0_6}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - d \cdot {^0_6}R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

# CALCULATING THE INDIVIDUAL JOINT ANGLES

I have used atan2 function to calculate theta1 .

```
theta1 = atan2(WC[1] , WC[0])
```




I have used the code below to calculate theta 2 and theta 3.

```
# SSS triangle for theta2 and theta3
        side_a = 1.501
        side_b = sqrt(pow((sqrt(WC[0] * WC[0] + WC[1] * WC[1]) - 0.35), 2) + pow((WC[2] - 0.75), 2))
        side_c = 1.25

        angle_a = acos((side_b * side_b + side_c * side_c - side_a * side_a) / (2 * side_b * side_c))
        angle_b = acos((side_a * side_a + side_c * side_c - side_b * side_b) / (2 * side_a * side_c))
        angle_c = acos((side_a * side_a + side_b * side_b - side_c * side_c) / (2 * side_a * side_b))

        theta2 = pi / 2 - angle_a - atan2(WC[2] - 0.75, sqrt(WC[0] * WC[0] + WC[1] * WC[1]) - 0.35)
        theta3 = pi / 2 - (angle_b + 0.036)
```

0.036 accounts for sag in link4 of -0.054m

I have used the code below to calculate the last three theta value.

```
R0_3 = T0_1[0:3,0:3] * T1_2[0:3,0:3] * T2_3[0:3,0:3]
        R0_3 = R0_3.evalf(subs={q1: theta1, q2: theta2, q3: theta3})

        R3_6 = R0_3.inv("LU") * ROT_EE

        # EULER angles from rotation Matrix
        theta4 = atan2(R3_6[2,2], -R3_6[0,2])
        theta5 = atan2(sqrt(R3_6[0,2] * R3_6[0,2] + R3_6[2,2] * R3_6[2,2]), R3_6[1,2])
        theta6 = atan2(-R3_6[1,1], R3_6[1,0])
```