Math 215 – Fall 2017

Theory Homework 6 – Assigned October 12th, due October 23th

Note: Remember that you must show your work to get full credit for a problem.

1. Prove that for all positive integers a, b, and c, that if $a \mid c$, $b \mid c$, and $\mathrm{GCD}(a, b) = 1$, then $ab \mid c$.

2. Euler's totient function is a function from the positive integers to the positive integers. The function is denoted by φ and $\varphi(n)$ returns number of positive integers k less then or equal to n such that $\mathrm{GCD}(k, n) = 1$.

   Suppose that p is a prime number what is $\varphi(p)$?

3. Let p be a prime number, and a be a positive integer. Prove that

   $$\varphi(p^a) = p^{a-1}(p-1).$$

4. Prove that for all positive integers a, b and k that $\mathrm{GCD}(k, ab) = 1$ if and only if $\mathrm{GCD}(k, a) = 1$ and $\mathrm{GCD}(k, b) = 1$.

5. As a consequence of the above problem, it can shown with group theory (in MATH 421) that if a and b are positive integers such that $\mathrm{GCD}(a, b) = 1$ then $\varphi(ab) = \varphi(a)\varphi(b)$.

   Let n be a positive integer. Using the above information, express $\varphi(n)$ without using the Euler totient function. Justify each step in your calculation.