Tridiagonal LU Decomposition and an Application

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tridiagonalLU.m is designed for finding LU decomposition of any tridiagonal matrix. The time complexity of the algorithm is $O(n)$, it exploits properties of tridiagonal matrix. (Recall that the time comlexity of original LU decomposition is $O(n^3)$)

application.m includes an application of tridiagonal LU decomposition. It is designed for solving the following two-point boundary value problem.

$$-\frac{d^2y}{dx^2} - (\pi^2 + 1)y = -cos(\pi x)$$

where $x \in (0, 1)$.

I divided the interval $(0, 1)$ into 1000 equal pieces. I set $n = 1000$, $h = \frac{1}{n}$ and $x_k = kh$ for $k = 1, 2, ..., 999$. The problem reduces to

$$-\frac{Y_{k+1} - 2Y_k + Y_{k-1}}{h^2} - (\pi^2 + 1)Y_k = -cos(\pi x_k)$$

for $k = 1, 2, ..., 999$. Also $Y_0 = Y_{1000} = 0$. Observe that this is a linear system of 999 equations.

My algorithm interprets the linear system as matrix multiplication, then it calculates tridiagonal LU decomposition. (since each equation includes only 3 y values, the matrix is tridiagonal) Then, it uses backward and forward substitutions to find the solution. (I exploited properties of tridiagonal matrices again and they take linear time) The time complexity of whole algorithm is still $O(n)$.