PHYS 582 SPRING 2023 HWA #3

1 We can add two types of potentials to the Dirac equation either as a Lorentz scalar in which case one has

$$\left[i\hbar \gamma^\mu \partial_\mu - mc + \mathbb{W}\right]\Psi = 0 \quad (A)$$

or via minimal electric coupling { via the covarient EM derivative}

$$\left[i\hbar \gamma^\mu \partial_\mu - \frac{e}{c}\gamma^\mu \mathcal{A}_\mu - mc\right]\Psi = 0 \quad (B)$$

For case (A) let $\mathbb{W}$ be a constant and find all the solutions be detailed.

For case (B) let $\mathcal{A}_0 = c\phi$ $\vec{\mathcal{A}} = 0$ and set ϕ to be a constant and find all the solutions.

You may of course get help from the free particle solutions (especially on page 50. of Ryder)

For the problem above discuss various cases W, ϕ positive, negative, strong, weak.

2 Consider (A) and write it as $i\hbar \frac{\partial \Psi}{\partial t} = H_{(A)} \Psi$ find $H_{(A)}$

Consider (B) and write it as $i\hbar \frac{\partial \Psi}{\partial t} = H_{(B)} \Psi$ find $H_{(B)}$

{ one generally denotes $\gamma^0\gamma^i \equiv \alpha^i$ and $\gamma^0 \equiv \beta$ in Bjorken & Drell }

3 Consider $i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$ with $H = H_{(B)}$ with $\mathcal{A}_\mu = 0$ {free particle}

Introduce the velocity operator via $[\vec{X}, H] = i\hbar \underbrace{\frac{d\vec{X}}{dt}}_{\equiv \vec{\mathbb{V}} \text{ velocity operator}}$

first argue that the eigenvalues of the velocity operator are $\pm c$. But surely the eigenfunctions of momentum we have found for the free particle do not show such pathology. Why do yo think?

Now to make a contest let us construct the velocity operator in the non-relativistic theory $H = \frac{p^2}{2m}$, and digress on the situation above.

4 Consider again $i\hbar \frac{\partial \psi}{\partial t} = H\psi$ with $H = H_{(B)}$ with $A_\mu = 0$.

Try to find all operators that commute with H.