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FORMAL LANGUAGES & AUTOMATA

3.1.1. Consider the grammar $G = (V, \Sigma, R, S)$, where

$$
\begin{aligned}
V &= \{a, b, S, A\},\\
\Sigma &= \{a, b\},\\
R &= \{S \to AA,\\
&\quad A \to AAA,.\\
&\quad A \to a,\\
&\quad A \to bA,\\
&\quad A \to Ab\}.
\end{aligned}
$$

(a) Which strings of $L(G)$ can be produced by derivations of four or fewer steps?
(b) Give at least four distinct derivations for the string $babbab$.
(c) For any $m, n, p > 0$, describe a derivation in G of the string $b^m ab^n ab^p$.

ANSWER:

a) **aa, baa, aba, aab, aaa**

b) **S → AA → bAA → bAAb → bAAbAb → babbAb → babbab**

S → AA → bAA → bAAb → bAAbAb → bAbbAb → babbab

S → AA → bAA → bAbA → babA → babAb → babbAb → babbab

S → AA → AAb → bAbA → baAb → babAb → babbAb → babbab

c) **S → AA**

$\to^{m} b^{m}AA$

$\to^{n} b^{m}Ab^{n}A$
$\to^{p} b^{m}Ab^{n}Ab^{p}$
$\to\ b^{m}ab^{n}Ab^{p}$
$\to\ b^{m}ab^{n}ab^{p}$

3.1.2. Consider the grammar (V, Σ, R, S), where V, Σ, and R are defined as follows:

$$\begin{aligned} V &= \{a, b, S, A\}, \\ \Sigma &= \{a, b\}, \\ R = \{&S \to aAa, \\ &S \to bAb, \\ &S \to e, \\ &A \to SS\}. \end{aligned}$$

Give a derivation of the string $baabbb$ in G. (Notice that, unlike all other context-free languages we have seen so far, this one is very difficult to describe in English.)

ANSWER:

S → bAb

→ bSSb

→ baAbSb

→ baSSaSb

→ baSaSb

→ baaSb

→ baabSAbb

→ baabSSbb

→ baabSbb

→ baabbb

3.1.3. Construct context-free grammars that generate each of these languages.

(a) $\{wcw^R : w \in \{a, b\}^*\}$

(b) $\{ww^R : w \in \{a, b\}^*\}$

(c) $\{w \in \{a, b\}^* : w = w^R\}$

ANSWER:

a) G=(v,∑,R,S) => V={a,b,S}

∑={a,b}

R = {S→aSa, S→bSb, S→c}

b) G=(v,∑,R,S) => V={a,b,S}
∑={a,b}
R = {S→aSa, S→bSb, S→c}

c) G=(v,∑,R,S) => V={a,b,S}
∑={a,b}
K = {S→aSa, S→bSb, S→a, S→b, S→c}

3.1.5. Consider the context-free grammar $G = (V, \Sigma, R, S)$, where

$$\begin{aligned}
V &= \{a, b, S, A, B\},\\
\Sigma &= \{a, b\},\\
R &= \{S \to aB,\\
&\quad S \to bA,\\
&\quad A \to a,\\
&\quad A \to aS,\\
&\quad A \to BAA,\\
&\quad B \to b,\\
&\quad B \to bS,\\
&\quad B \to ABB\}.
\end{aligned}$$

(a) Show that $ababba \in L(G)$.
(b) Prove that $L(G)$ is the set of all nonempty strings in $\{a, b\}$ that have equal numbers of occurrences of a and b.

ANSWER:

a) S→aB → abS → abaB → ababS → ababbA → ababba
b) w(S) = 0
c) w(a) = w(A) = 1
d) w(b) = w(B) = -1
e) w(S) = 0 = w(aB)
f) w(S) = 0 = w(aB)
g) w(A) = 1 = w(a)
h) w(A) = 1 = w(aS)
i) w(A) = 1 = w(BAA)
j) w(B) = -1 = w(b)
k) w(B) = -1 = w(bS)
l) w(B) = -1 = w(ABB)