Series of Exercises # 6

1. Solve the following LP problem using the revised dual-simplex method

$$
\begin{aligned}
\max z = \quad -5x_1 \quad - \quad 35x_2 \quad - \quad 20x_3 & \\
\text{s.t.} \qquad x_1 \quad - \quad x_2 \quad - \quad x_3 &\le -2 \\
-x_1 \quad - \quad 3x_2 \qquad\qquad\quad &\le -3 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

2. Consider the LP problem

$$
\begin{aligned}
\max z = \quad 2x_1 \quad - \quad x_2 \quad + \quad x_3 & \\
\text{s.t.} \qquad x_1 \quad + \quad x_2 \quad + \quad x_3 &\le 6 \\
-x_1 \quad + \quad 2x_2 \qquad\qquad\quad &\le 4 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

Let x_4 (resp. x_5) be the slack variable of the first (resp. second) inequality

(a) Is the basis $\{x_1, x_5\}$ optimal?

(b) Suppose c_2 is replaced by $c_2 + \Delta$. Find the range of values of Δ which will make $\{x_1, x_5\}$ remain optimal.

(c) Suppose c_1 is replaced by $c_1 + \Delta$. Find the range of values of Δ which will make $\{x_1, x_5\}$ remain optimal.

(d) Suppose $b = \begin{pmatrix} 6 & 4 \end{pmatrix}^T$ is replaced by $b = \begin{pmatrix} 6 + \Delta & 4 \end{pmatrix}^T$. Find the range of values of Δ which will make $\{x_1, x_5\}$ remain optimal.

3. A workshop manufactures four kinds of furniture: bookcases, desks, chairs and bedframes.

- A bookcase takes three hours of labor, one unit of metal and four units of wood.
- A desk takes two hours of labor, one unit of metal and three units of wood.
- A chair takes one hour of labor, one unit of metal and three units of wood.
- A bedframe takes two hours of labor, one unit of metal and four units of wood.

There are 420 units of wood, 117 units of metal and 225 hours of labor available per day. Knowing that a bookcase brings a net profit of $19, a desk brings a net profit of $13, a chair brings a net profit of $12 and a bedframe brings a net profit oof $17, how is the workshop going to maximize its daily net profit?

(a) Show that the optimal solution calls for 39 bookcases, 48 chairs, and 30 bedframes to be produced per day.

(b) Solve the following variations:

    i. the net profit brought in by each desk increases from $13 to $15;

    ii. the availability of metal increases from 117 to 125 per day;

    iii. the company may also produce coffee tables, each of which requires three hours of work, one unit of metal, two units of wood, and brings in a net profit of $14;

    iv. the number of chairs produced must be at most five times the numbers of desks;

    v. the demand for wood per chair increases from three to four units.