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FORMAL LANGUAGES & AUTOMATA

Page 8 and 9 ;

1.1.1. Determine whether each of the following is true or false.

(a) $\emptyset \subseteq \emptyset$
(b) $\emptyset \in \emptyset$
(c) $\emptyset \in \{\emptyset\}$
(d) $\emptyset \subseteq \{\emptyset\}$
(e) $\{a, b\} \in \{a, b, c, \{a, b\}\}$
(f) $\{a, b\} \subseteq \{a, b, \{a, b\}\}$
(g) $\{a, b\} \subseteq 2^{\{a,b,\{a,b\}\}}$
(h) $\{\{a, b\}\} \in 2^{\{a,b,\{a,b\}\}}$
(i) $\{a, b, \{a, b\}\} - \{a, b\} = \{a, b\}$

ANSWER:

(a) True

(b) False

(c) True

(d) True

(e) True

(f) True

(g) False

(h) True

(i) False

1.1.2. What are these sets? Write them using braces, commas, and numerals only.

(a) $(\{1, 3, 5\} \cup \{3, 1\}) \cap \{3, 5, 7\}$
(b) $\bigcup\{\{3\}, \{3, 5\}, \bigcap\{\{5, 7\}, \{7, 9\}\}\}$
(c) $(\{1, 2, 5\} - \{5, 7, 9\}) \cup (\{5, 7, 9\} - \{1, 2, 5\})$
(d) $2^{\{7,8,9\}} - 2^{\{7,9\}}$
(e) $2^{\emptyset}$

ANSWER:

(a) {3,5}

(b) {3,5,7}

(c) {1,2,7,9}

(d) {{8},{7,8},{8,9},{7,8,9}}

(e) {∅}

1.1.4. Let $S = \{a, b, c, d\}$.

(a) What partition of S has the fewest members? The most members?

(b) List all partitions of S with exactly two members.

ANSWER:

(a) Fewest Members:

{{a,b,c,d}}

(b) Most Members:

{{a},{b},{c},{d}}

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1.3.1. Let $R = \{(a, c), (c, e), (e, e), (e, b), (d, b), (d, d)\}$. Draw directed graphs representing each of the following.

(a) R

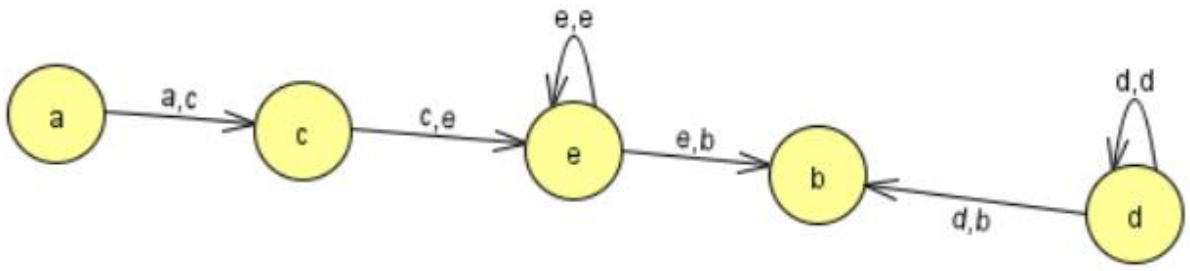
(b) R^{-1}

(c) $R \cup R^{-1}$

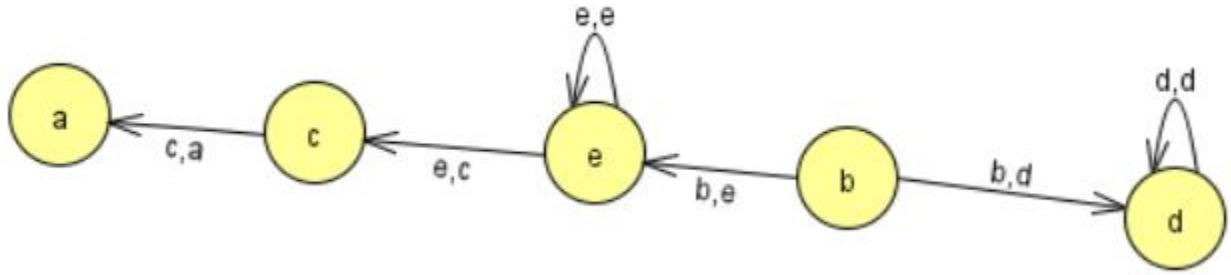
(d) $R \cap R^{-1}$

ANSWER:

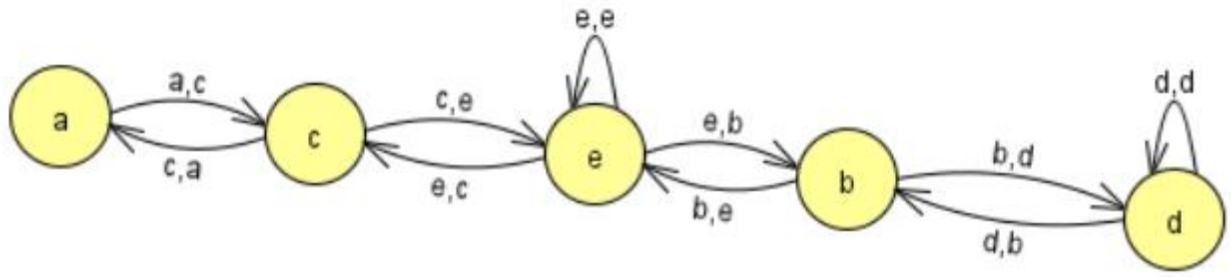
(a)



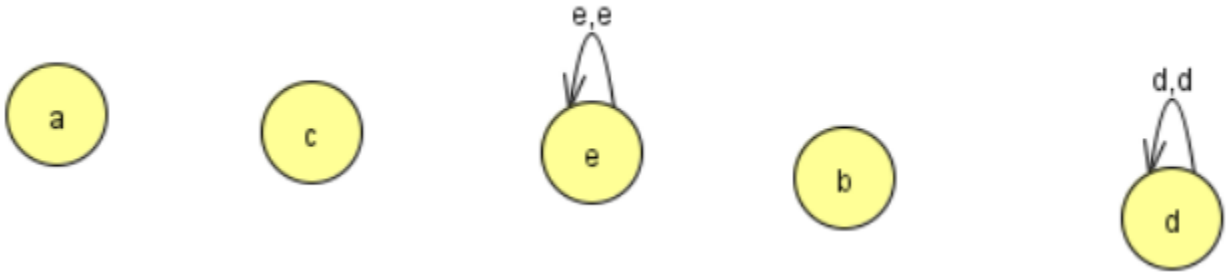
(b)



(c)



(d)



1.3.2. Let R and S be the binary relations on $A = \{1, \ldots, 7\}$ with the graphical representations shown in the next page.

(a) Indicate whether each of R and S is (i) symmetric, (ii) reflexive, and (iii) transitive.

(b) Repeat (a) for the relation $R \cup S$.

ANSWER:

(a)

R: Not reflexive, transitive, and symmetric

S: Not reflexive and transitive. On the other hand it is symmetric.

(b)

R U S: it is reflexive

1.3.4. Let A be a nonempty set and let $R \subseteq A \times A$ be the empty set. Which properties does R have?

(a) Reflexivity.

(b) Symmetry.

(c) Antisymmetry.

(d) Transitivity.

ANSWER:

(a) R is not Reflexive

(b) R is symmetric

(c) R is also Antisymmetric

(d) R is Transitive

1.3.7. Let R_1 and R_2 be any two partial orders on the same set A. Show that $R_1 \cap R_2$ is a partial order.

ANSWER:

R1 ∩ R2 is reflexive, antisymmetric and transitive.

1.3.9. Under what circumstances does a directed graph represent a function?

ANSWER:

In directed graphs, the edge emerging from each node represents a function.