LabReport1
Lenses and Optical Instruments

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Abstract

This experiment was focused on building 2 types of telescopes (Kepler-type and Galileo-type), a microscope, an eyeglass lens prototype, and it also had as one of the goals the following: verification of geometrical optics laws. Telescopes were magnifying image by approximately a factor of 6 (Snellen chart could be clearly observed from a hallway-order distances), Kepler Telescope gave an inverted image, while Galileo Telescope gave a normal image. Microscope was magnifying an image by a factor of more than 50, which gave the ability to observe the dog flea in detail through such a microscope. The calculated values had a range intersection with empirically gotten values, thus laws of geometric optics are supported by this experiment.

Contents

1 Introduction and Theory

In this experiment laws of geometric optic are being tested and the magnifying capabilities of optical intruments are being researched. This experiment considers situations where *optical objects* - anything that the light rays come from - are much larger than the wavelength of light and where light is propagated by light rays. Therefore, trigonometric and geometric equations are used to describe the behavior of light rays. Such a description of optics is called *geometric optics.* On the other hand, there is a position where rays seem to converge other than the object which is called the *image position.* This position is classified into two different types: *a real image* and *a virtual image.* The main difference is that a real image can be projected on a screen, while a virtual image can be seen only by the observer.

The main element of optical instruments is the *lens.* If we draw a line through its center and normal to the plane of the lens, then it is called *an optical axis.* This line consists of a point – *the focal point* - through which all parallel to the optical axis light rays focused by the lens pass. The distances from the lens to a focal point, to an object, and to an image are called the *focal length*, *object distance*, and *image distance* respectively.

The main rays of optical instruments are the *focal ray*, *the parallel ray*, and *the central ray.* Using these rays will help to construct the image. The central ray passing through the center of the lens is not deflected, while the other two rays are deflected: the parallel ray starts running to a focal point and the focal ray starts running parallel to the optical axis after passing the lens. (As shown in Fig. 1)

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Figure 1: Image construction by three principal rays

From the laws of similar triangles,

$$\frac{y'}{s'} = \frac{y}{s} \quad \textit{and} \quad \frac{y}{f} = \frac{y'}{s'-f} \quad \textit{or} \quad \frac{y'}{y} = \frac{s'-f}{f} \tag{1}$$

where y' is the image size, y is the object size, s' is the distance between the lens and image, and s is the distance between the lens and object. *The Lens Formula* is derived by converting equations (1)

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{2}$$

If any two lenses are combined and the distance between them is much smaller than their focal lengths, their total focal length can be calculated by:

$$\frac{1}{f_{tot}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2} \tag{3}$$

where f_1 and f_2 is the focal lengths of the first and the second lenses respectively, f_{tot} is the total focal length of the combination of two lenses, and d is the distance between the two lenses.

1. **Lenses**
   A lens is a optical device that focuses or disperses a light beam by means of refraction. The most simple way to determine the focal length of a lens is to create an image of any object using the lens, calculate the object length and image length, and using equation (2) calculate the focal length.

2. **Optical Instruments**
   Optical instruments are the devices which process light wave to enhance an image for a more clear view. The use of optical instruments, such as a *microscope* or *telescope*, usually makes things bigger and helps us to see in a more detailed manner. The use of converging lenses makes things appear larger, and on the other hand, diverging lenses always get you smaller images. The normal closest distance for a young adult is about 25cm (the near point) from the eye. Bringing the image closer than 25cm will blur it.

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   Figure 2: Angular size of objects

   - *The simple magnifier*
     It is usually a single convergent lens placed between the page and the eye to give a magnified image. In this case, the *magnifying power* or *angular magnification* is measured as the ratio of the angular size of the virtual image to the angular size of the object, where angular size means apparent size as seen from an observer (Fig. 2). Using a simple magnifier creates a virtual image that has a large angular size than the object has.

     $$M = \frac{\theta'}{\theta} \tag{4}$$

   - *Microscope*
     A microscope is an instrument used to see objects that are too small to be seen by the normal eye. A typical microscope consists of two lenses: *an objective lens* and *an ocular (or eyepiece) lens.* The first lens is placed close to the object and create a real and magnified image of the object. The second one is used to magnify the image further and to create a virtual image at infinity. The product of the magnifications of the objective and of the ocular gives a total magnification of the microscope.

     $$M = M_{ob} \cdot M_{oc} = \frac{s'_{ob}}{f_{ob}} \cdot \frac{25cm}{f_{oc}} \tag{5}$$

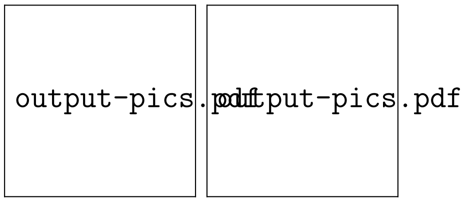

Figure 3: Microscope

If the image is being projected on the screen, then the magnification formula for microscope is rewritten as follows:

$$M = M_{ob} \cdot M_{oc} = \frac{s'_{ob}}{f_{ob}} \cdot \frac{s'_{oc}}{f_{oc}} \tag{6}$$

- *Telescope*
  A telescope is an optical instrument that gathers and focuses light, from the visible part of the electromagnetic spectrum, to create a magnified image for direct view. Mainly, telescopes are used to enlarge the retinal image of a distant object. Typically, it also has two lenses: *an objective lens* and *an ocular (or eyepiece) lens*. However, in this case, the object is located far away from the telescope, therefore, rays from the object enter the lens almost as parallel rays. That is why their image lays on the focal plane of the objective lens. Using an ocular increases magnification and results in an angular magnification of **the Kepler telescope**. This type of telescope consists of only two convex lenses and has a magnification of

$$M_{Kep} = -\frac{f_{ob}}{f_{oc}} \tag{7}$$

Such telescopes create an inverted image of the object. On the other hand, **the Galilei telescopes** produces upright images. This result is achieved by placing a concave lens between the real first image produced by the objective lens and objective lens.

$$M_{Gal} = \frac{f_{ob}}{f_{oc}} \tag{8}$$

2 Experimental and Procedure

2.1 Main procedures

2.1.1 Calibration

Lights were dimmed, in order to improve picture sharpness classification capabilities in the experiment. All elements on the optical bench were carefully aligned (same height, direction of holders perpendicular to the optical axis, lamp direction parallel to the optical axis). Condenser lens in the lamp was moved into position, from which lamp gives sharp image on the infinity (practically speaking, on e.g. the wall at the sufficiently long distance in comparison to typical experiment distances (distance to wall ≈ a couple of meters))

2.1.2 Actual experimental procedures

2.1.2.1 Measuring the focal length of a lens The focal length of a lens was measured using the Lens Formula (2) in combination with following setup: Focal length measurement setup

Figure 4: Focal length measurement setup

2.1.2.2 Measuring the focal length of a system of 2 lenses The focal length of a 2-lens system (following lens combinations (from left to right on the schemes) were used: (100 mm; 200 mm) and (100 mm; -200 mm)) was calculated using the Formula for the closely situated Lenses (3). This result was used to calculate theoretical distance from system of lenses to a sharp image, and was compared to the length result from an experimental setup: Double convex-convex lens system

Figure 5: Double lens (convex+convex) setup

For combination of convex and concave lens system equations are the same (but it's important to pay attention signs of values, that are being plugged into formulas), and setup is similar: Double convex-concave lens system

Figure 6: Double lens (convex+concave) setup

2.1.2.3 Creation of Optical Instruments

2.1.2.3.1 Microscope The microscope was built using following lenses - (50 mm - objective; 20 mm - ocular) in a following setup: Microscope setup

Figure 7: Microscope setup

The cover glass with a dog flea was observed through microscope. The microscope image was focused on the screen.

2.1.2.3.2 Kepler Telescope The Kepler Telescope was built using following lenses - (300 mm - objective; 50 mm - eyepiece) in a following setup: Kepler Telescope setup

Figure 8: Kepler telescope setup

Inverted enlarged image of Snellen chart (eye chart) was observed through Kepler telescope.

2.1.2.3.3 Galileo Telescope The Galileo Telescope was built using following lenses (300 mm - objective; -50 mm - ocular) in a following setup: Galileo Telescope setup

Figure 9: Galileo telescope setup

Enlarged normal image of Snellen chart (eye chart) was observed through Galileo telescope.

2.2 Main equipment

For optical instruments construction the following parts were used:

- Experimental lamp with built-in condenser lens and the power supply
- Set of converging and diverging lenses
- Screen
- Optical bench with holders and ruler for holder position determination
- Arrow slit, cover glass with a dog flea

3 Results and Data Analysis

3.1 Lenses

First setup (Focal Length Measurement) gave us data, represented in Table 1(1).

	d_1 [cm]	l_1 [cm]	F [cm] (rounded)
A	20	20	10.00
	25	17	10.12
	30	15	10
	23	18	10.10
	28	15.5	9.98
B	28	31	14.72
	40	23.5	14.80
	35	25.5	14.75
	24	38	14.71
	30	29	14.75

Table 1: Raw measurements and their focus lengths based on the formula (2) (Unrounded values for F can be found in appendix C)

Based on this raw data and Lens Formula Focal Lengths of lenses A and B were calculated: ($\Delta d_1 = 0.01\,cm$, $\Delta l_1 = 0.5\,cm$ (because the sharpness of image can't be detected ideally by sight - 0.5 cm represents range of positions of which image was sharp) then using the error propagation formula (9) we get ΔF for each row in a table, and then again using (9) we combine all values of F ($F_{fin} = \frac{\sum_{i=1}^{n} F_i}{n}$) and all respective errors ($\Delta F_{fin} = \frac{1}{n}\sqrt{(\sum_{i=1}^{n} \Delta F_i)^2}$) into the final value $< F > \pm\Delta F$)

$$< F_A >= (10.039 \pm 0.082)\, cm$$
$$\varepsilon_A = 0.81\%$$

1: Focus length of lens A

$$< F_B >= (14.745 \pm 0.065)\, cm$$
$$\varepsilon_B = 0.44\%$$

2: Focus length of lens B

Next 3 setups (System of Lenses Measurement - First Lens, System of Lenses Measurement - First+Second Lenses, System of Lenses Measurement - First+Third Lenses) gave us the next 3 tables of raw data - (2, 3, 4).

F_1 [cm]	d_1 [cm]	l_1 [cm]
10	20	19.9

Table 2: One-lens system

F_1 [cm]	F_2 [cm]	d_1 [cm]	d_2 [cm]	l_2 [cm]
10	20	20	4	12.5

Table 3: Convex-convex lens system

F_1 [cm]	F_3 [cm]	d_1 [cm]	d_2 [cm]	l_3 [cm]
10	-20	20	4	50.8

Table 4: Convex-concave lens system

Here we use the Lens Formula (2) to get theoretical estimates of values l_1, l_2, l_3 as follows:

$$\frac{1}{d_1} + \frac{1}{l_1} = \frac{1}{F} \Longrightarrow l_1 = \frac{d_1 \cdot F}{d_1 - F}$$

$$\frac{1}{d_1} + \frac{1}{l_1} = \frac{1}{F_{system}} \overset{(3)}{=} \frac{1}{F_1} + \frac{1}{F_2} - \frac{d_2}{F_1 \cdot F_2} \Longrightarrow l_2 = \frac{1}{\frac{1}{F_1} + \frac{1}{F_2} - \frac{d_2}{F_1 \cdot F_2} - \frac{1}{d_1}}$$

$$l_3 = \frac{1}{\frac{1}{F_1} + \frac{1}{F_3} - \frac{d_2}{F_1 \cdot F_3} - \frac{1}{d_1}} \;(Analogously)$$

Respective errors calculation was performed using Error Propagation Formula (9).

$$\Delta l_1 = (\frac{F}{d_1 - F})^2 \cdot \Delta d_1$$

$$\Delta l_2 = \frac{1}{\frac{1}{F_1} + \frac{1}{F_2} - \frac{d_2}{F_1 \cdot F_2} - \frac{1}{d_1}} \sqrt{(\frac{\Delta d_2}{F_1 \cdot F_2})^2 + (\Delta d_1)^2}$$

$$\Delta l_3 = \frac{1}{\frac{1}{F_1} + \frac{1}{F_3} - \frac{d_2}{F_1 \cdot F_3} - \frac{1}{d_1}} \sqrt{(\frac{\Delta d_2}{F_1 \cdot F_3})^2 + (\Delta d_1)^2}$$

Putting all calculations together we get the final values (once again distances, which were calculated empirically, but are related to the position of the screen, have 0.5 cm error, or even 1 cm in convex-concave case (and they are related to the question - which screen position gives sharp image - that's what makes their error higher than just ruler instrumental error)).

$$< l_{1_{exp}} >= (19.9 \pm 0.5)\, cm$$
$$\varepsilon_{l_{1_{exp}}} = 2.6\%$$

3: Explicitly measured distance from lens to the screen

$$< l_{1_{theor}} >= (20.0 \pm 0.1)\, cm$$
$$\varepsilon_{l_{1_{theor}}} = 0.5\%$$

4: Theoretically calculated distance from lens to the screen

$$< l_{2_{exp}} >= (12.5 \pm 0.5)\, cm$$
$$\varepsilon_{l_{2_{exp}}} = 4\%$$

5: Explicitly measured distance from convex-convex lenses system to the screen

$$< l_{2_{theor}} >= (12.5 \pm 1.3)\, cm$$
$$\varepsilon_{l_{2_{theor}}} = 11\%$$

6: Theoretically calculated distance from convex-convex lenses system to the screen

$$< l_{3_{exp}} >= (50.8 \pm 1.0)\, cm$$
$$\varepsilon_{l_{3_{exp}}} = 2\%$$

7: Explicitly measured distance from convex-concave lenses system to the screen

$$< l_{3_{theor}} >= (50.0 \pm 5.1)\, cm$$
$$\varepsilon_{l_{3_{theor}}} = 11\%$$

8: Theoretically calculated distance from convex-concave lenses system to the screen

The significant source of error of the experimental approach is the sharpness of image uncertainty - which makes errors of direct measurements larger. On the other hand lens formula has a lot of inverses of variables in it, which is one of the factors why theoretical calculations have higher errors than direct measurement (i. e. final formulae for theoretical values are complicated, which in turn multiplies error propagation). In order to make theoretical approach more precise we need to mainly tweak the quality of measurement of d_1, because d_2 is involved in the smallest by value term, thus not making a great impact on errors of theoretical method.

3.2 Eyeglasses

All three systems that we've created (100 mm; 100 mm+200 mm; 100 mm-200 mm) in the end do all have their optical power greater than zero. This means that they all resemble converging systems, therefore they could only be used as lenses in glasses of farsighted people (shortsighted people need negative diopters, so that the image from glasses would become closer to them than the original object)

3.3 Microscope

Details of microscope were set up in such a way, that in order to focus the image, one had to move the lamp with the object (which is similar in principle to what one would do with the professional optical microscope - the gear should be rotated, which will change distance of object from microscope, and in the right position person is able to see focused magnified image of an object)
To measure s'_{obj} temporary screen was put in-between the lenses in the position where image of a dog flea is focused on this Temp-Screen (Microscope setup). Then the length of an image dog flea was measured, as well as the length of a dog flea in a glass cover (B)
In the next table raw information from Microscope setup is provided (5).

F_1 [cm]	F_2 [cm]	x_1 [cm]	x_2 [cm]	s'_{obj} [cm]	s'_{oc} [cm]	$L_{image\,dog\,flea}$ [cm]	$L_{0\,dog\,flea}$ [cm]
5	2	7.7	20	18	30	8	0.2

Table 5: Microscope setup raw data

Using these measurements Magnification of the Microscope was calculated (in experimental and theoretical way).
Experimental way - divide size of image by size of starting object. Theoretical comes

from angular magnification knowledge and geometry (6).

$$M_{exp} = \frac{L_{image\,dog\,flea}}{L_{0\,dog\,flea}}$$

$$\Delta M_{exp} = \sqrt{(\frac{1}{L_{0\,dog\,flea}}\Delta L_{image\,dog\,flea})^2 + (\frac{L_{image\,dog\,flea}}{L^2_{0\,dog\,flea}}\Delta L_{0\,dog\,flea})^2}$$

$$M_{theor} = \frac{s'_{obj} \cdot s'_{oc}}{F_1 \cdot F_2}$$

$$\Delta M_{theor} = \sqrt{(\frac{s'_{oc}}{F_1 \cdot F_2}\Delta s'_{obj})^2 + (\frac{s'_{obj}}{F_1 \cdot F_2}\Delta s'_{oc})^2}$$

$$< M_{exp} >= 40 \pm 21$$
$$\varepsilon_{M_{exp}} = 53\%$$

9: Theoretically calculated distance from convex-concave lenses system to the screen

$$< M_{theor} >= 54 \pm 3$$
$$\varepsilon_{M_{theor}} = 4.3\%$$

10: Theoretically calculated distance from convex-concave lenses system to the screen

3.4 Kepler Telescope

Kepler telescope was set up on the optical bench in the lab hallway, and was pointed to the eyechart on the opposite end of the hallway (Kepler Telescope setup). Telescope gave inverted magnified image of the eyechart (B). To get sizes of object and its image, images were taken by smartphone and were zoomed to the same maximum zoom in phone's photo editor ($\approx$ x10 zoom) and then were measured using the ruler (these results are specified in the raw data table 6)
Magnification of Kepler Telescope was calculated in 2 ways: [experimental] - divide image size by object size, [theoretical] - divide focal length of objective by focal length of ocular (Kepler Telescope Magnification formula). Respective errors were calculated using formula 9.

$$M_{exp} = \frac{S_{eyediagram\,image}}{S_{eyediagram\,0}}$$

$$\Delta M_{exp} = \sqrt{(\frac{1}{S_{eyediagram\,0}}\Delta S_{eyediagram\,image})^2 + (\frac{S_{eyediagram\,image}}{S_{eyediagram\,0}}\Delta S_{eyediagram\,0})^2}$$

$$M_{theor} = \frac{F_2}{F_1}$$

$F_{objective\,k}$ [cm]	$F_{eyepiece\,k}$ [cm]	$S_{eyediagram\,0}$ [cm]	$S_{eyediagram\,image}$ [cm]
30	5	1	6.2

Table 6: Kepler Telescope setup raw data

Final results are calculated, based on raw data (6) and respective formulae.

$$< M_{theor} >= 6$$

11: Theoretically calculated magnification of Kepler Telescope

(Error value is very small for M_{theor}, we assume that focal lengths of lenses are given by manufacturers with much lower errors (automated factory level precision), compared to other measurements in the lab)

$$< M_{exp} >= 6.2 \pm 0.4$$
$$\varepsilon_{M_{exp}} = 6.3\%$$

12: Experimentally calculated magnification of Kepler Telescope

3.5 Galileo Telescope

The procedures for the Galileo Telescope were the same as for the Kepler Telescope (Galileo Telescope setup), as well as calculation procedure (formulae for the absolute value of magnification are the same for both telescopes (compare Kepler Telescope Magnification formula and Galileo Telescope Magnification formula - they are only different in sign, because Kepler telescope inverts image and Galileo Telescope doesn't)).
Galileo telescope gave the normal magnified image of the eyechart (B).

$F_{objective\,k}$ [cm]	$F_{eyepiece\,k}$ [cm]	$S_{eyediagram\,0}$ [cm]	$S_{eyediagram\,image}$ [cm]
30	-5	1	5.8

Table 7: Galileo Telescope setup raw data

Final results are calculated, based on raw data (7) and respective formulae.

$$< M_{theor} >= 6$$

13: Theoretically calculated magnification of Galileo Telescope

(Error value is very small for M_{theor}, we assume that focal lengths of lenses are given by manufacturers with much lower errors (automated factory level precision), compared to other measurements in the lab)

$$< M_{exp} >= 5.8 \pm 0.4$$
$$\varepsilon_{M_{exp}} = 6\%$$

14: Experimentally calculated magnification of Galileo Telescope

4 Error analysis

This is the most important formula for error evaluation in the experiments

$$\Delta y = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i} \Delta x_i \right)^2} \tag{9}$$

Error Propagation Formula [1]

Relative error of value z is denoted as ε_z, it gives the comprehension on how precise the measurement is

$$\varepsilon_z = \frac{\Delta z}{< z >} \tag{10}$$

Relative Error Formula

Extra notes on error calculation are in the appendix A.

5 Discussion

The first experiment finds out the focuses of lenses A and B. Experimentally, distances between the lens and the object and between the lens and an image of the object were found. However, since it is difficult to find the right position of a screen, which gives a sharp image by plain sight, it was reasonable to specify this as the range of possible values where the image is sharp in a form of enlarged error of direct measurement Moreover, doing several measurements increased the accuracy of the data. Using such an approach gave more data which gave more confidence in the final data.

In the second experiment, a microscope was created by using different two lenses. Since any microscope is used to enlarge small objects to a screen, cover glass with a dog flea – which had a very small size – was used to show the magnified image. A dog flea had a very small size and the measurement with the ruler wasn't very precise. That is why the range of the magnification was so large. However, even with the large range in the magnification, theoretical, and experimental range of values intersect. The dog flea's size could be measured by using the telephone camera: taking the picture of the dog flea and using the phone's camera scale, measure the size of the dog flea more accurately. Such a way would give more accurate value in comparison to plain ruler direct measurement.

Kepler and Galileo telescopes are used to see an object at long distances. In both cases, magnification was calculated in two ways: dividing image size by object size and dividing the focal length of the objective by focal length of ocular. In this case, the object and the image sizes were measured by using the phone's camera scale which gave more accurate values. (The range is not as large as in the experiment with the microscope.) Since the object measurements as well as image measurements had their instrumental error, experimentally calculated magnification was presented with error range.

In these experiments, most of the experimental results almost matched the results based on formulas. It means that the experiments support correctness of theoretical knowledge.

Another way of determining focal lengths of lenses is to find a position where the highlighted field on the screen has the smallest area, instead of finding the position where an image will be sharp. It is easier to find such a position (using light detector, for example, this process can be automated) and on this position screen will be on the distance of lens' focal length from a lens.

6 Conclusion

In this experiment, the focal length of unknown two lenses, the total focal length of the microscope, and the total focal length of the Kepler and Galileo telescopes were found using several methods. In cases with microscope and telescope, experimental values were confirming the theoretical values. Moreover, in cases with unknown lenses and telescope, the error ranges were small ($\varepsilon < 10\%$). This means that data from the experiment was obtained with good precision. To avoid inaccuracy with measurements of the small objects, by using device such as phone's camera and zooming into the picture higher accuracy can be obtained.
Experimental results confirm geometrical optics laws.

A Error notes

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(a) Part 1

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(b) Part 2

Figure 10: Error notes

B Optical instrument photos

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(a) Snellen chart (viewed from Kepler Telescope)

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(b) Snellen chart (viewed from Galileo Telescope)

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(c) Snellen chart (viewed by plain sight)

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(d) Dog flea image from microscope on the screen

Figure 11: Optical instruments

C Raw data tables (unrounded)

F [cm] (unrounded)
10
10.11905
10
10.09756
9.977011
14.71186
14.80315
14.75207
14.70968
14.74576

Table 8: Unrounded raw data (F in the focal length measurement setup)

References

[1] Prof. Dr. Jürgen Fritz. *Error Analysis Booklet*. Bremen: Jacobs University, 2019.