LabReport3
Ideal Gas

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Abstract

This experiment was focusing on the Ideal Gas Law.

In this experiment the amount of substance of gas in the enclosed region in the syringe was determined by fixing the temperature of the gas, and then varying the volume of gas and measuring the respective pressure values of gas. A value $\langle n \rangle = (1.858 \pm 0.031)$ mmol ($\varepsilon_{\langle n \rangle} = 1.7\%$) has been obtained, which agrees with the expected value $n_{expected} \approx 1.859$ mmol.

Additionally, the Thermal Tension was measured by fixing the gas' volume and then varying its temperature and recording the respective pressure values of gas. The value $\beta_{T^\circ} = (3.71 \pm 0.14) \cdot 10^{-3}$ K^{-1} ($\varepsilon_{\beta_{T^\circ}} = 3.8\%$) was achieved, this value covers the expected value $\beta_{T^\circ_{expected}} = 3.661 \cdot 10^{-3}$ K^{-1} with the uncertainty range.

Contents

1 Introduction and Theory

An *ideal gas* is a theoretical model of a gas. The gas in this model is composed of many randomly moving point particles (their size is negligible in comparison to size of container in which gas is situated) that are not interacting chemically between each other. A lot of real gases behave like an ideal gas at the sufficient pressures and temperatures. For example, helium, oxygen, nitrogen, hydrogen, air and carbon dioxide can be treated as ideal gases (at standard pressure and temperature). Usually, a gas behaves like an ideal gas at high temperature and at low pressure due to the fact that inter-molecular forces do not influence movement of particles significantly and the sizes of the particles become negligible in comparison to the typical size of total region which gas occupies.

The kinetic gas theory can help to describe how pressure, volume, temperature, and an amount of substance of gas are related - which can be observed in experiments. The equation that describes the relationship between these variables is called *Ideal Gas Law* and is written as

$$PV = nRT \tag{1}$$

Ideal Gas Law

Here, P is absolute pressure $P = P_{atm} + P_{relative}$, V is a volume, n is the amount of substance of a gas, given as a number of moles (one mole is equal to $6.022 \cdot 10^{23}$ particles), R is the gas constant ($8.3143 \frac{J}{K \cdot mol}$), and T is the absolute temperature. Note that the equation is valid only for ideal gases which are assumed to consist of point-like particles with a negligible volume where particles interact with each other only by elastic collisions. Another thing to be noted is that if the volume, pressure, and temperature of different ideal gases are the same, then the amount of substance of these gases is always the same.

To compare experiments and calibrate instruments, standard values for gases were introduced: one mole of a gas ($n = 1$) at the standard condition has pressure $P^{\ominus} = 1013.25$ hPa, temperature $T^{\ominus} = 273.15$ K, and the volume of $V^{\ominus} = 0.02241$ m^3.

1.1 The dependence of pressure on volume at a constant temperature

The absolute pressure exerted by a given mass of an ideal gas is inversely proportional to the volume it occupies if the temperature and the amount of gas remain unchanged within a closed system. The relationship was confirmed by Boyle and Mariotte. It is known as *Boyle's law*. Mathematically, the relationship can be stated as:

$$P \propto \frac{1}{V} \quad P \cdot V = const \quad \text{at } T = const \tag{2}$$

Boyle's Law

1.2 The dependence of pressure on temperature at a constant volume

When the pressure of a gas with constant mass is held constant, the Kelvin temperature and the volume will be in direct proportion. The relationship was investigated by *Charles* and mathematically, it can be written as

$$P \propto T \quad \frac{P}{T} = const \quad \text{at } V = const \tag{3}$$

Charles' Law

1.3 The dependence of volume on temperature at a constant pressure

The pressure of a gas of fixed mass and fixed volume is directly proportional to the gas' absolute temperature. The direct proportion is known as *Gay-Lussac's law* or *Charles' 2nd law* and it can be expressed as:

$$V \propto T \quad \frac{V}{T} = const \quad \text{at } P = const \tag{4}$$

Gay-Lussac's Law

For these individual laws the respective proportionality constants can be defined. One such constant is known as *thermal tension* that can be established in consideration of changes in pressure with the changes in temperature. The *thermal tension* is defined as

$$\beta_{T^\ominus} = \frac{1}{P_{T^\ominus}} \left(\frac{\partial P}{\partial T} \right)_{V,n} \tag{5}$$

Thermal tension

where $P_{T^\ominus}$ is the pressure at the standard temperature. Note that the *thermal tension* remains the same for different ideal gases. This theoretical value $\beta_{T^\ominus}$ of can be calculated as follows:

$$\left. \begin{array}{c} P \overset{(1)}{=} \frac{nRT}{V} \Rightarrow \left(\frac{\partial P}{\partial T} \right)_{V,n} = \frac{nR}{V} \\ P_{T^\ominus} = \frac{nRT^\ominus}{V} \end{array} \right\} \implies \beta_{T^\ominus} = \frac{V}{nRT^\ominus} \cdot \frac{nR}{V} = \frac{1}{T^\ominus} = 3.661 \cdot 10^{-3} \text{ K}^{-1} \tag{6}$$

Thermal tension theoretical value

2 Experimental and Procedure

2.1 Calibration

The atmospheric pressure value has been taken from a barometer: $P_{atm} = (107 \pm 5)$ hPa.

The following setup was established - (Fig. 1), and the tightness of the sensor mounting was checked by changing the volume of gas slowly with the syringe and then fixing it at some point. If the pressure readings didn't fluctuate, but instead remained constant, then setup was considered hermetically sealed (Sealing of pressure sensor was repeatedly done until the setup was hermetic).
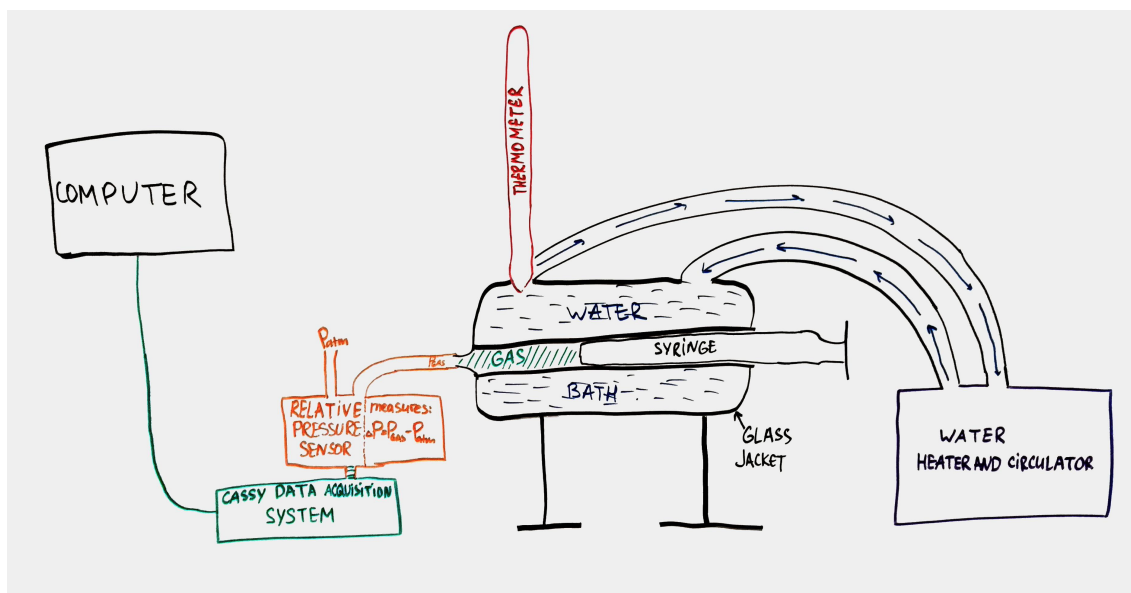
Figure 1: Experimental Setup Scheme
(Computer, Cassy, Relative pressure sensor, Syringe in water bath
with thermometer, water heater and circulator)

Then water circulation was switched on, and the temperature on thermometer was checked to be constant for 5 minutes. After the temperature has established the setup was ready for the experiment.

2.2 Main procedures

This experiment consists of 2 different parts: in the first part the number of gas moles is being determined by measuring pressure in dependence of volume (Boyle's Law - (2)), in the second part the value of thermal tension is being verified by measuring pressure in dependence of temperature (Charles' Law - (3)).

2.2.1 Boyle's Law

The temperature reading was taken from the thermometer: $T_0 = (20 \pm 1)$ K. Starting value for gas volume after calibration was $V_0 = (45 \pm 1)$ ml (at this volume relative pressure is zero). In this part of the experiment temperature remained constant (at the value T_0). The volume of gas has been changed in diapason from -12 to +12 ml and respective data, corresponding to each of new volumes of gas, from relative pressure sensor was saved.

2.2.2 Charles' Law

After the first part the syringe position has been reset - and at the starting volume $V_0 = (45 \pm 1)$ ml the relative pressure was still nearby zero ($P_{rel_{V_0}} = (4 \pm 1)$ hPa), thus system was considered to have successfully kept its Air-tightness. The position of syringe was secured with the holder (the syringe was always being checked to be on the starting spot at each measurement point), therefore the volume of gas remained constant through this part of experiment (at the value $V_0 = (45 \pm 1)$ ml). The water circulation was kept switched on, water heater was turned on and was set to 30°C at first (first measurement was made at this temperature point after the temperature indication on the thermometer had been

remaining constant (at the value 30°C) for 3 minutes). Then, consecutively, some water was exchanged with enough ice in the water reservoir (where water heater was situated) for the temperature to drop, and the temperature on the heater had been being changed to a new value 4-6°C lower. After each change of temperature on the heater, the temperature was considered to be established after it has been constant on the thermometer for 3 minutes. The gas temperature has been lowered all the way down to 4°C in the end (this was the lowest temperature point). The relative pressure data from relative pressure sensor was recorded for each respective gas temperature value.

3 Results and Data Analysis

3.1 Boyle's Law results

During the first part of the experiment (Boyle's Law) the following raw data was achieved:

V [ml]	ΔV [ml]	P_{rel} [hPa]	ΔP_{rel} [hPa]	P [hPa]	ΔP [hPa]	n [mmol]
48	1	-60	1	947	6	1.865
51	1	-110	1	897	6	1.877
54	1	-156	1	851	6	1.885
57	1	-196	1	811	6	1.897
45	1	0	1	1007	6	1.859
42	1	46	1	1053	6	1.815
39	1	142	1	1149	6	1.839
36	1	230	1	1237	6	1.827
33	1	322	1	1329	6	1.799

Table 1: Raw data from measurements at constant temperature

The absolute pressure is gotten from relative pressure as follows: $P = P_{rel} + P_{atm}$. The maximal error ΔP is therefore obtained as follows: $\Delta P = \Delta P_{rel} + \Delta P_{atm}$. From Ideal Gas Law (1) formula for n is achieved:

$$n = \frac{PV}{RT} \text{ (for this part of experiment, specifically, } T = T_0 = (20 \pm 1) \text{ K)} \qquad (7)$$

Denote number of measurements as N for convenience.
The mean value of n was calculated using the following formula:

$$\langle n \rangle = \frac{\sum_{i=1}^{N} n_i}{N} \qquad (8)$$

Using Error Propagation Formula (20) error of n was calculated:

$$\Delta n = n \cdot \sqrt{\left(\frac{\Delta P}{P}\right)^2 + \left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta T}{T}\right)^2} \qquad (9)$$

By applying Error Propagation Formula (20) to each individual error Δn the formula for average error of mean is achieved: $\Delta \langle n \rangle_{avg} = \frac{1}{N} \sqrt{\sum_{i=1}^{N} (\Delta n_i)^2}$

The partial derivatives of n with respect to P, V and T, multiplied by error of the respective quantity are given below:

$$\left| \left(\frac{\partial n}{\partial P} \right)_{V,T} \cdot \Delta P \right| = \frac{V}{RT} \Delta P \tag{10}$$

$$\left| \left(\frac{\partial n}{\partial V} \right)_{P,T} \cdot \Delta V \right| = \frac{P}{RT} \Delta V \tag{11}$$

$$\left| \left(\frac{\partial n}{\partial T} \right)_{P,V} \cdot \Delta T \right| = \frac{PV}{RT^2} \Delta T \tag{12}$$

Partial derivatives, multiplied by respective quantity error

For the statistical error of n the following formula was used:

$$\Delta \langle n \rangle_{stat} = t \cdot \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (n_i - \langle n \rangle)^2} \tag{13}$$

with student coefficient $t = 2.306$ for $N = 9$ measurements, confidence level $\alpha = 95\%$ [1]

Statistical error formula

Final error for $\langle n \rangle$ is combined from statistical and propagated error as a root of sum of squares of each error: $\Delta \langle n \rangle = \sqrt{(\Delta \langle n \rangle_{avg})^2 + (\Delta \langle n \rangle_{stat})^2}$

The final value for n:

$$\langle n \rangle = (1.858 \pm 0.031) \text{ mmol}$$

$$\varepsilon_{\langle n \rangle} = 1.7\%$$

Number of moles of gas in a system

3.2 Charles' Law results

Second part of the experiment gave the following raw data:

T [°C]	ΔT [°C]	P_{rel} [hPa]	ΔP_{rel} [hPa]	P [hPa]	ΔP [hPa]
4	1	-50	1	957	6
8	1	-38	1	969	6
14	1	-18	1	989	6
20	1	4	1	1011	6
25	1	22	1	1029	6
30	1	40	1	1047	6

Table 2: Raw data measurements at constant volume

The pressure values P over T are plotted on the following graph:
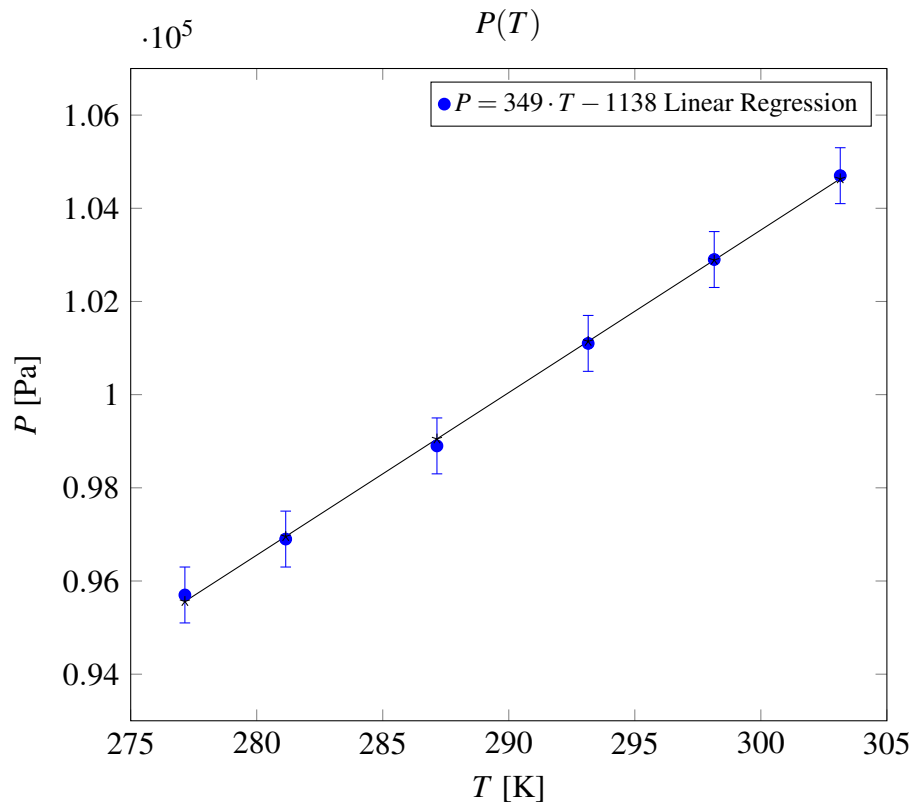

Figure 2: P dependence on absolute temperature of gas T

From Ideal Gas Law (1):

$$P = \frac{nR}{V}T \xrightarrow{\text{(denoting } T_{rel}=T-T^{\ominus})} \frac{nR(T^{\ominus} + T_{rel})}{V} = \frac{nRT^{\ominus}}{V} + \frac{nRT_{rel}}{V} \xrightarrow{\text{(using } P_{T^{\ominus}} = \frac{nRT^{\ominus}}{V} \text{ (6))}}$$

$$\xrightarrow{\text{(using } P_{T^{\ominus}} = \frac{nRT^{\ominus}}{V} \text{ (6))}} P = P_{T^{\ominus}} + \frac{nR}{V}T_{rel} \xrightarrow{\text{(using } \left(\frac{\partial P}{\partial T}\right)_{V,n} = \frac{nR}{V} \text{ (6))}} P(T_{rel}) = P_{T^{\ominus}} + \left(\frac{\partial P}{\partial T}\right)_{V,n} T_{rel}$$

Therefore graph $P(T_{rel})$ is perfect for determination of values $P_{T^{\ominus}}$ and $\left(\frac{\partial P}{\partial T}\right)_{V,n}$ as these values are intercept and slope of this graph respectively.

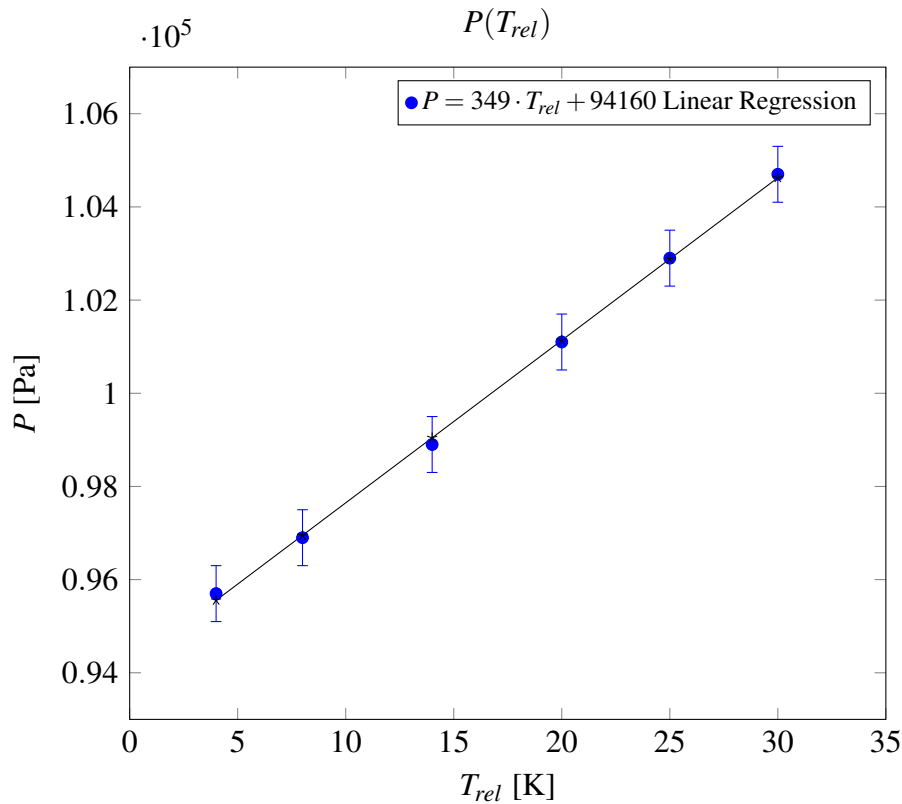The pressure values P over T_{rel} are plotted on the following graph:

Figure 3: P dependence on relative temperature of gas $T_{rel} = T - T^{\ominus}$

The slope (a), its error (Δa), intercept (b) and its error (Δb) were obtained using following formulae [1]:

$$a = \frac{\sum_i (T_{rel_i} - \langle T_{rel} \rangle)(P_i - \langle P \rangle)}{\sum_i (T_{rel_i} - \langle T_{rel} \rangle)^2} \tag{14}$$

$$b = \langle P \rangle - a\langle T_{rel} \rangle \tag{15}$$

$$\Delta a = a \cdot t \sqrt{\frac{1}{N-2} \frac{1-R^2}{R^2}} \tag{16}$$

with student coefficient $t = 2.571$ for $N = 6$ measurements, confidence level $\alpha = 95\%$

$$\Delta b = a \sqrt{\sum_i T_{rel_i}^2} \cdot t \sqrt{\frac{1}{N(N-2)} \frac{1-R^2}{R^2}} \tag{17}$$

where correlation coefficient is defined as follows: $R = \frac{\sum_i (T_{rel_i} - \langle T_{rel} \rangle) \cdot (P_i - \langle P \rangle)}{\sqrt{\sum_i (T_{rel_i} - \langle T_{rel} \rangle)^2 \cdot \sum_i (P_i - \langle P \rangle)^2}}$

The final value for $a = \left(\frac{\partial P}{\partial T}\right)_{V,n}$:

$$a = (349 \pm 14)\frac{\text{Pa}}{\text{K}}$$

$$\varepsilon_a = 3.8\%$$

Slope of $P(T_{rel})$ graph - $\left(\frac{\partial P}{\partial T}\right)_{V,n}$

The final value for $b = P_{T^\ominus}$:

$$b = (94160 \pm 250)\text{Pa}$$

$$\varepsilon_b = 0.27\%$$

Intercept of $P(T_{rel})$ graph - $P_{T^\ominus}$

Achieved values now can be inserted into the defining formula for $\beta_{T^\ominus}$ (5):

$$\beta_{T^\ominus} = \frac{1}{P_{T^\ominus}}\left(\frac{\partial P}{\partial T}\right)_{V,n} \tag{18}$$

Using Error Propagation Formula (20) the error $\Delta\beta_{T^\ominus}$ was achieved:

$$\Delta\beta_{T^\ominus} = \beta_{T^\ominus}\sqrt{\left(\frac{\Delta P_{T^\ominus}}{P_{T^\ominus}}\right)^2 + \left(\frac{\Delta\left(\frac{\partial P}{\partial T}\right)_{V,n}}{\left(\frac{\partial P}{\partial T}\right)_{V,n}}\right)^2} \tag{19}$$

The final value for $\beta_{T^\ominus}$:

$$\beta_{T^\ominus} = (3.71 \pm 0.14)\cdot 10^{-3}\ \text{K}^{-1}$$

$$\varepsilon_{\beta_{T^\ominus}} = 3.8\%$$

Thermal tension

4 Error analysis

4.1 Formulae

The next formula is the most important formula for error evaluation in the experiments:

$$\Delta y = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial y}{\partial x_i}\Delta x_i\right)^2} \qquad (20)$$

Error Propagation Formula [1]

Relative error of value z is denoted as ε_z, it gives the comprehension on how precise the measurement is

$$\varepsilon_z = \frac{\Delta z}{\langle z \rangle} \qquad (21)$$

Relative Error Formula

4.2 Boyle's Law

For comparison of error sources formulas 10, 11, 12 were used. For the first part of experiment (Table 1) the major error of source for n comes from V:

n [mmol]	Error from P [mmol]	Error from V [mmol]	Error from T [mmol]
1.865	0.012	0.039	0.007
1.877	0.013	0.037	0.007
1.885	0.014	0.035	0.007
1.897	0.015	0.034	0.007
1.859	0.012	0.042	0.007
1.815	0.011	0.044	0.007
1.839	0.010	0.048	0.007
1.827	0.009	0.051	0.007
1.799	0.009	0.055	0.007

Table 3: Error contributions from different variables

The statistical error for n: $\Delta\langle n \rangle_{stat} = 0.026$ mmol is larger than the propagated error $\Delta\langle n \rangle_{avg} = 0.016$ mmol.

4.3 Charles' Law

The errors for pressure values were the same for every recorded value (because it was only influenced by instrumental error of barometer, which measured atmospheric pressure, and by instrumental error of relative pressure sensor), therefore for error of the slope and intercept of graph $P(T_{rel})$ statistical error formulae were used. The major error source for $\beta_{T^{\ominus}}$ comes from the slope ($\varepsilon_{slope} = 3.8\% \gg \varepsilon_{intercept} = 0.27\%$).

5   Discussion and Conclusion

The first part of the experiment (Boyle's Law) finds out the amount of substance of gas in the syringe. The measured value is $\langle n \rangle = (1.858 \pm 0.031)$ mmol. The expected value of n for $T = T_0 = 20$ K, $V = V_0 = 45$ ml and $P = P_{atm} = 100700$ Pa is $n_{expected} \approx 1.859$ mmol. The uncertainty range of measured value covers expected value, as expected. This measurement was performed with a good precision ($\varepsilon_{\langle n \rangle} = 1.7\%$). The major error source of the measurement was the error of V - gas volume. Therefore, the experiment precision could be increased by getting a longer syringe with smaller diameter, which will allow the volume marks to be more frequent and thus, more precise.

In the second part of the experiment (Charles' Law) the Thermal Tension value was being checked. Measured value is $\beta_{T^{\ominus}} = (3.71 \pm 0.14) \cdot 10^{-3}$ K^{-1}. The measured value agrees with the expected theoretical value $\beta_{T^{\ominus}_{expected}} = 3.661 \cdot 10^{-3}$ K^{-1}. This measurement was conducted with worse precision due to the more complicated calculation formula, although the precision is still decent ($\varepsilon_{\beta_{T^{\ominus}}} = 3.8\%$). The major factor, which influences error is the error of the graph slope calculation, which is influenced by measurements of temperature and pressure. Thus, a new relative pressure sensor (because the one, which was used, was not on its first year of usage - it could've been outdated) would increase reliability of measurement, as well as more precise thermometer (but temperature error is the least error source - so it is the last priority).

The one thing which could improve all measurements on the setup (Figure 1) is the improvement of a connection between syringe and the relative pressure sensor, specifically the improvement of impermeability of this connection. The more tight this connection is, the less leakage of gas is happening, and consequently the less distortion is being brought into the experiment (so that condition $n = const$ holds better).

References

[1]   Prof. Dr. Jürgen Fritz. *Error Analysis Booklet*. Bremen: Jacobs University, 2019.