Series of Exercises # 3

1. Solve the following LP problem using the simplex method

$$\begin{array}{lrcrcr} \text{maximize} & 3x_1 & + & x_2 & & \\ \text{subject to} & x_1 & - & x_2 & \leq & -1 \\ & -x_1 & - & x_2 & \leq & -3 \\ & 2x_1 & + & x_2 & \leq & 4 \\ & x_1 & & & \geq & 0 \\ & & & x_2 & \geq & 0 \end{array}$$

2. Solve the following LP problem using the simplex method

$$\begin{array}{lrcrcr} \text{maximize} & 3x_1 & + & x_2 & & \\ \text{subject to} & x_1 & - & x_2 & \leq & -1 \\ & -x_1 & - & x_2 & \leq & -3 \\ & 2x_1 & + & x_2 & \leq & 2 \\ & x_1 & & & \geq & 0 \\ & & & x_2 & \geq & 0 \end{array}$$

3. Solve the following LP problem using the simplex method

$$\begin{array}{lrcrcr} \text{maximize} & 3x_1 & + & x_2 & & \\ \text{subject to} & x_1 & - & x_2 & \leq & -1 \\ & -x_1 & - & x_2 & \leq & -3 \\ & 2x_1 & - & x_2 & \leq & 2 \\ & x_1 & & & \geq & 0 \\ & & & x_2 & \geq & 0 \end{array}$$

4. Solve the following LP problem using the simplex method

$$\begin{array}{lrcrcrcrcr} \text{minimize} & 3x_1 & + & x_2 & & & - & 4x_4 & & \\ \text{subject to} & 2x_1 & + & x_2 & + & 3x_3 & - & 4x_4 & \leq & 2 \\ & x_1 & + & x_2 & - & 2x_3 & + & 3x_4 & \geq & 3 \\ & -x_1 & + & x_2 & + & x_3 & + & 2x_4 & = & 7 \\ & & & & & & & x_1, x_4 & \leq & 0 \\ & & & & & & & x_2, x_3 & \geq & 0 \end{array}$$

5. Consider the following LP problem

$$
\begin{array}{lrcrcl}
\text{maximize} & -x_1 & + & 8x_2 & & \\
\text{subject to} & x_1 & + & x_2 & \geq & 1 \\
 & -x_1 & + & 6x_2 & \leq & 3 \\
 & & & x_2 & \leq & 2 \\
 & x_1 & & & \geq & 0 \\
 & & & x_2 & \geq & 0
\end{array}
$$

   (a) Solve the problem geometrically

   (b) Solve the problem by the two-phase simplex method. Show that the points generated by the first phase correspond to basic solutions of the original system

6. Prove or disprove: A feasible dictionary whose last row reads $z = z^* + \sum \bar{c}_j x_j$ describes an optimal solution if and only if $\bar{c}_j \leq 0$ for all j

7. Solve the following LP problem

$$
\begin{array}{lrcrcrcrcl}
\text{minimize} & 10x_1 & - & 57x_2 & - & 9x_3 & - & 24x_4 & & \\
\text{subject to} & \frac{1}{2}x_1 & - & \frac{11}{2}x_2 & - & \frac{5}{2}x_3 & + & 9x_4 & \leq & 0 \\
 & \frac{1}{2}x_1 & - & \frac{3}{2}x_2 & - & \frac{1}{2}x_3 & + & x_4 & \leq & 0 \\
 & x_1 & & & & & & & \leq & 1 \\
 & & & & & & & x_1, x_2, x_3, x_4 & \geq & 0
\end{array}
$$

   (a) using the perturbation method to resolve degeneracy

   (b) using Bland's rule to resolve degeneracy