Series of Exercises # 4

1. Write the dual problem of the following LP problem

$$
\begin{array}{lrcrcrcrcr}
\max & 4x_1 & + & 3x_2 & + & 7x_3 & + & 9x_4 & & \\
\text{s.t.} & 2x_1 & - & 4x_2 & + & x_3 & & & \leq & -1 \\
 & x_1 & + & 5x_2 & + & x_3 & + & x_4 & \leq & 16 \\
 & x_1 & & & + & x_3 & & & \leq & 5 \\
 & 2x_1 & + & 4x_2 & & & - & x_4 & \leq & 8 \\
 & & & x_2 & - & 3x_3 & + & x_4 & \leq & 0 \\
 & & & -4x_2 & & & + & 3x_4 & \leq & 4 \\
 & 5x_1 & + & 2x_2 & - & 3x_3 & + & 6x_4 & \leq & 19 \\
 & & & & & x_1, x_2, x_3, x_4 & & & \geq & 0
\end{array}
$$

2. Write the dual problem of the following LP problem

$$
\begin{array}{lrcrcrcr}
\min & 5x_1 & + & 2x_2 & + & 6x_3 & & \\
\text{s.t.} & 2x_1 & + & x_2 & + & x_3 & \geq & 5 \\
 & & & x_2 & + & 2x_3 & \geq & 4 \\
 & x_1 & & & + & x_3 & \geq & 4 \\
 & & & & & x_1, x_2, x_3 & \geq & 0
\end{array}
$$

3. Write the dual problem of the following LP problem

$$
\begin{array}{lrcrcrcr}
\max & 3x_1 & + & 2x_2 & + & 5x_3 & & \\
\text{s.t.} & 5x_1 & + & 3x_2 & + & x_3 & = & -8 \\
 & 4x_1 & + & 2x_2 & + & 8x_3 & \leq & 1 \\
 & 6x_1 & + & 7x_2 & + & 3x_3 & \geq & 1 \\
 & x_1 & & & & & \leq & 4 \\
 & & & & & x_3 & \geq & 0
\end{array}
$$

4. Write the dual of the following LP problem

$$
\begin{array}{lrcrcrcrcr}
\text{maximize} & 2x_1 & + & 4x_2 & + & x_3 & + & x_4 & & \\
\text{subject to} & x_1 & - & x_2 & + & 3x_3 & - & x_4 & = & 2 \\
 & x_1 & + & 2x_2 & + & 2x_3 & + & 2x_4 & \geq & 6 \\
 & 3x_1 & + & 4x_2 & + & 5x_3 & - & 3x_4 & \leq & 2 \\
 & x_1 & & & & & & & \geq & 0 \\
 & & & x_2 & & & & & \geq & 0 \\
 & & & & & x_3 & & & \leq & 0
\end{array}
$$

5. Write the dual of the following LP problem

$$\begin{array}{ll} \text{maximize} & z \\ \text{subject to} & z + \sum_{i=1}^{m} a_{ij} x_i \leq 0 \quad \text{for } j = 1, 2, \ldots, n \\ & \sum_{i=1}^{m} x_i = 1 \\ & x_i \geq 0 \quad \text{for } i = 1, 2, \ldots, m \end{array}$$

6. Consider the following LP problem:

$$\begin{array}{lrcrcrcl} \text{maximize} & 3x_1 & + & 2x_2 & & & & \\ \text{subject to} & 3x_1 & + & 2x_2 & \leq & 8 \\ & x_1 & - & x_2 & \leq & 1 \\ & x_1 & + & x_2 & \leq & 3 \\ & x_1 & & & \geq & 0 \\ & & & x_2 & \geq & 0 \end{array}$$

   (a) Write the dual LP problem

   (b) Check that $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is primal feasible and $\mathbf{y} = \begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{5}{2} \end{bmatrix}$ is dual feasible.

   What can you conclude?

7. Consider the following LP problem

$$\begin{array}{lrcrcrcrcrcrcl} \max & 18x_1 & - & 7x_2 & + & 12x_3 & + & 5x_4 & & & + & x_6 & & \\ \text{s.t.} & 2x_1 & - & 6x_2 & + & 2x_3 & + & 7x_4 & + & 3x_5 & + & 8x_6 & \leq & 1 \\ & -3x_1 & - & x_2 & + & 4x_3 & - & 3x_4 & + & x_5 & + & 2x_6 & \leq & -2 \\ & 8x_1 & - & 3x_2 & + & 5x_3 & - & 2x_4 & & & + & 2x_6 & \leq & 4 \\ & 4x_1 & & & + & 8x_3 & + & 7x_4 & - & x_5 & + & 3x_6 & \leq & 1 \\ & 5x_1 & + & 2x_2 & - & 3x_3 & + & 6x_4 & - & 2x_5 & - & x_6 & \leq & 5 \\ & & & & & & & & & x_1, x_2, x_3, x_4, x_5, x_6 & & & \geq & 0 \end{array}$$

   Is $\begin{bmatrix} 2 & 4 & 0 & 0 & 7 & 0 \end{bmatrix}^T$ optimal?

8. Consider the following LP problem

$$\begin{array}{lrcrcrcrcrcl} \max & 8x_1 & - & 9x_2 & + & 12x_3 & + & 4x_4 & + & 11x_5 & & \\ \text{s.t.} & 2x_1 & - & 3x_2 & + & 4x_3 & + & x_4 & + & 3x_5 & \leq & 1 \\ & x_1 & + & 7x_2 & + & 3x_3 & - & 2x_4 & + & x_5 & \leq & 1 \\ & 5x_1 & + & 4x_2 & - & 6x_3 & + & 2x_4 & + & 3x_5 & \leq & 22 \\ & & & & & & & x_1, x_2, x_3, x_4, x_5 & & & \geq & 0 \end{array}$$

   Is $\begin{bmatrix} 0 & 2 & 0 & 7 & 0 \end{bmatrix}^T$ optimal?

9. Consider the following LP problem

$$\begin{array}{lrcrcl} \max & 3x_1 & + & 2x_2 & & \\ \text{s.t.} & 3x_1 & + & 2x_2 & \leq & 18 \\ & x_1 & & & \leq & 4 \\ & & & x_2 & \leq & 6 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

Is $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ optimal?

10. Consider the following LP problem

$$\begin{array}{lrcrcrcrcl} \max & 6x_1 & + & x_2 & - & x_3 & - & x_4 & & \\ \text{s.t.} & x_1 & + & 2x_2 & + & x_3 & + & x_4 & \leq & 5 \\ & 3x_1 & + & x_2 & - & x_3 & & & \leq & 8 \\ & & & x_2 & + & x_3 & + & x_4 & = & 1 \\ & & & & & & & x_3, x_4 & \geq & 0 \end{array}$$

Is $\begin{bmatrix} 3 & -1 & 0 & 2 \end{bmatrix}^T$ optimal?