Signals and System
Lab Report 3

Fourier Series and Fourier Transform

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Contents

1 Introduction

During the experiment, different analog modulation techniques will be studied. The properties of double-sideband (DSB) modulation, double-sideband suppressed carrier (DSB-SC) modulation, and single-sideband (SSB) modulation and their frequency spectra will be investigated.
Amplitude modulation (AM) is a modulation technique used in electronic communication, most commonly for transmitting messages with a radio wave. Sinusoidal amplitude modulation is one of the most common forms of amplitude modulation. The main idea is to use information signal to vary the amplitude of a sinusoidal signal. Later, some of the most important amplitude modulation techniques will be described.

2 Prelab AM Modulation

2.1 Problem 1: Single frequency Amplitude Modulation

1. The modulated signal is expressed as follows:

$$y(t) = A_c[1 + kA_m cos(2\pi f_m t)]cos(2\pi f_c t)$$

where $m = kA_m$. So, the amplitude of $y(t)$ can be derived as

$$A_c[1 + mcos(2\pi f_m t)]$$

Note that it reaches maximum when $cos(2\pi f_m t) = 1$ and minimum when $cos(2\pi f_m t) = -1$. Thus,

$$\begin{aligned} A_{max} &= A_c[1+m] \\ A_{min} &= A_c[1-m] \end{aligned} \quad \Rightarrow \quad \begin{aligned} A_{max} + A_{min} &= 2A_c \\ A_{max} - A_{min} &= 2mA_c \end{aligned} \quad \Rightarrow \quad \frac{A_{max} + A_{min}}{A_{max} - A_{min}} = \frac{2mA_c}{A_c}$$

$$m = \frac{A_{max} + A_{min}}{A_{max} - A_{min}}$$

2. The modulated signal is expressed as follows:

$$y(t) = A_c[1 + mcos(2\pi f_m t)]cos(2\pi f_c t)$$

Knowing that $cos\alpha \cdot cos\beta = 1/2[cos(\alpha + \beta) + cos(\alpha - \beta)]$ gives:

$$y(t) = A_c cos(2\pi f_c t) + \frac{mA_c}{2} cos(2\pi f_m t - 2\pi f_c t) + \frac{mA_c}{2} cos(2\pi f_m t + 2\pi f_c t)$$

where the first term is the carrier signal, the second term is the lower sideband, and the third one is the upper sideband. The average power delivered to a resistor by the AM signal is compromised of these three terms.

$$P_{tot} = \frac{V_c^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

By using RMS values,

$$P_c = \frac{V_c^2}{R} = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

$$P_{LSB} = P_{USB} = \frac{V_{SB}^2}{R} = \frac{\left(\frac{mA_c/2}{\sqrt{2}}\right)^2}{R} = \frac{m^2A_c^2}{8R} = \frac{m^2}{4}P_c$$

$$P_{tot} = P_c + \frac{m^2}{4}P_c + \frac{m^2}{4}P_c = \frac{2+m^2}{2}P_c$$

$$r_P = \frac{P_s}{P_{tot}} = \frac{P_{USB} + P_{USB}}{P_{tot}} = \frac{P_c m^2/2}{P_c(2+m^2)/2} = \frac{m^2}{2+m^2}$$

3. A modulation index of 100% means that $m = 1$. So,

$$r_P = \frac{m^2}{2+m^2} = \frac{1}{2+1} = \frac{1}{3} = 33.33\%$$

4. The modulated signal is expressed as follows:

$$y(t) = A_c[1 + k(2 + A_m cos(2\pi f_m t))]cos(2\pi f_c t)$$

Knowing $m = A_m/A_c = kA_m$ gives $2kA_c = 2$

$$y(t) = [A_c + 2 + mcos(2\pi f_m t)]cos(2\pi f_c t)$$

$$y(t) = (A_c + 2)cos(2\pi f_c t) + \frac{mA_c}{2}cos(2\pi f_m t - 2\pi f_c t) + \frac{mA_c}{2}cos(2\pi f_m t + 2\pi f_c t)$$

where the first term is the carrier signal, the second term is the lower sideband, and the third one is the upper sideband. The average power delivered to a resistor by the AM signal is compromised of these three terms.

$$P_{tot} = \frac{V_c^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

By using RMS values,

$$P_c = \frac{V_c^2}{R} = \frac{\left(\frac{A_c+2}{\sqrt{2}}\right)^2}{R} = \frac{(A_c+2)^2}{2R}$$

$$P_{LSB} = P_{USB} = \frac{V_{SB}^2}{R} = \frac{\left(\frac{mA_c/2}{\sqrt{2}}\right)^2}{R} = \frac{m^2A_c^2}{8R}$$

$$P_{tot} = \frac{(A_c+2)^2}{2R} + \frac{m^2A_c^2}{8R} + \frac{m^2A_c^2}{8R} = \frac{2(A_c+2)^2 + m^2A_c^2}{4R}$$

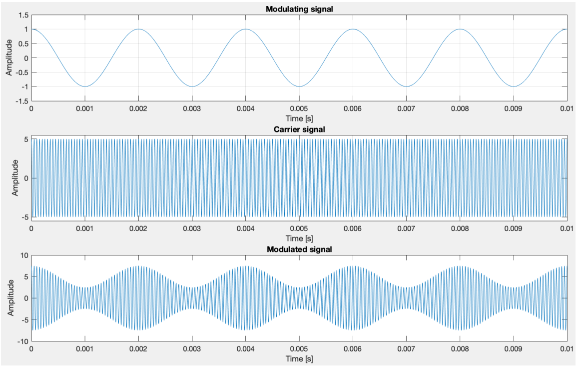
$$r_P = \frac{P_s}{P_{tot}} = \frac{P_{USB} + P_{USB}}{P_{tot}} = \frac{m^2A_c^2/(4R)}{(2(A_c+2)^2 + m^2A_c^2)/(4R)} = \frac{m^2A_c^2}{(2(A_c+2)^2 + m^2A_c^2)}$$

$$r_P = \frac{1}{2(5+2)^2+1} = \frac{1}{99} = 1.01\%$$

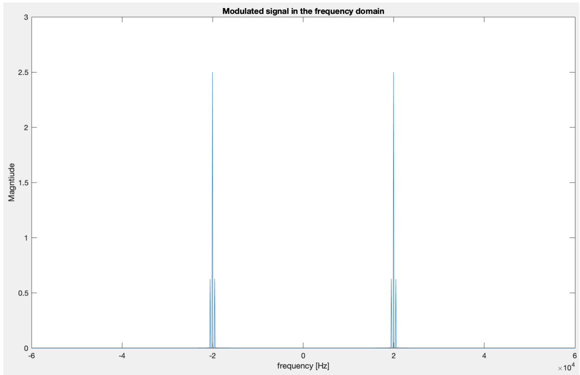
Note that DC offset and modulation index contributes to the total power. Thus, to maximize the ratio, the modulation index should be increased and the DC offset should be decreased.

2.2 Problem 2: Amplitude Demodulation

1. The plots of the modulated signal in time and in frequency domain are shown in Figure 1. Also, item 6 shows the Matlab code.



(a) in time somain



(b) in frequency domain

Figure 1: Modulation signal

2. The Bode diagram is represented in Figure 2 with the Matlab code shown in item 6.

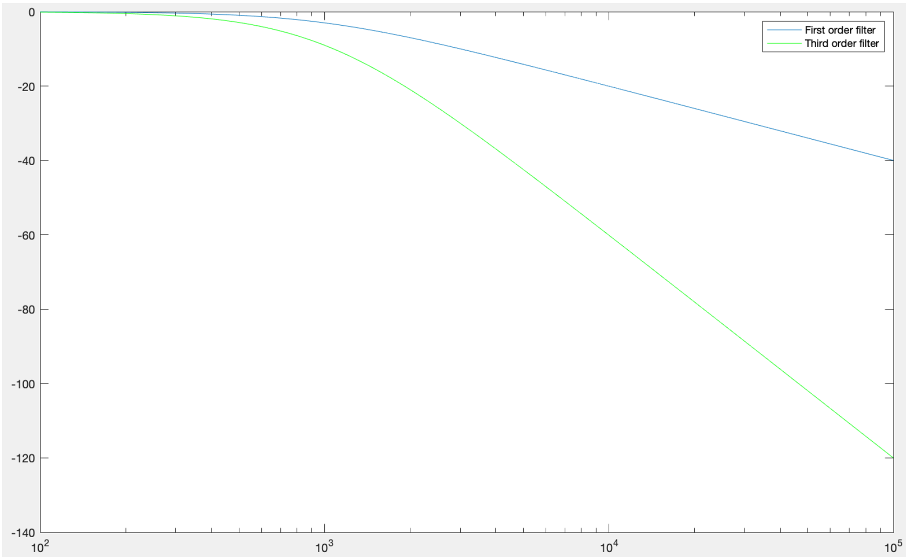


Figure 2: Bode diagram of first and third order filters

3. The plot of the rectified and the demodulated signals are illustrated in Figure 3 with the Matlab code shown in item 6.

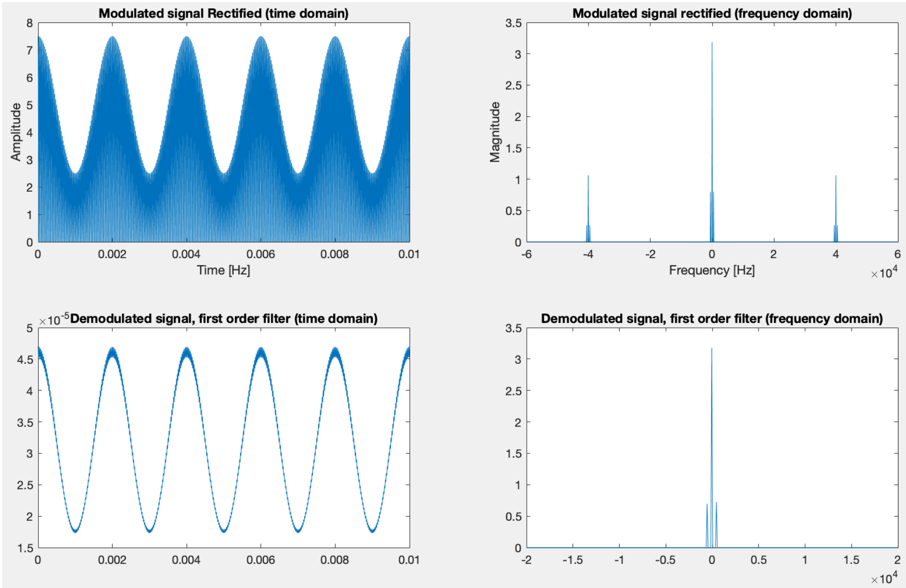


Figure 3: Bode diagram of first and third order filters

4. The plot of the demodulated signal after changing the order of the filter from 1 to 3 is shown in Figure 4 with the Matlab code in item 6.

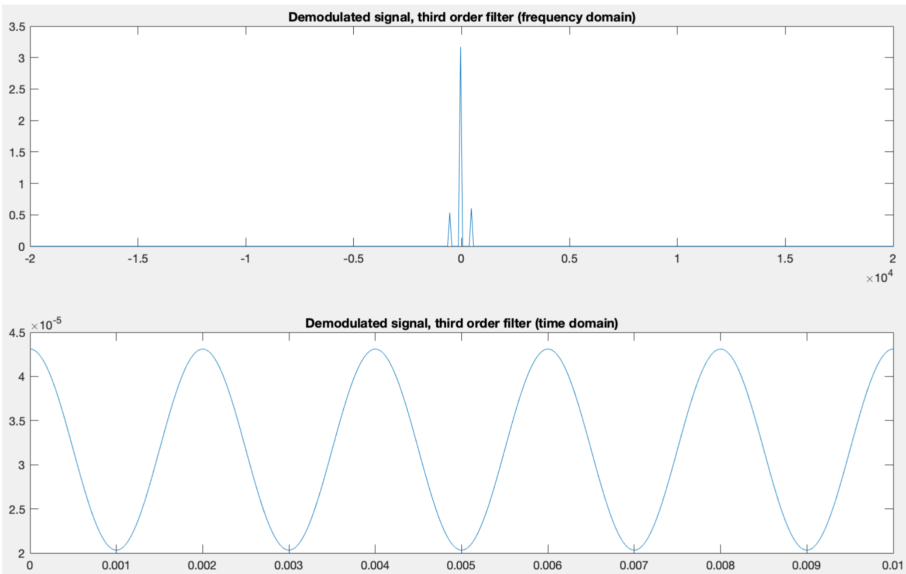


Figure 4: Bode diagram of first and third order filters

5. In the example above, it can be seen that the third order filter gives demodulation with higher quality than the first order one. In the frequency domain, it is really hard to tell the difference. However, when applying the inverse Fourier transform, third order one has the lower distortion of the information signal. It means that the higher the order of the filter, the lower the distortion.

6. The Matlab code used in this problem:

```
%% Variables
Fs = 10* 10^6;

freq_c = 20*10^3;
A = 5;

freq_m = 500;

m = 0.5;

t = 0:1/Fs:0.01;

x = cos(2*pi*freq_m*t);
c = A*cos(2*pi*freq_c*t);

y = A .* (1+m.*x) .* cos(2*pi*freq_c.*t);

%% Time domain

figure(1);

subplot(3,1,1);
plot(t,x);
ylim([-1.5 1.5]);
hold on
grid on

title('Modulating signal');
xlabel('Time [s]');
ylabel('Amplitude');

subplot(3,1,2);
plot(t,c);
ylim([-5.5 5.5]);

title('Carrier signal');
xlabel('Time [s]');
ylabel('Amplitude');

subplot(3,1,3);
plot(t,y);

title('Modulated signal');
xlabel('Time [s]');
ylabel('Amplitude');

%%  Frequency domain

figure(2);

Fnyq = Fs/2;
```

```
L = length(y);

Y = fftshift(fft(y)/L);
L = length(Y);
f = (-L/2:L/2-1) * Fs/L;

plot(f,abs(Y));
xlim([-6e04 6e04]);

title('Modulated signal in the frequency domain');
xlabel('frequency [Hz]');
ylabel('Magntiude');

%% Bode diagram
figure(3);

freq = 100:1:100*10^3;

w = 2*pi*freq;

R = 10*10^3 / (2*pi);

C = 100*10^-9;

order1 = 1 ./ ((w.*R*C*1i) + 1);
order3 = (order1).^3;

semilogx(freq,20*log10(abs(order1)));
hold on
semilogx(freq,20*log10(abs(order3)),'g');

legend('First order filter','Third order filter');

%% Rectified and demodulated signal - first order

figure(4);

% Modulated signal rectified - time domain
rec = abs(y);

subplot(2,2,1);
plot(t,rec);

title('Modulated signal Rectified (time domain)');
xlabel('Time [Hz]');
ylabel('Amplitude');

% Modulated signal rectified - frequency domain
rectspec = fftshift(fft(rec)/length(rec));

subplot(2,2,2);
plot(f,abs(rectspec));

title('Modulated signal rectified (frequency domain)');
```

```
xlabel('Frequency [Hz]');
ylabel('Magnitude');

% Demodulated signal rectified - frequency domain
xlim([-6e4,6e4]);

demod_order1_freq = abs((1./((2*pi.*f*R*C*1i)+1)).*abs(rectspec));

subplot(2,2,4);
plot(f,abs(demod_order1_freq));
xlim([-2e4,2e4]);

title('Demodulated signal, first order filter (frequency domain)');

% Demodulated signal rectified - time domain
demod_order1_time=ifft(demod_order1_freq);

subplot(2,2,3);
plot(t,abs(demod_order1_time));

title('Demodulated signal, first order filter (time domain)');

%% demodulated signal - third order

figure(5);

demod_order3_freq = abs((1./((2*pi.*f*R*C*1i)+1).^3).*(rectspec));


subplot(2,1,1);
plot(f,abs(demod_order3_freq));
xlim([-2e4,2e4]);

title('Demodulated signal, third order filter (frequency domain)');

demod_order3_time = ifft(demod_order3_freq);

subplot(2,1,2);

plot(t,abs(demod_order3_time));

title('Demodulated signal, third order filter (time domain)');
```

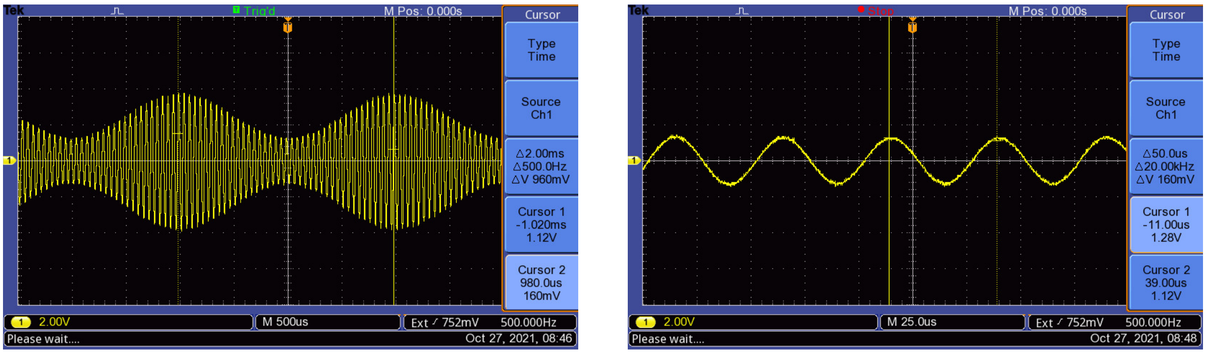
3 Execution AM Modulation

3.1 Problem 1: AM modulated Signals in Time Domain

First of all, the signal generator was connected to the oscilloscope with the following settings:

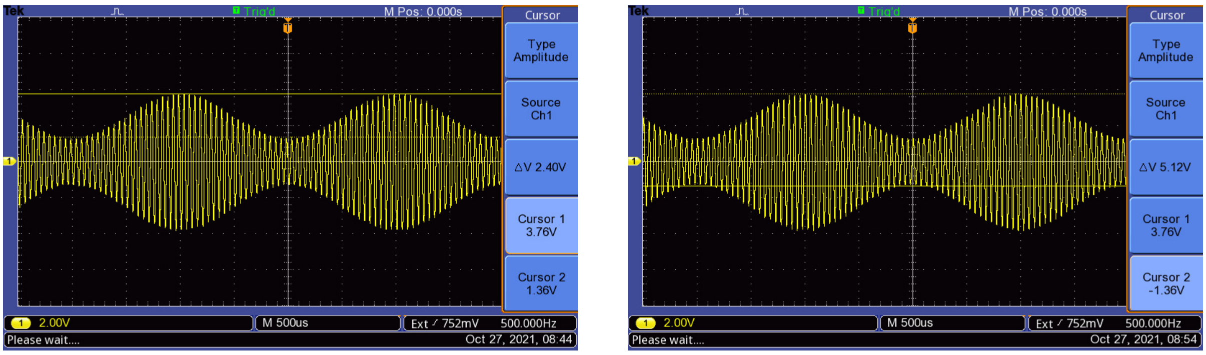
Signal Shape = Sine
Modulation = AM
Carrier Frequency = 20 KHz
Carrier Amplitude = 10 V_{PP}
Modulation Frequency = 500 Hz
Modulation Index = 50%

Then, the frequency and amplitude properties of the modulated signal are obtained. The results are shown in Figure 5 and in Figure 6



(a) Modulating signal frequency (b) Carrier signal frequency

Figure 5: Signal



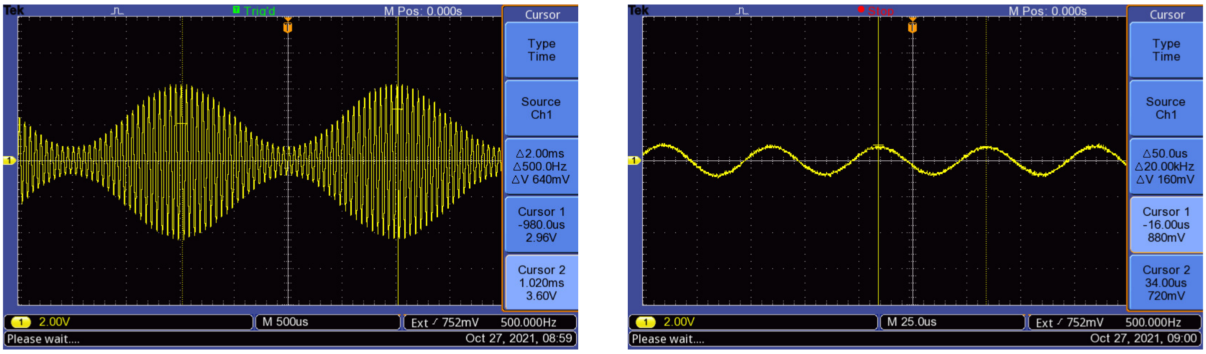
(a) $A_{max} + A_{min}$ (b) $A_{max} - A_{min}$

Figure 6: A_{max}, A_{min}

The frequencies of carrier signal and modulating signal are 20KHz and 500Hz respectively. Also, amplitudes of these signals are 2.56V and 1.2V respectively.

$$A_{max} + A_{min} = 5.12V \qquad A_{max} - A_{min} = 2.40V$$
$$A_c + A_m = 3.76V \qquad A_c - A_m = 1.36V$$
$$\frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{2.40}{5.12} = 46.88\%$$

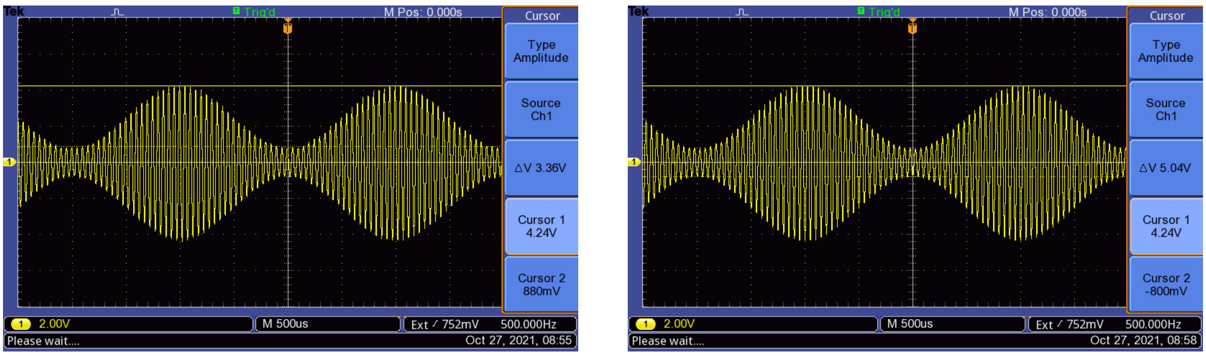
The modulation index was changed to 70% at the generator and the same process was repeated again.



(a) Modulating signal frequency

(b) Carrier signal frequency

Figure 7: Signal



(a) $A_{max} + A_{min}$

(b) $A_{max} - A_{min}$

Figure 8: A_{max}, A_{min}

The modulation index was changed to 120% at the generator. The overmodulated signal can be observed in Figure 9.

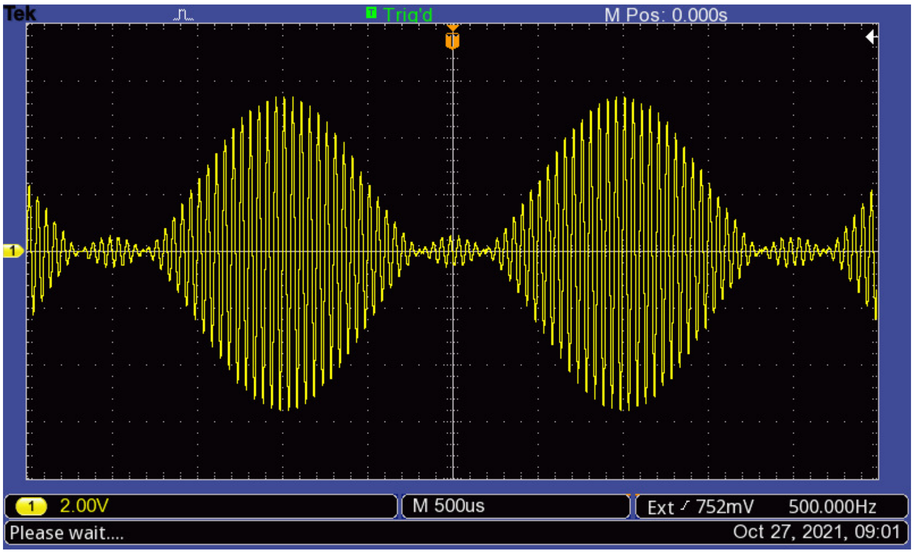


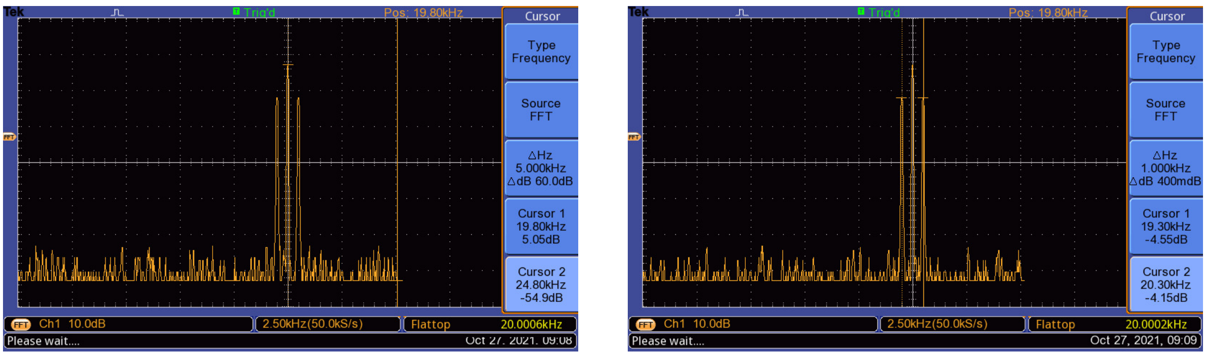
Figure 9: Overmodulated signal

3.2 Problem 2: AM Modulated Signals in Frequency Domain

The same setup as before with 70% modulation index was used in this problem. In the oscilloscope, the frequency domain of the signal was displayed. The obtained hard-copies are shown in Figure 10 and in Figure 11. From there, the following numbers can be seen:

	f_c	$f_c - f_m$	$f_c + f_m$
Frequency [Hz]	19.80K	19.30K	20.30K
Magnitude [dB]	5.05	-4.15	-4.15

Table 1: Measured values



(a) Carrier frequency at cursor 1 (b) $f_c - f_m$ at cursor 1 and $f_c + f_m$ at cursor 2

Figure 10: Signal

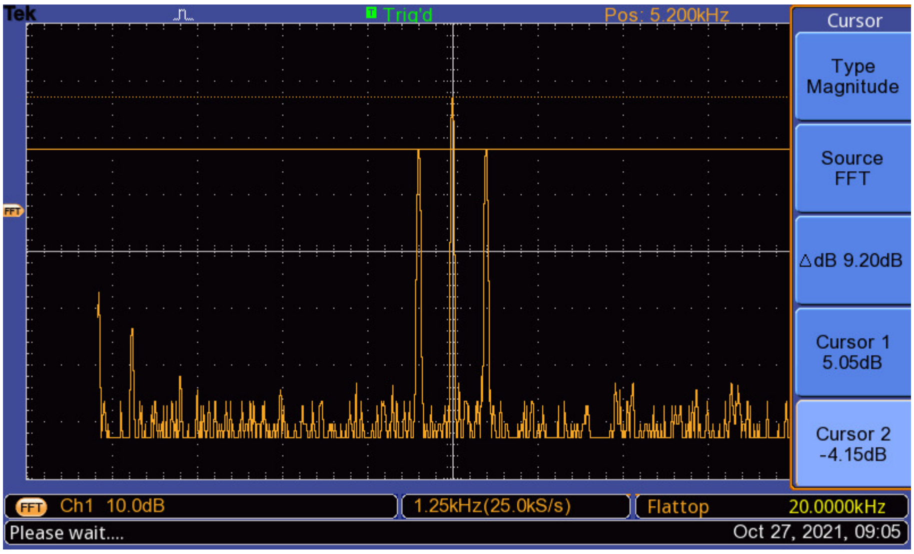


Figure 11: Magnitude of $f_c - f_m$ and $f_c + f_m$ at cursor 1 and magnitude of f_c at cursor 2

3.3 Problem 3: Demodulation of a message signal

Firstly, the circuit shown in Figure 12 was assembled in the breadboard. As a input, the signal generator with the following settings was used.

Signal Shape = Sine
Modulation = AM
Carrier Frequency = 20 KHz
Carrier Amplitude = 10 V_{PP}
Modulation Frequency = 500 Hz
Modulation Index = 50%

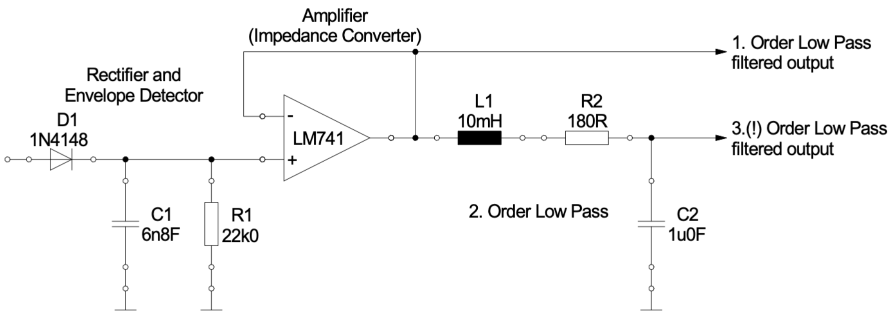
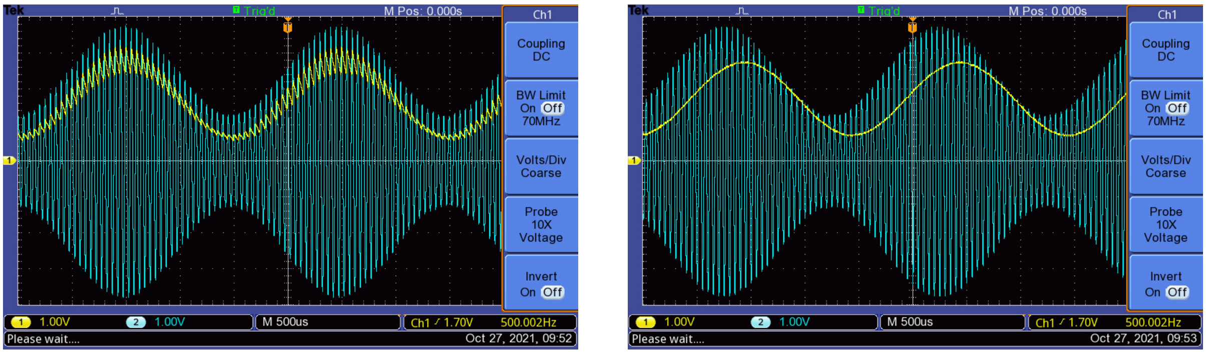


Figure 12: Circuit

Then, the oscilloscope was used to display AM modulated signal with the first order filter output. The hard-copy is shown in Figure 13 and in Figure 14. Then, the hard-coy of the FFT of the 3rd order filter output was taken (Figure 15).

$$
\begin{aligned}
V_{pp} &= 2.08V \\
V_{max} &= 2.76V
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
A_m &= V_{pp}/2 = 1.04V \\
V_{DC} &= V_{max} - A_m = 2.76 - 1.04 = 1.72V
\end{aligned}
$$



(a) AM modulated signal and 1st order filter output

(b) AM modulated signal and 3rd order filter output

Figure 13: Signal

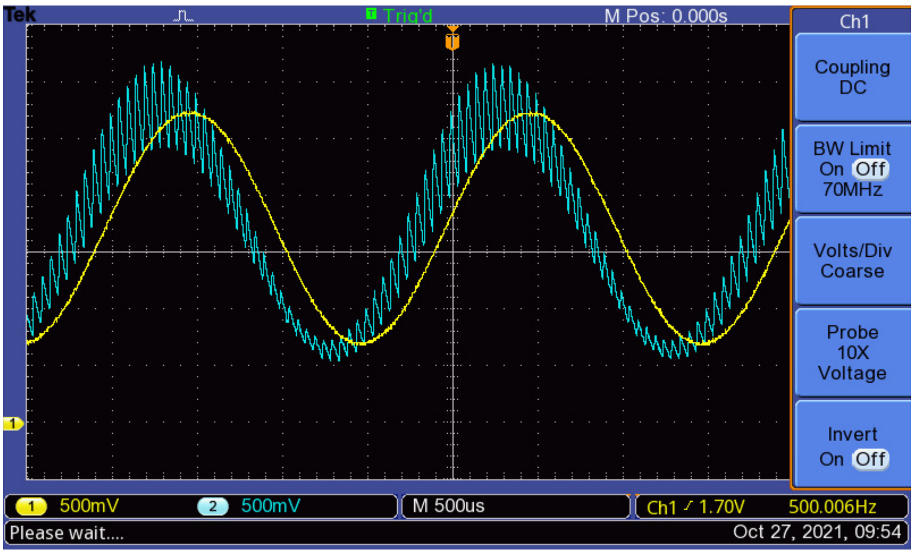


Figure 14: Comparison of the 1st and 3rd order filter outputs

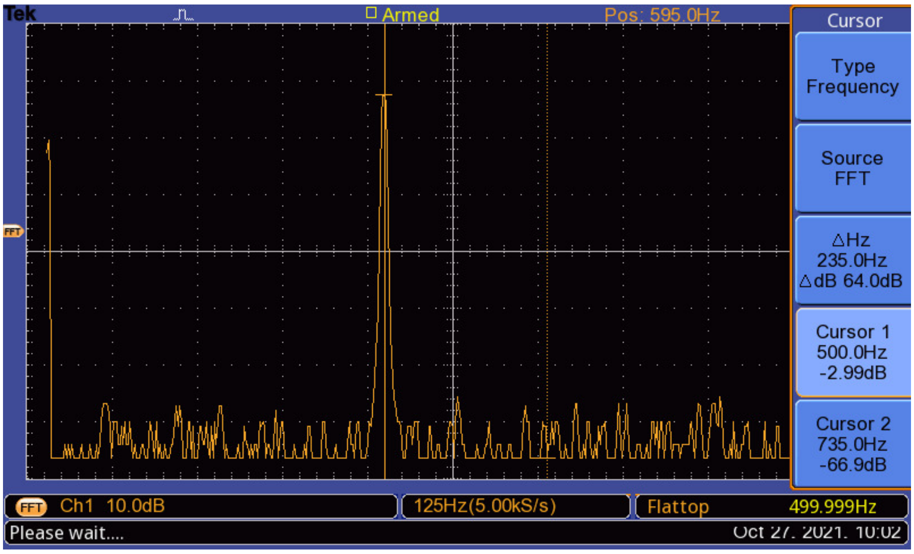


Figure 15: FFT of the 3rd order filter output

4 Evaluation AM Modulation

4.1 Problem 1: AM modulated Signals in Time Domain

1. In the Prelab of this experiment, the relation between the modulation index and the relative magnitudes of the frequency components is derived as follows:

$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

2. In the execution part, the following modulation indexes are obtained:

$$m_{50} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{2.40}{5.12} = 46.88\%$$
$$m_{70} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{3.36}{5.04} = 66.67\%$$

Thus, the calculated modulation indexes (44.88% and 66.67%) are slightly different from the expected (50% and 70%). However, relative errors is around 5%, which is acceptable in this case. The main source of errors are the generator - which can not generate the exact desired signal, the resolution of the oscilloscope - numbers were read using the cursors, and the internal resistors of used instruments.

3. Modulation index is the measure of extent to which the parameter of the carrier is varied in accordance with the modulating signal. If the modulation percentage is more than 100%, it is called overmodulation. In this case, the carriers experience a 180 degree phase reversal, which causes additional sidebands. Thus, some part of the envelope is cut off and information is lost.

4.2 Problem 2: AM Modulated Signals in Frequency Domain

1. The modulated signal is represented by three peaks in the frequency domain. A center peaks with the largest magnitude is the carrier frequency, f_c, other two peaks with the same magnitude are the carrier frequencies, $f_c + f_m$ and $f_c - f_m$.
   Theoretically, from the Prelab of the experiment, the following numbers can be obtained:

$$\begin{aligned} f_c &= 20KHz \\ f_m &= 500Hz \end{aligned} \quad \Rightarrow \quad \begin{aligned} f_c + f_m &= 20.5KHz \\ f_c - f_m &= 19.5KHz \end{aligned}$$

   Experimentally, from Figure 10, the following numbers are recorded:

$$f_c = 19.80KHz \qquad f_c - f_m = 19.30KHz \qquad f_c + f_m = 20.30KHz$$

   The numbers are pretty close. However, there are still some errors because of the signal generator and resolution of the oscilloscope.

2. Since there is a presence of the carrier frequency components in the FFT of the signal, the generator creates a DSB amplitude modulation. In the case of DSB-SC modulation, this component would be vanished.

3. Changing the carrier frequency will result in shifting the spectrum either to higher frequencies or to lower frequencies and in shifting the base spectrum. However, if the modulating frequency is still negligible compared to the carrier frequency, the envelope will stay the same in the time domain.

4. Changing the message frequency will not affect the entire spectrum. However, the modulating signal components will shift and two side peaks will move apart from the carrier frequency. Thus, it causes the change in bandwidth of the modulating signal.

5. From the hard-copy, the following numbers are extracted:

$$\begin{aligned} A_{FFT-c} &= 5.05dB \\ A_{FFT-m+c} &= 5.05dB \end{aligned} \quad \Rightarrow \quad A_c = 1.79V A_m = 1.24V$$

   The modulation index is given by:

$$m = \frac{A_m}{A_c} = \frac{1.24}{1.79} = 69.66\%$$

   Thus, the modulation index is very close to the theoretical one, which is 70%.

4.3 Problem 3 Demodulation of a message signal

1. From Figure 13, it can be concluded that third order filter provides a clear envelope, whereas the envelope from the first order filter has some noise or distortion. This is because the high order filters filter the high frequency signals better, removing the high frequency components that distorted the first order output.

2. Comparing Figure 16 with Figure 13 gives very similar results. In the simulation and in the experiment, the distortion from the first order filter was observed. There is a phase shift in the hard-copy from the oscilloscope for both cases, but it is clearly visible for the third order filter.

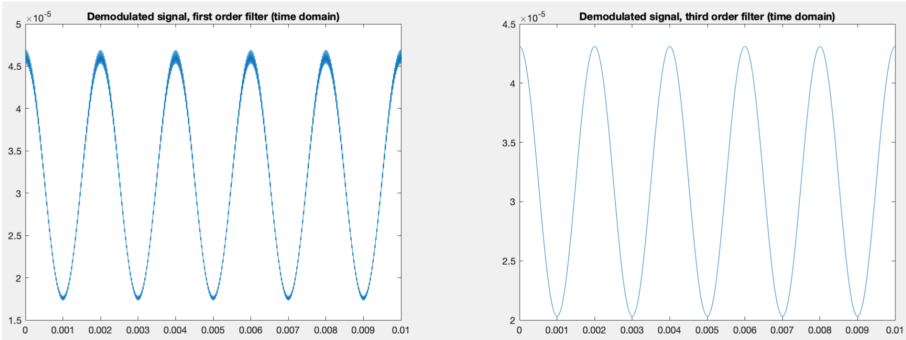


Figure 16: Simulation of first and third order filter outputs

5 Conclusion

The main idea of this experiment was to analyse the techniques of amplitude modulation. The first part was about observing the effects of the modulation index. We observed how changing modulation index changes the AM waveform and its envelope. In the experiment, modulation index of 50%, 70%, and 120% were used. Experimentally obtained numbers matched the expected ones. Then, it was concluded that modulation index should be below 100% to avoid overmodulation and loss of information.
Additionally, the frequency domain components of the AM signal were observed from oscilloscope. The shape of the spectrum was confirmed with the ones from Matlab. We concluded that the signal generator was generating a DSB. Also, three peaks were seen: Carrier frequency f_c at the center and $f_c - f_m$, $f_c + f_m$ side peaks.
Finally, AM signal was passed through the demodulating circuit. It was concluded that higher order filters provide a better demodulation by comparing the first order and third order filters. We observed that high order filters cancel high frequency components better.
However, there were some small error during the experiment. It was mainly because of the generator and resolution of the oscilloscope. The generator can not provide the exactly desired signal. Also, to read numbers from the oscilloscope, cursors were used. It gave some inaccurate numbers.

6 Prelab FM Modulation

6.1 Problem 1: Frequency Modulator

A sinusoidal modulation signal:

$$m(t) = 4cos(8000\pi t)$$

is applied to an FM modulator with frequency deviation constant $K_f = 10KHz/V$.
Frequency deviation:

$$\Delta f = K_f A_m = 10000 \times 4 = 40KHz$$

Frequency modulation index:

$$\beta_f = \frac{\Delta f}{f_m} = \frac{40000}{8000\pi/2\pi} = 10$$

6.2 Problem 2: FM signal in the frequency domain

Magnitudes in dB_{rms} are displayed in Figure 17 with Matlab code shown in Figure 18.

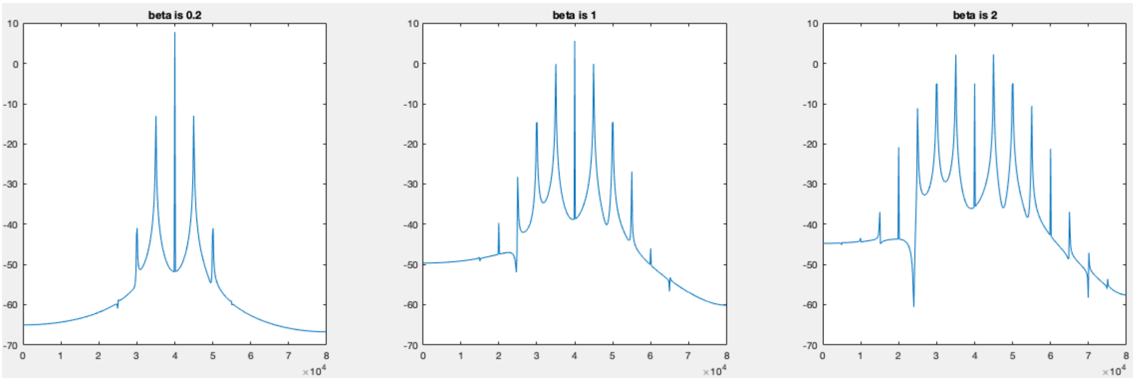


Figure 17

```
Ac = 2.5;
fc = 4e04;
fm = 5e03;
bf = 2; %vary between 0.2, 1, 2
Fs = 4*fc;
Ts = 1/Fs;
N = 1000;
t = 0:Ts:(N-1)*Ts;
s = Ac .* cos(2.*pi.*fc.*t + bf.*sin(2.*pi.*fm.*t));
L = length(s);
S = fft(s, L);
S = S(1:L/2);
dBVrms = mag2db(2.*abs(S)/L);
f = (0:L/2-1) .* Fs/L;
B = 2*fm*(bf+1);
plot(f, dBVrms);
```

Figure 18

$$B(0.2) = 12KHz \qquad B(0.2) = 20KHz \qquad B(0.2) = 30KHz$$

in KHz	15	20	25	30	35	40	45	50	55	60	65
$\beta_f = 0.2$	-	-	-	-40.98	-13	7.872	-12.99	-41	-	-	-
$\beta_f = 1$	-	-39.68	-28.22	-14.61	-0.10	5.64	-0.06	-14.57	-26.88	-46.02	-
$\beta_f = 2$	-36.97	-20.83	-11.13	-5.02	2.23	-4.982	-2.32	-4.94	10.54	-21.26	-36.91

Table 2: Measured values

6.3 Problem 3: Frequency demodulation

The FM signal is applied to an LC circuit which results in an amplitude and frequency-modulated signal. The slope detector consists of a circuit where the resonance frequency is a little bit different from the carrier frequency of the FM modulated signal. It means that FM signal sits on the slope of the responsive curve. The output signal from this passes to an envelope detector.

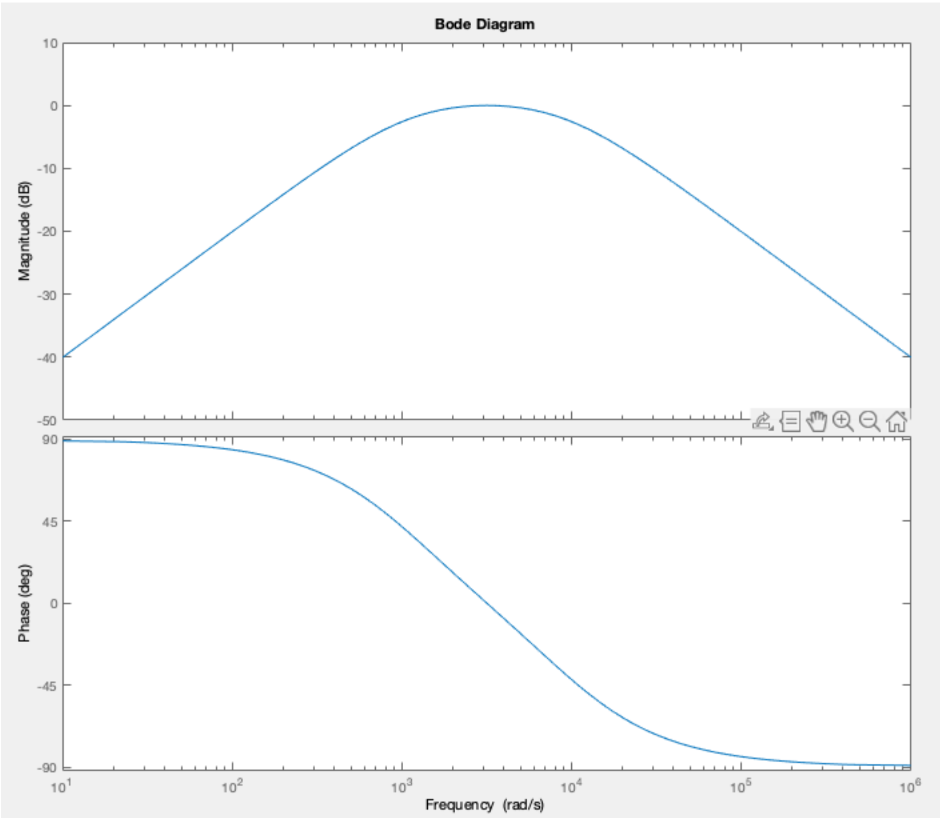


Figure 19

7 References

References

[1] CO-520-B Signals and Systems Lab
Instructors: Uwe Pagel and Mojtaba Joodaki.