Lab Report 2

Ohm's Law

Thevenin's and Norton's Theorem

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1 Introduction and Theory

In the first experiment, Ohm's Law was studied. Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference across the two points. This proportionality is called resistance.

$$I = \frac{V}{R}$$

Formula 1

For a strict implementation of this rule, the temperature and the resistance R have to be constant. In this case, the behavior is called "ohmic."
Additionally, in the first part of the experiment, different resistors such as copper wire, metal film, PTC resistor, and NTC resistor were studied.
In the second experiment, practical usage of Thevenin's and Norton's theorems was shown. Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load. While Norton's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single current and parallel resistance connected to a load.

2 Ohm's Law

2.2 Resistance of a copper wire

2.2.1 Objective

A copper wire can be resistor in a circuit with resistance given by the formula:

$$R = \rho \frac{l}{A}$$

Formula 2

where ρ is resistivity which varies from varies from material to material, l is the length of the wire, and A is the cross-sectional area of the wire. In this experiment, wire with $0.0195\,\frac{\Omega\,\mathrm{mm}^2}{\mathrm{m}}$ resistivity, 1m length, and $0.25\mathrm{mm}^2$ cross-sectional area was used. Since the resistance of the wire is very low, Kelvin (4-wire) resistance measurement method was used.

2.2.2 Experimental Set-up and Preparation

The following circuit was wired up. ELABO ammeter was set to 2A range, Tenma voltmeter to mV range, and the wire was connected to 1A current source in series.

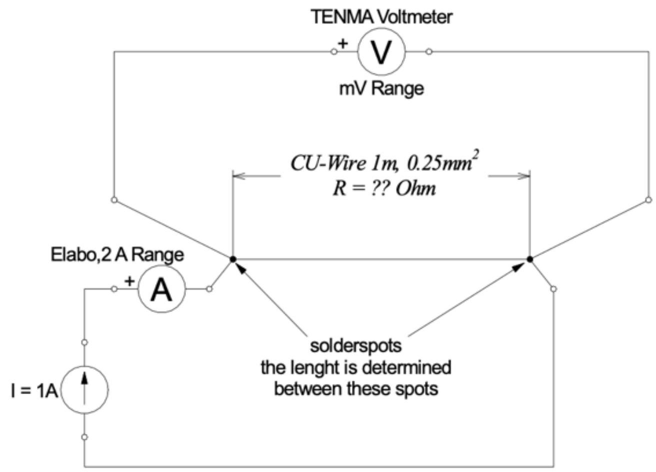


Figure 1

After preparing the circuit, voltage and current were recorded. Then, Tenma was changed to resistance mode and the resistance of the wire was measured. The table 1 shows all recorded numbers.

2.2.3 Results

Reading Voltage, V	Reading Current, A	Reading Resistance, Ohm
0.068	1.015	0.190

Table 1

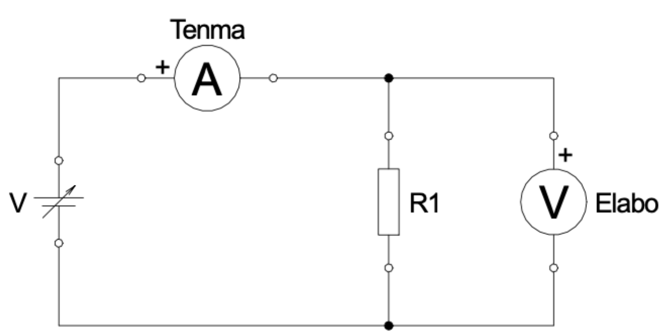
2.3 Resistance of a metal film resistor

2.3.1 Objective

In this experiment, a metal film resistor was used as an ohmic resistor. In real-life, a metal film resistor cannot be an ohmic resistor. However, in this case, due to the narrow limits, a metal film resistor can be taken as constant. To show the behavior of a metal film resistor, the data at different voltages was documented.

2.3.2 Experimental Set-up and Preparation

The following circuit was constructed for this experiment. The supplied voltage was varied from 0V to 24V in 2V steps, and at every step, the current and the voltage were recorded. Then, Formula 1 gave the resistance of the used resistor.



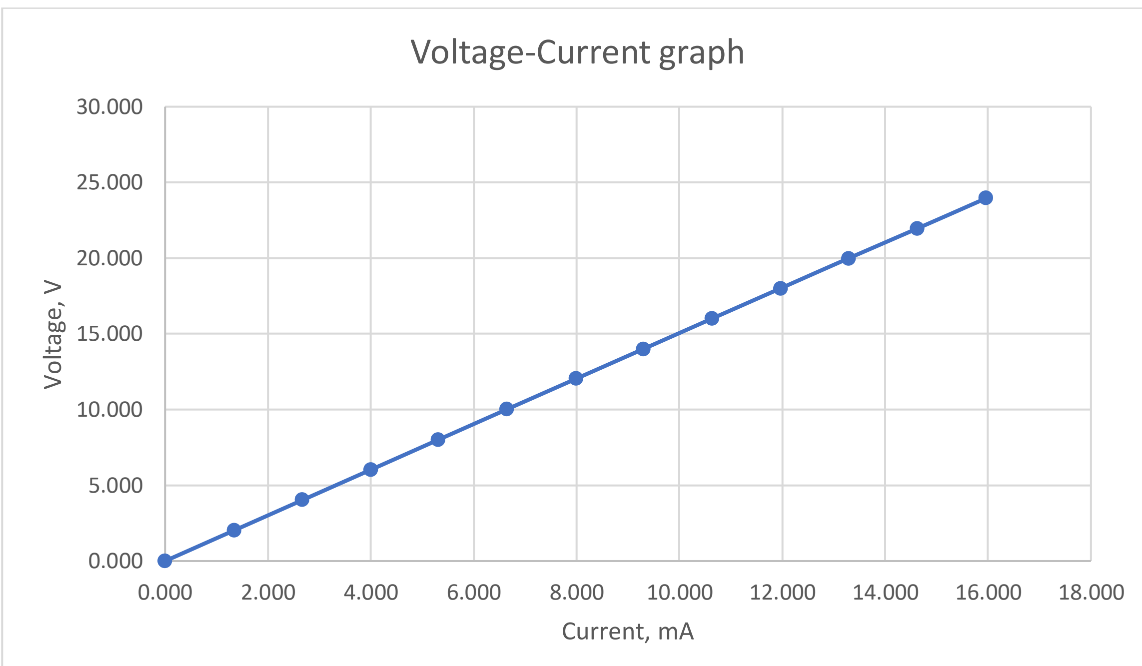
$$R_1 = 1k50\,\Omega$$

Figure 2

2.3.3 Results

Supplied Voltage, V	Reading Voltage, V	Reading Current, mA	Resistance, Ohm
0	0.000	0.000	Undefined
2	2.037	1.351	1507.772
4	4.025	2.670	1507.491
6	6.028	4.000	1507.038
8	8.000	5.306	1507.727
10	10.014	6.643	1507.451
12	12.048	7.998	1506.377
14	14.006	9.302	1505.698
16	16.016	10.638	1505.546
18	18.009	11.968	1504.763
20	19.987	13.286	1504.365
22	21.970	14.633	1501.401
24	23.960	15.965	1500.783

Table 2



Graph 1

2.4 Resistance of a PTC resistor

2.4.1 Objective

The thermistor is a temperature-sensing device whose resistance is a function of its temperature. One of the types of thermistors is PTC (positive temperature coefficient). The resistance of a PTC thermistor increases as the temperature increases. The following formula gives the resistance of a PTC resistor at a temperature lower than 150℃:

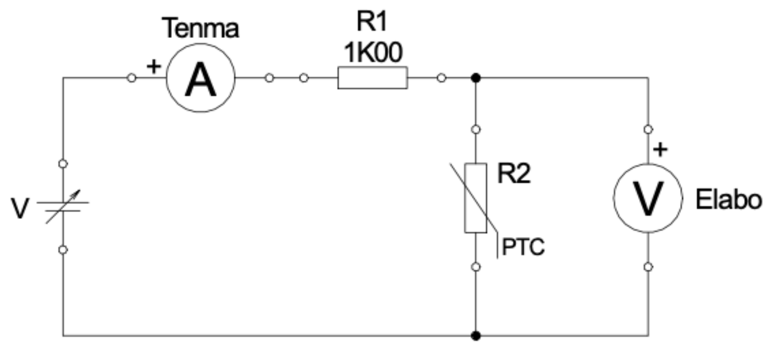
$$R_T = R_{25} \times (1 + \alpha \times \Delta T)$$

Formula 3

where R_T is the resistance at temperature T, R_{25} is the resistance at the reference temperature which is, in this case, 25°C (since in this experiment a nickel thin film thermistor used as a PTC resistor, $R_{25} = 1500\Omega$), ΔT is the difference in temperature between reference temperature and actual temperature, and α is the temperature coefficient (for a nickel thin is $3.8724 \times 10^{-3}\ K^{-1}$).

2.4.2 Experimental Set-up and Preparation

The following circuit was constructed for this experiment. The supplied voltage was varied from 0V to 24V in 2V steps with a 2min time interval to allow the temperature at the PTC resistor to be stable, and at every step, the current and the voltage were recorded. Then, Formula 1 gave the resistance of the used resistor.



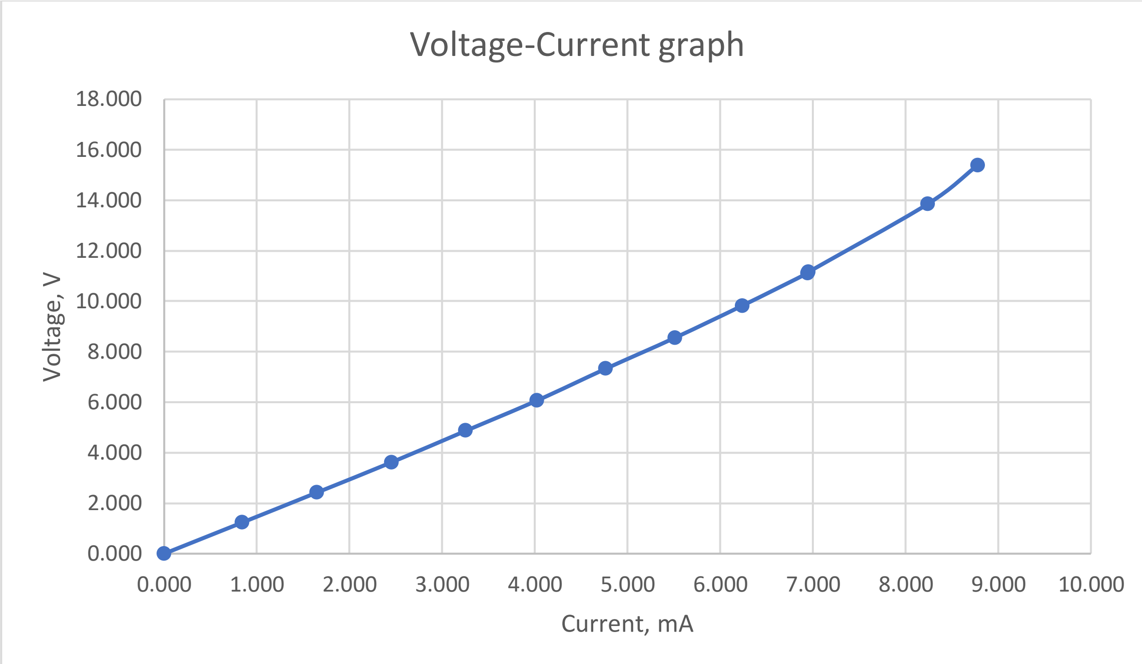
$$R_2 = 1k50\,\Omega$$

Figure 3

2.4.3 Results

Supplied Voltage, V	Reading Voltage, V	Reading Current, mA	Resistance, Ohm
0	0.000	0.000	Undefined
2	1.230	0.839	1466.031
4	2.425	1.650	1469.697
6	3.621	2.454	1475.550
8	4.868	3.255	1495.545
10	6.063	4.021	1507.834
12	7.330	4.767	1537.655
14	8.561	5.516	1552.030
16	9.828	6.240	1575.000
18	11.119	6.947	1600.547
20	11.155	6.950	1605.036
22	13.860	8.242	1681.631
24	15.390	8.778	1753.247

Table 3



Graph 2

2.5 Resistance of a NTC resistor

2.5.1 Objective

A thermistor is a temperature-sensing device whose resistance is a function of its temperature, as mentioned in the previous section. Another type of thermistors is NTC (negative temperature coefficient). The resistance of an NTC thermistor decreases as temperature increases, and this type seems to be the most commonly used thermistor. The following formula gives the resistance of the NTC resistor:

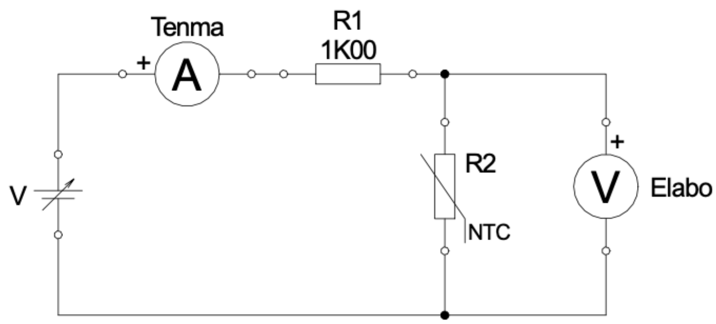
$$R_T = R_{25} \times e^{\beta\left(\frac{1}{T}-\frac{1}{T_0}\right)}$$

Formula 4

where R_T is the resistance at temperature T, R_{25} is the resistance at the reference temperature, T is the actual temperature, T_0 is the reference temperature which in this experiment is $273.15° + 25° = 298.15°\text{K}$, and β is a constant depending on the material. The NTC used in this part of the experiment has a constant of 3560K.

2.5.2 Experimental Set-up and Preparation

The following circuit was wired up for this experiment. The supplied voltage was varied from 0V to 24V in 2V steps with a 2min time interval to allow the temperature at the NTC resistor to be stable, and at every step, the current and the voltage were recorded. Then, Formula 1 gave the resistance of the used resistor.



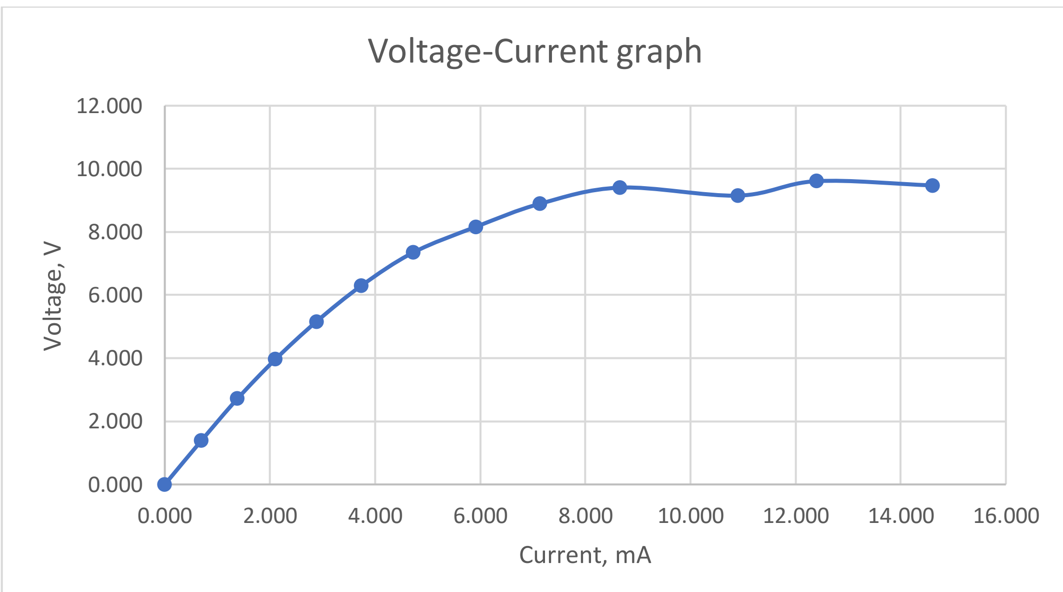
$$R_2 = 1k50\,\Omega$$

Figure 4

2.5.3 Results

Supplied Voltage, V	Reading Voltage, V	Reading Current, mA	Resistance, Ohm
0	0.000	0.000	Undefined
2	1.401	0.702	1995.726
4	2.715	1.381	1965.967
6	3.965	2.104	1884.506
8	5.163	2.887	1788.362
10	6.306	3.747	1682.946
12	7.351	4.728	1554.780
14	8.165	5.922	1378.757
16	8.895	7.140	1245.798
18	9.406	8.668	1085.141
20	9.152	10.912	838.710
22	9.609	12.400	774.919
24	9.474	14.610	648.460

Table 4



Graph 3

3 Thevenin's and Norton's Theorem

3.1 A linear network

3.1.1 Objective

In this part, the voltage and the current between terminals A-B were recorded. In the next sections, these values would be needed to find Thevenin and Norton equivalents.

3.1.2 Experimental Set-up and Preparation

The following circuit was built in this part with the voltage supply 15V. Both Tenma and ELABO multimeters was switched to voltage measurement mode.

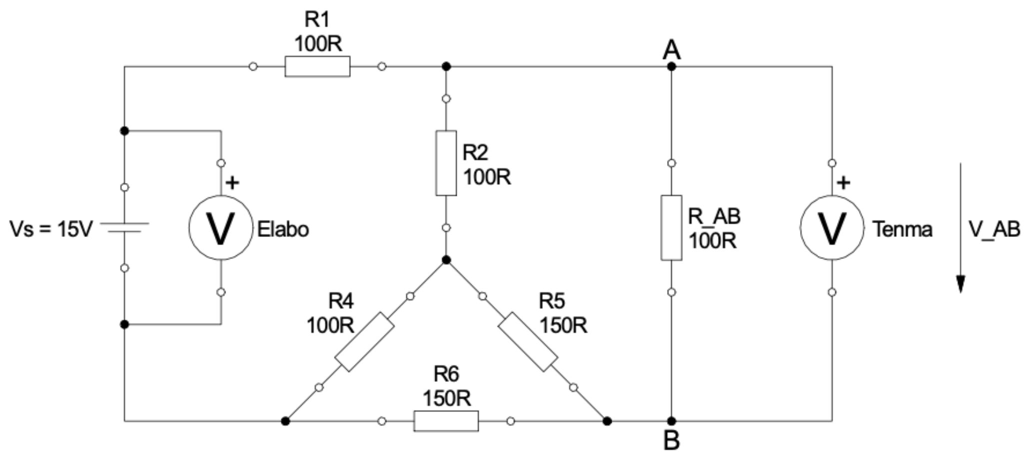


Figure 5

3.1.3 Results

Supplied Voltage, V	Reading Voltage, V (ELABO)	Reading Voltage, V (Tenma)
15.00	15.00	3.34

Table 5

3.2 Determine Thevenin's and Norton's parameters

3.2.1 Objective

In this part, the load was replaced by a short-circuit and an open-circuit to get V_{Th} and I_N. However, to get R_{Th} and R_N, the voltage source was replaced by a short-circuit.

3.2.2 Experimental Set-up and Preparation

First, the load was removed and a voltage between terminals A-B was recorded. This value gives V_{Th}. Then, the terminals A-B was short-circuited and the Tenma was switched to ammeter mode. In this case, Tenma showed the I_N. Finally, the voltage supply was replaced by a short-circuit, and the Tenma multimeter was switched to measure the resistance between terminals A-B. All values were documented at the best range.

3.2.3 Results

Thevenin Voltage, V	Norton Current, mA	Resistance, Ohm
7.47	57.11	124.72

Table 6

3.3 Determine $\boldsymbol{V_{AB}}$ using Thevenin's circuit

3.3.1 Objective

In this part, it was shown that Thevenin's circuit is equivalent to the initial circuit. To do so, data from the previous section was used.

3.3.2 Experimental Set-up and Preparation

Thevenin's equivalent of the circuit in Figure 5 was constructed, which is shown in the following figure. The Thevenin resistance was set to 125Ω, the closest to the Thevenin's resistance that was founded in the previous section. Also, $R_{AB} = 100\Omega$ and voltage supply should be approximately 7.47V.

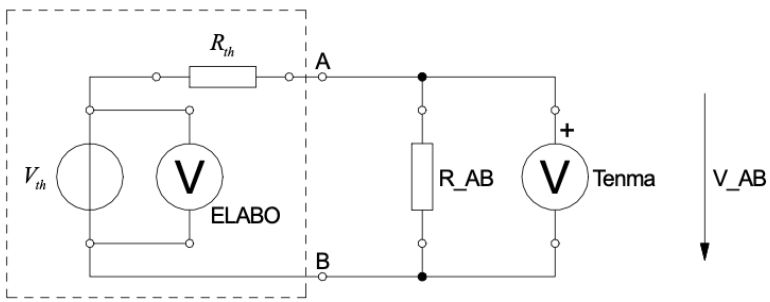


Figure 6

3.3.3 Results

Supplied Voltage, V	Reading Voltage, V
7.466	3.321

Table 7

As expected, the reading voltage is very close to the reading voltage from Table 5.

3.4 Determine $\boldsymbol{V_{AB}}$ using Norton's circuit

3.4.1 Objective

In this part, it was shown that Norton's circuit is equivalent to the initial circuit. To do so, data from the 3.2 Determine Thevenin's and Norton's parameters section was used.

3.4.2 Experimental Set-up and Preparation

Norton's equivalent of the circuit in Figure 5 was constructed, which is shown in the following figure. The Norton resistance was set to 125Ω, the closest to Norton's resistance. Also, $R_{AB} = 100\Omega$ and the current supply should be approximately 57.11V.

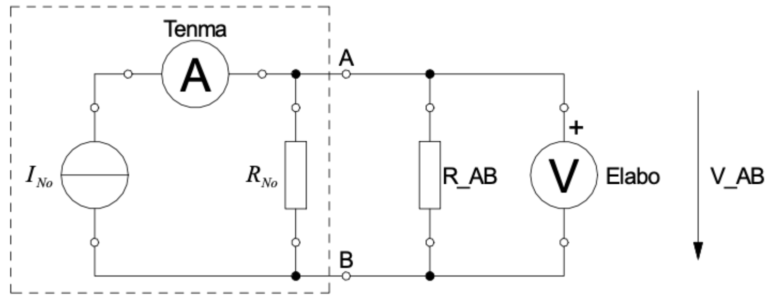


Figure 7

3.4.3 Results

Supplied Current, mA	Reading Voltage, V
57.18	3.16

Table 8

It is clearly seen that the reading voltage is slightly different from Table 5. This is due to the fact that our Norton's Current is not accurate. Theoretically, is has to be around 60mA.

3.5 Evaluation

3.5.1 A linear network

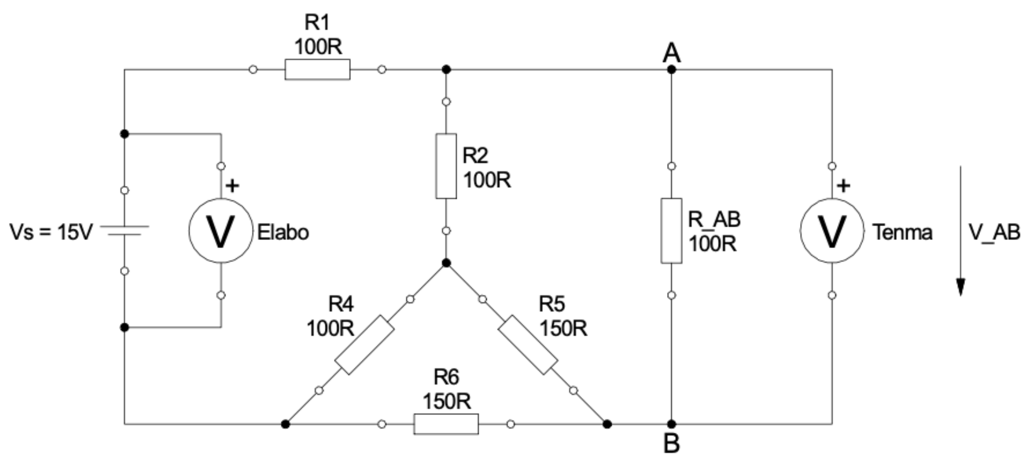
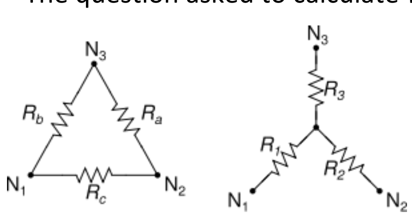


Figure 8

The question asked to calculate V_{AB}. First, delta-to-wye transformation was applied to the circuit.



$$R_1 = \frac{R_b R_c}{R_a+R_b+R_c} \qquad R_2 = \frac{R_a R_c}{R_a+R_b+R_c}$$

$$R_3 = \frac{R_a R_b}{R_a+R_b+R_c}$$

Formula 5

Figure 9

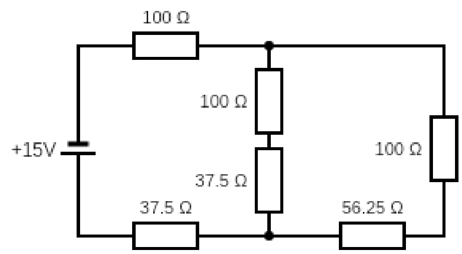


Figure 10

Let the current in the first loop be i_1 and the current in the second loop be i_2. Applying KVL to these two loops gives the following two equations:

$$-15 + 100i_1 + 100(i_1 - i_2) + 37.5(i_1 - i_2) + 37.5i_1 = 0$$
$$100(i_2 - i_1) + 37.5(i_2 - i_1) + 100i_2 + 56.25i_2 = 0$$

Solving these two equations for i_1 and i_2 gives that $i_1 = 0.261\text{A}$ and $i_2 = 0.333\text{A}$. It means the voltage through the load resistor is equal to $i_2 \times 100\Omega = 0.333\text{A} \times 100\Omega = 3.33\text{V}$.

3.5.2 Thevenin's and Norton's equivalents

From the theory, it is known that Thevenin's resistance and Norton's resistance are equal. To find the resistance, all independent sources have to be turned off. After removing all independent sources from the circuit shown in Figure 8, the following circuit can be obtained:

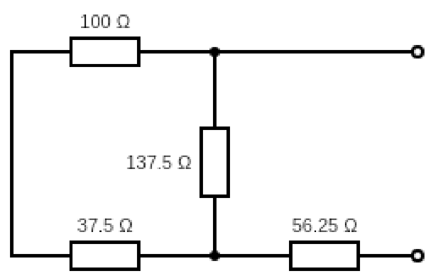


Figure 11

In the first loop, there are two resistors which are connected by series. Applying the formula for the series resistors, the following circuit was derived:

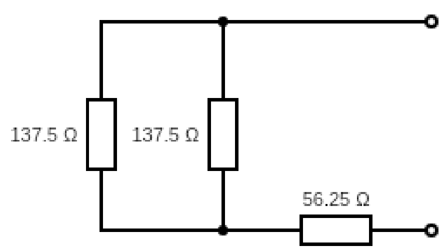


Figure 12

Now, there are two resistors in parallel connection. After connecting them, there will be only two resistors in series connection.

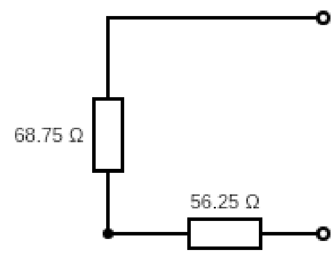


Figure 13

Now, it can be easily obtained that the resistance between the terminals A-B is 125Ω.

To obtain Thevenin's voltage, the terminals A-B have to be open-circuited.

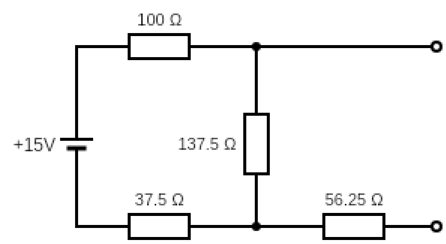


Figure 14

It can be seen that the voltage between terminals A-B is equal to the voltage through the resistor 137.5Ω. Let the current in the first loop be i. Then, according to the KVL:

$$-15 + 100i + 137.5i + 37.5i = 0$$

Solving this equation gives $i = \frac{3}{55}$. Now, to calculate the voltage through the resistor with resistance of 137.5Ω, Formula 1 was used.

$$V = Ri = 137.5 \times \frac{3}{55} = 7.5V$$

It gives the Thevenin's Voltage of $7.5V$.

To calculate the Norton's current, the terminals A-B have to be replaced by short-circuit and the current throughout the terminals A-B is needed.

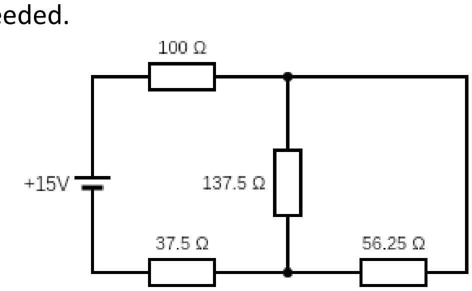


Figure 15

Let the current in the first loop be i_1 and the current in the second loop be i_2. Then, the Norton's current will be equal to i_2. KVL for these two loops gives the following two equations:

$$-15 + 100i_1 + 137.5(i_1 - i_2) + 37.5i_1 = 0$$
$$137.5(i_2 - i_1) + 56.25i_2 = 0$$

From these two equations, it can be obtained that $i_1 = 0.085A$ and $i_2 = 0.06A$. Therefore, the Norton's current is $0.06A$ (or $60mA$)

	Thevenin's Voltage, V	Norton's Current, mA	Resistance, Ohm	Load Voltage, V
Measured	7.47	57.11	124.72	3.34
Calculated	7.5	60	125	3.33

Table 9

Absolute error for the ELABO voltmeter can be calculated by using the following formula:

$$\Delta \mathrm{E} = 0.03\% \times (Reading) + 0.01\% \times (Range)$$

Formula 6

Relative error for all instruments can be calculated by using the following formula:

$$\mathrm{E}_{\mathrm{rel}} = \frac{\Delta E}{Reading} \times 100\%$$

Formula 7

The methodical error can be calculated by using the following formula:

$$\text{Methodical Error} = \frac{Reading - True}{True} \times 100\%$$

Formula 8

Absolute error of the Tenma ammeter for the range 40mA-10A with the resolution of 0.001A can be calculated as:

$$0.5\% \times (Reading) + 30 \times (Resolution)$$

Formula 9

Absolute error of the Tenma voltmeter for the range 400mV-4V with the resolution of 0.0001V can be found by using the following formula:

$$0.05\% \times (Reading) + 5 \times (Resolution)$$

Formula 10

To calculate the absolute error of the Tenma ohmmeter in the range 0Ω - 400Ω, the following formula with the resolution 0.01Ω was used:

$$0.3\% \times (Reading) + 8 \times (Resolution)$$

Formula 11

For all the following tables Formula 7 and Formula 8 gives the relative errors and methodical errors respectively. As a true value, theoretically calculated numbers were used.

In the first part, ELABO and Tenma voltmeters were used to measure the voltage. Therefore, Formula 6 with the range 20V and Formula 10 were used to calculate absolute error.

	Reading, V	Ab. Er., V	Re. Er., %	True Value, V	Me. Er., %
Voltage (ELABO)	15.00	0.0065	0.04%	15.00	0.00%
Voltage (Tenma)	3.34	0.0022	0.06%	3.33	0.30%

Table 10

While in the second part, ELABO was used as a voltmeter with the range 20V, Tenma was used as ammeter and ohmmeter. Thus, for the ELABO Formula 6 was used and for the Tenma Formula 9 and Formula 11 were applied.

	Reading	Ab. Er.	Re. Er., %	True Value	Me. Er., %
Thevenin's Voltage, V (ELABO)	7.47V	0.0042	0.06%	7.5V	-0.40%
Norton's Current, A (Tenma)	57.11mA	0.2357mA	0.41%	60mA	4.82%
Resistance, Ohm (Tenma)	124.72Ω	0.1174Ω	0.09%	125Ω	-0.22%

Table 11

For the next part, only voltmeters were used. Therefore, in calculations Formula 6 with 20V range and Formula 10 were used.

	Reading, V	Ab. Er., V	Re. Er., %	True Value, V	Me. Er., %
Supplied Voltage, (ELABO)	7.47	0.0042	0.06%	7.5	-0.45%
Reading Voltage, (Tenma)	3.32	0.022	0.07%	3.33	-0.26%

Table 12

Finally, to calculate the absolute error of the Tenma ammeter Formula 9 was applied and to calculate the absolute error of the ELABO voltmeter Formula 6 was used with 20V range.

	Reading	Ab. Er.	Re. Er.	True Value	Me. Er
Supplied Current, (Tenma)	57.11mA	0.2357mA	0.41%	60mA	-4.72%
Reading Voltage, (ELABO)	3.16V	0.0029V	0.09%	3.33V	-5.11%

Table 13

Almost all relative errors and methodical errors are less than 1% except the methodical errors for the Norton's Current in Table 11 and, therefore, supplied current and reading voltage in Table 13. Such high errors could be since our instruments have internal resistance and in the case of the Tenma ammeter, this internal resistance isn't small enough to be insignificant compared to the whole circuit's resistance. Thus, our experimental value for the Norton's Current wasn't accurate. It can be seen that theoretical value (expected value) and experimental value are very different. Therefore, in the last part of the experiment, the Reading Voltage wasn't accurate too. In Table 13, it should be mentioned that the relative errors are very small which means that recorded values are precise, while the methodical errors are about 5%.

4 Conclusion

The experiment showed practically Ohm's Law, Thevenin's and Norton's Theorems. Moreover, theoretical and experimental values were compared to better understand the influence of these Laws and Theorems in real-life circuits. Some methodical errors could be avoided by using very small internal resistance in instruments. During the experiment, we have learned how to get values for Thevenin and Norton equivalents.

5 References

Pagel, U., & Joodaki, M. (Fall Semester 2020). *Lab Manual General Electrical Engineering.* Jacobs University Bremen.
All about Circuit. (n.d.). Retrieved from Thevenin's and Norton's Theorem: https://www.allaboutcircuits.com/