$$r = e^{\theta}$$

Horizontal

$$\frac{dy}{d\theta}$$

$$y = r \sin\theta$$

$$y = e^{\theta} \sin\theta$$

$$\frac{dy}{d\theta} = e^{\theta} \sin\theta + e^{\theta} \cos(\theta)$$

$$e^{\theta} (\sin\theta + \cos(\theta)) = 0$$

$$\sin\theta + \cos(\theta) = 0$$

$$\tan(\theta) = -1$$

$$\boxed{\theta = -\frac{\pi}{4} + n\pi}$$

$$(e^{\theta}, \theta) \longrightarrow = (e^{-\frac{\pi}{4} + n\pi}, -\frac{\pi}{4} + n\pi)$$

Vertical $\qquad \dfrac{dx}{d\theta} = e^{\theta}\cos\theta - e^{\theta}\sin(\theta)$

$\cos\theta - \sin(\theta) = 0 \qquad\qquad => \cos(\theta) = \sin(\theta)$

$$\tan\theta = 1 \qquad => \theta = \dfrac{\pi}{4} + n\pi$$

$\left( e^{\frac{\pi}{4}+n\pi}, \ \dfrac{\pi}{4} + n\pi \right)$