Electricity and Magnetism Review

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Contents

1 Chapter 21: The Electric Field I: Discrete Charge Distributions

1.1 Charge

There are two kinds of charge, positive and negative. Charges of like sign repel, those of opposite sign attract.

Quantization
Charge is quantized — it always occurs in integer multiples of the fundamental charge unit e. The charge of the electron is $-e$ and that of the proton is $+e$.

Magnitude

$$e = 1.60 \times 10^{-19} C \tag{1}$$

Conservation
Charge is conserved. When charged particles are created or annihilated, the total amount of charge carried by the created or annihilated particles is zero.

1.2 Conductors and Insulators

In metals, about one electron per atom is delocalized (free to move about the entire material). In insulators, all the electrons are bound to nearby atoms.

Ground
A very large conductor (such as Earth) that can supply or absorb a virtually unlimited amount of charge is called a ground.

1.3 Charge by Induction

To charge a conductor buy induction: connect a ground to the conductor, hold an external charge near the conductor (to attract or repel the conduction electrons), then disconnect the conductor from the ground, and finally move the external charge away from the conductor.

1.4 Coulomb's Law

The force exerted by point charge q_1 on point charge q_2 a distance r_{12} away is given by

$$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} \tag{2}$$

where unit vector $\hat{r}_{12}$ points from q_1 towards q_2.

Coulomb constant

$$k = 8.99 \times 10^9 N \cdot m^2/C^2 \tag{3}$$

1.5 Electric Field

The electric field due to a system of charges at a point is defined as the net force $\vec{F}$, exerted by those charges on a very small positive test charge q_0, divided by q_0:

$$\vec{E} = \frac{\vec{F}}{q_0} \tag{4}$$

Due to a point charge

$$\vec{E}_{iP} = \frac{kq_i}{r_{iP}^2}\hat{r}_{iP} \tag{5}$$

Due to a system of point charges

The Electric field at P due to several charges is the vector sum of the fields at P due to the individual charges:

$$\vec{E}_P = \sum_i \vec{E}_{iP} \tag{6}$$

1.6 Electric Field Lines

The electric field can be represented by electric field lines that emanate from positive charges and terminate on negative charges. The strength of the electric field is indicated by the density of the electric field lines.

1.7 Dipole

A dipole is a system of two equal but opposite charges separated by a small distance.

Dipole moment

$$\vec{p} = q\vec{L} \tag{7}$$

where $\vec{L}$ is the position of the positive charge relative to the negative charge.

Field due to dipole

The electric field strength far from a dipole is proportional to the magnitude of the dipole moment and decreased with the cube of the distance.

Torque on a dipole

In a uniform electric field, the net force on a dipole is zero, but there is a torque that tends to align the dipole in the direction of the field.

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{8}$$

Potential energy of a dipole

$$U = -\vec{p} \cdot \vec{E} + U_0 \tag{9}$$

Where U_0 is usually taken to be zero.

1.8 Polar and Nonpolar Molecules

Polar molecules, such as H_2O and HCL, have permanent dipole moments because their centers of positive and negative charge do not coincide. They behave like simple dipoles in an electric field. Nonpolar molecules do not have permanent dipole moments, but they acquire induced dipole moments in the presence of an electric field.

2 Chapter 22: The Electric Field II: Continuous Charge Distributions

2.1 Electric Field for Continuous Charge Distribution

$$\vec{E} = \int d\vec{E} = \int \frac{k\hat{r}}{r^2} dq \quad \text{(Coulomb's law)} \tag{10}$$

where $dq = \rho dV$ for a charge distributed throughout a volume, $dq = \sigma dA$ for a charge distributed on a surface, and $dq = \lambda dL$ for a charge distributed along a line.

2.2 Electric Flux

$$\phi = \lim_{\Delta A_i \to 0} \sum_i \vec{E}_i \cdot \hat{n}_i \Delta A_i = \int_S \vec{E} \cdot \hat{n} dA \tag{11}$$

2.3 Gauss's Law

$$\phi_{\text{net}} = \oint_S \vec{E} \cdot \hat{n} dA = \oint_S E_n dA = \frac{Q_{\text{inside}}}{\epsilon_0} \tag{12}$$

The net outward electric flux through a closed surface equals the net charge within the surface divided by ϵ_0.

2.4 Coulomb Constant k and Electric Constant (Permittivity of Free Space) ϵ_0

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 N \cdot m^2/C^2 \tag{13}$$

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} C^2/(N \cdot m^2) \tag{14}$$

2.5 Coulomb's Law and Gauss's Law

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \tag{15}$$

$$\phi_{\text{net}} = \oint_S E_n dA = \frac{Q_{\text{inside}}}{\epsilon_0} \tag{16}$$

2.6 Discontinuity of E_n

At a surface having a surface charge density of σ, the component of the electric field normal to the surface is discontinuous by σ/ϵ_0.

$$E_{n+} - E_{n-} = \frac{\sigma}{\epsilon_0} \tag{17}$$

2.7 Charge on a Conductor

In electrostatic equilibrium, the charge density is zero throughout the material of the conductor. All excess or deficit charge resides on the surfaces of the conductor.

2.8 $\vec{E}$ Just Outside a Conductor

The resultant electric field just outside the surface of a conductor is normal to the surface and has the magnitude σ/ϵ_0, where σ is the local surface charge density on the conductor:

$$E_n = \frac{\sigma}{\epsilon} \tag{18}$$

2.9 Electric Fields for Selected Uniform Charge Distributions

Of a line charge of infinite length

$$E_R = 2k\frac{\lambda}{R} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{R} \tag{19}$$

On the axis of a chrged ring

$$E_z = \frac{kQz}{\left(z^2 + a^2\right)^{3/2}} \tag{20}$$

On the axis of a charged disk

$$E_z = \text{sign}(z) \cdot \frac{\sigma}{2\epsilon_0} \left[1 - \left(1 + \frac{R^2}{z^2}\right)^{-1}\right] \tag{21}$$

Of a charged infinite plane

$$E_z = \text{sign}(z) \cdot \frac{\sigma}{2\epsilon_0} \tag{22}$$

Of a charged thin spherical shell

$$E_r = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad r > R \tag{23}$$

$$E_r = 0 \quad r < R \tag{24}$$

3 Chapter 23: Electric Potential

3.1 Potential Difference

The potential different $V_b - V_a$ is defined as the negative of the work per unit charge done by the electric field on a test charge as it moves from point a to point b.

$$\Delta V = V_b - V_a = \frac{\Delta U}{q_0} = -\int_a^b \vec{E} \cdot \vec{dl} \tag{25}$$

Potential difference for infinitesminal displacements

$$dV = -\vec{E} \cdot \vec{dl} \tag{26}$$

3.2 Electric Potential

Potential due to a point charge

$$V = \frac{kq}{r} - \frac{kq}{r_{\text{ref}}} \quad (V = 0 \text{ if } r = r_{\text{ref}}) \tag{27}$$

Coulomb potential

$$V = \frac{kq}{r} \quad (V = 0 \text{ if } r = \text{inf}) \tag{28}$$

Potential due to a system of point charges

$$V = \sum_i \frac{kq_i}{r_i} \quad (V = 0 \text{ if } r_i = \text{inf}, i = 1, 2, \dots) \tag{29}$$

Potential due to continuous charge distribution

$$V = \int \frac{kdq}{r} \quad (V = 0 \text{ if } r = \text{inf}) \tag{30}$$

Where dq is an increment of charge and r is the distance from the increment to the field point. This expression can be used only if the charge distribution is contained in an finite volume so that the potential can be chosen to be zero at infinity.

Continuity of electric potential
The potential function V is continuous everywhere in space.

3.3 Computing the Electric Field from the Potential

The electric field points in the direction of the most rapid decrease in the potential. The charge in potential when a test charge undergoes a displacement $\vec{dl}$ is given by

$$E_{\text{tan}} = -\frac{dV}{dl} \tag{31}$$

Gradient
A vector that points in the direction of the greatest rate of change in a scalar function and that has a magnitude equal to the derivative of that function, with respect to the distance in that direction, is called the gradient of the function. $\vec{E}$ is the negative gradient of V.

Potential a function of x alone

$$E_X = -\frac{dV(x)}{dx} \tag{32}$$

Potential a function of r alone

$$E_r = -\frac{dV(r)}{dr} \tag{33}$$

3.4 Units

V and ΔV
The SI unit of potential and potential difference is the volt (V):

$$1V = 1J/C \tag{34}$$

Electric field

$$1N/C = 1V/m \tag{35}$$

Electron volt

The electron volt (eV) is the charge in potential energy of a particle of charge e as it moves from a to b where $V_b - V_a = 1$ volt:

$$1eV = 1.6 \times 10^{-19} C \cdot V = 1.6 \times 10^{-19} J \tag{36}$$

3.5 Potential Energy of Two Point Charges

$$U = q_0 V = \frac{k q_0 q}{r} \quad (U = 0 \text{ if } r = \text{inf}) \tag{37}$$

3.6 Potential Functions

On the axis of a uniformly charged ring

$$V = \frac{kQ}{\sqrt{z^2 + a^2}} \quad (V = 0 \text{ if } |z| = \text{inf}) \tag{38}$$

On the axis of a uniformly charged disk

$$V = 2\pi k \sigma |z| \left(\sqrt{1 + \frac{R^2}{z^2}} - 1 \right) \quad (V = 0 \text{ if } |z| = \text{inf}) \tag{39}$$

For an infinite plane of charge

$$V = V_0 - 2\pi k \sigma |x| \quad (V = V_0 \text{ if } x = 0) \tag{40}$$

For a spherical shell of charge

$$V = \begin{cases} \frac{kQ}{r} & r \geq R \\ \frac{kQ}{R} & r \leq R \end{cases} \quad (V = 0 \text{ if } r = \text{inf}) \tag{41}$$

For an infinite line charge

$$V = 2k\lambda \ln \frac{R_{\text{ref}}}{R} \quad (V = 0 \text{ if } r = R_{\text{ref}}) \tag{42}$$

3.7 Charge on a Nonspherical Conductor

On a conductor of arbitrary shape, the surface charge density σ is greatest at points where the radius of curvature is smallest.

3.8 Dielectric Breakdown

The amount of charge that can be placed on a conductor is limited by the fact that molecules of the surrounding medium undergo dielectric breakdown at very high electric fields, causing the medium to become a conductor.

Dielectric strength

The dielectric strength is the magnitude of the electric field at which dielectric breakdown occurs. The dielectric strength of dry air is

$$E_{\max} \approx 3 \times 10^6 V/m = 3MV/m \tag{43}$$

3.9 Electrostatic Potential Energy

The electrostatic potential energy of a system of point charges is the work needed to bring the charges from a infinite separation to their final positions.

Of point charges

$$U = \frac{1}{2}\sum_{i=1}^{n} q_i V_i \tag{44}$$

Of a conductor with charge Q at potential V

$$U = \frac{1}{2}QV \tag{45}$$

Of a system of conductors

$$U = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i \tag{46}$$

4 Chapter 24: Capacitance

4.1 Capacitor

A capacitor is a device for storing charge and energy. It consists of two conductors that are insulated from each other and carry equal and opposite charges.

4.2 Capacitance

Definition of capacitance

$$C = \frac{Q}{V} \tag{47}$$

Single conductor

Q is the conductor's total charge, V is the conductor's potential relative to its surroundings.

Capacitor

Q is the magnitude of the charge on either conductor, V is the magnitude of the potential difference between the conductors.

Of an isolated spherical conductor

$$C = 4\pi\epsilon_0 R \tag{48}$$

Of a parallel-plate capacitor

$$C = \frac{\epsilon_0 A}{d} \tag{49}$$

Of a cylindrical capacitor

$$C = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{R_2}{R_1}\right)} \tag{50}$$

Energy stored in a capacitor

$$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 \tag{51}$$

Energy density of an electric field

$$u_e = \frac{1}{2}\epsilon_0 E^2 \tag{52}$$

4.3 Equivalent Capacitance

Parallel capacitors

When devices are connected in parallel, the voltage drop is the same across each.

$$C_{\text{eq}} = C_1 + C_2 + \cdots \tag{53}$$

Series capacitors

When capacitors are in series, the voltage drop add. If the total charge on each connected pair of plates is zero, then:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \tag{54}$$

4.4 Dielectrics

Macroscopic behavior

A nonconducting material is called a dielectric. When a dielectric is inserted between the plates of a charged, electrically isolated capacitor, the electric field between the plates is weakened and the capacitance is thereby increased by the factor κ, which is the dielectric constant.

Microscopic view

The electric field in the dielectric of a capacitor is weakened because the molecular dipole moments (either preexisting or induced) tend to align with the applied field and thereby produce a second electric field inside the dielectric that opposes the applied field. The aligned dipole moment of the dielectric is proportional to the applied field.

Electric field inside

$$E = \frac{E_0}{\kappa} \tag{55}$$

Effect on capacitance

$$C = \kappa C_0 \tag{56}$$

Permittivity ϵ

$$\epsilon = \kappa \epsilon_0 \tag{57}$$

Uses of a dielectric

1. Increases capacitance
2. Increases dielectric strength
3. Physically separates conductors

5 Chapter 25: Electric Current and Direct-Current Circuits

5.1 Electric Current

Electric current is the rate of flow of electric charge through a cross-sectional area.

$$I = \frac{\Delta Q}{\Delta t} \tag{58}$$

in the limit that Δt approaches zero.

Drift velocity

In a conducting wire, electric current is the result of the slow drift of negatively charged electrons that are accelerated by an electric field in the wire and then collide with the lattice ions. Typical drift speeds of electrons in wires are of the order of a few millimeters per second. For mobile charges moving in the positive direction,

$$I = qnAv_d \tag{59}$$

where $q = -e$, n is the number density of free electrons, A is the cross-sectional area, and v_d is the drift speed.

Current density

The current density $\vec{\jmath}$ is related to the drift velocity by

$$\vec{\jmath} = qn\vec{v}_d \tag{60}$$

The current I through a cross-sectional surface is the flux of the current density through the surface.

5.2 Resistance

Definition of resistance

$$R = \frac{V}{I} \tag{61}$$

Resistivity, ρ

$$R = \rho\frac{L}{A} \tag{62}$$

Temperature coefficient of resistivity, α

$$\alpha = \frac{\frac{\rho - \rho_0}{\rho_0}}{T - T_0} \tag{63}$$

5.3 Ohm's Law

For ohmic materials, the resistance does not depend on either the current or the potential drop:

$$V = IR, \quad R \text{ constant} \tag{64}$$

5.4 Power

Supplied to a device or segment

$$P = IV \tag{65}$$

Delivered to a resistor

$$P = IV - I^2R = \frac{V^2}{R} \tag{66}$$

5.5 Emf

Source of emf

A device that supplies electric energy to a circuit.

Power supplied by an ideal emf source

$$P = I\mathcal{E} \tag{67}$$

5.6 Battery

Ideal

An ideal battery is a source of emf that maintains a constant potential difference between its two terminals, independent of the current through the battery.

Real

A real battery can be considered as an ideal battery in series with a small resistance, called its internal resistance.

Terminal voltage

$$V_a - V_b = \mathcal{E} - Ir \tag{68}$$

where in the battery the positive direction is the direction of increasing potential.

Total energy stored

$$E_{\text{stored}} = Q\mathcal{E} \tag{69}$$

5.7 Equivalent Resistance

Resistors in series

$$R_{\text{eq}} = R_1 + R_2 + \cdots \tag{70}$$

Resistors in parallel

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \tag{71}$$

5.8 Kirchhoff's Rules

1. When any closed loop is traversed, the algebraic sum of the changes in potential around the loop must equal zero.

2. At any junction (branch point) in a circuit where the current can divide, the sum of the current into the junction must equal the sum of the currents out of the junction.

5.9 Measuring Devices

Ammeter

An ammeter is a very low resistance device that is placed in series with a circuit element to measure the current in the element.

Voltmeter

A voltmeter is a very high resistance device that is placed in parallel with a circuit element to measure the potential difference across the element.

Ohmmeter

An ohmmeter is a device containing a battery connected in series with a galvanometer and resistor that is used to measure the resistance of a circuit element placed across its terminals.

5.10 Discharging a Capacitor

Charge on the capacitor

$$Q(t) = Q_0 e^{-\frac{t}{RC}} = Q_0 e^{-\frac{t}{\tau}} \tag{72}$$

Current in the circuit

$$I = -\frac{dQ}{dt} = \frac{V_0}{R} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{\tau}} \tag{73}$$

Time constant

$$\tau = RC \tag{74}$$

5.11 Charging a Capacitor

Charge on the capacitor

$$Q = C\mathcal{E}\left[1 - e^{-\frac{t}{RC}}\right] = Q_f\left(1 - e^{-\frac{t}{\tau}}\right) \tag{75}$$

Current in the cirucit

$$I = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{\tau}} \tag{76}$$

6 Chapter 26: The Magnetic Field

6.1 Magnetic Force

On a moving charge

$$\vec{F} = q\vec{v} \times \vec{B} \tag{77}$$

On a current element

$$d\vec{F} = Id\vec{l} \times \vec{B} \tag{78}$$

Unit of magnetic field

The SI unit of magnetic filds is the tesla (T). A commonly used unit is the gauss (G), which is related to the tesla by

$$1G = 10^{-4}T \tag{79}$$

6.2 Motion of Point Charges

A particle of mass m and charge q moving with speed v is a plane perpendicular to a uniform magnetic field moves in a circular orbit. The period and frequency of the circular motion are independent of the radius of the orbit and of the speed of the particle.

Newton's second law

$$qvB = m\frac{v^2}{r} \tag{80}$$

Cyclotron period

$$T = \frac{2\pi m}{qB} \tag{81}$$

Cyclotron frequency

$$f = \frac{1}{T} = \frac{qB}{2\pi m} \tag{82}$$

6.3 Current Loops

Magnetic dipol moment

$$\vec{\mu} = NIA\hat{n} \tag{83}$$

Torque

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{84}$$

Potential energy of a magnetic dipole

$$U = -\vec{\mu} \cdot \vec{B} \tag{85}$$

Net force

The net force on a current loop in a *uniform* magnetic field is zero.

6.4 The Hall Effect

When a conducting strip carrying a current is placed in a magnetic field, the magnetic force on the charge carriers causes a searation of charge called the Hall effect. This results in a voltage V_H, called the Hall voltage. The sign of the charge carriers can be determined from a measurement of the sign of the Hall voltage, and the number of carriers per unit colume can be determined from the magnitude of V_H.

Hall voltage

$$V_H = E_H w = v_d B w = \frac{|I|}{nte} B \tag{86}$$

7 Chapter 27: Sources of Magnetic Field

7.1 Magnetic Field $\vec{B}$

Due to a moving point charge

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \tag{87}$$

where $\hat{r}$ is a unit vector that points to the field point P from the charge q moving with velocity $\vec{v}$, and μ_0 is a constant of proportionality called the magnetic constant (the permeability of empty space):

$$\mu_0 = 4\pi \times 10^{-7} T \cdot m/A = 4\pi \times 10^{-7} N/A^2 \tag{88}$$

Due to a current element

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} \tag{89}$$

On the axis of a current loop

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(z^2+R^2)^{\frac{3}{2}}} \tag{90}$$

Inside a long solenoid, far from the ends

$$B_z = \mu_0 n I \tag{91}$$

where n is the number of turns per unit length.

Due to a straight-wire segment

$$B = \frac{\mu_0}{r\pi}\frac{I}{R}(\sin\theta_2 - \sin\theta_1) \tag{92}$$

Where R is the perpendicular distance to the wire and θ_1 and θ_2 are the angles subtended at the field point by the end of the wire.

Inside the loops of a tightly wound toriod

$$B = \frac{\mu_0}{2\pi}\frac{NI}{r} \tag{93}$$

7.2 Magnetic Field Lines

Magnetic lines neither begin nor end. Either they form closed loops or they continue indefinitely.

7.3 Gauss's Law for Magnetism

$$\phi_{m\text{ net}} = \oint_S \vec{B} \cdot \hat{n} dA = \oint_S B_n dA = 0 \tag{94}$$

7.4 Magnetic Poles

Magnetic poles always occur in north-south pairs. Isolated magnetic poles have not been found.

7.5 Ampère's Law

$$\oint_C \vec{B} \cdot d\vec{l} = \oint_C B_t dl = \mu_0 I_C \tag{95}$$

where C is any closed curve.

Validity of Ampr̂e's law
: Ampère's law is valid only if the currents are steady and continuous. It can be used to derive expressions for the magnetic field for situations with a high degree of symmetry, such as a long, straight, current-carrying wire or a long, tightly wound solenoid.

7.6 Magnetism in Matter

Matter can be classified as paramagnetic, ferromagnetic, or diamagnetic.

Magnetization
: A magnetized material is described by its magnetization vector $\vec{M}$, which is defined as the magnetic dipole moment per unit volume of the material:

$$\vec{M} = \frac{d\vec{\mu}}{dV} \tag{96}$$

The magnetic field due to a uniformly magnetized cylinder is the same as if the cylinder carried a current per unit length of magnitude M on its surface. This current, which is due to the intrinsic motion of the atom charges in the cylinder, is called an amperian current.

7.7 $\vec{B}$ in Magnetic Materials

$$\vec{B} = \vec{B}_{\text{app}} + \mu_0 \vec{M} \tag{97}$$

Magnetic susceptibility X_m

$$\vec{M} = \chi_m \frac{\vec{B}_{\text{app}}}{\mu_0} \tag{98}$$

For paramagnetic materials, χ_m is a small positive number that depends on temperature. For diamagnetic materials, it is a small negative constant independent of temperature. For superconductors, $\chi_m = -1$. For ferromagnetic materials, the magnetization depends not only on the magnetizing current but also on the past history of the material.

Relative permeability

$$\vec{B} = K_m \vec{B}_{\text{app}} \tag{99}$$

where

$$K_m = 1 + \chi_m \tag{100}$$

7.8 Atomic Magnetic Moments

$$\vec{\mu} = \frac{q}{2m}\vec{L} \tag{101}$$

where $\vec{L}$ is the orbital angular momentum of the particle.

Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} A \cdot m^2 = 9.27 \times 10^{-24} J/T \tag{102}$$

Due to the orbital motion of an electron

$$\vec{\mu}_l = -\mu_B \frac{\vec{L}}{\hbar} \tag{103}$$

Due to electron spin

$$\vec{\mu}_S = -2\mu_B \frac{\vec{S}}{\hbar} \tag{104}$$

7.9 Paramagnetism

Paramagnetic materials have permanent atomic magnetic moments that have random directions in the absence of an applied magnetic field. In an applied field these dipoles are aligned with the field to some degree, producing a small contribution to the total field that adds to the applied field. The degree of alignment is small except in very strong fields and at very low temperatures. At ordinary temperatures, thermal motion tends to maintain the random directions of the magnetic moments.

7.10 Ferromagnetism

Ferromagnetic materials have small regions of space called magnetic domains in which all the permanent atomic magnetic moments are aligned. When the material is unmagnetized, the direction of alignment in one domain is independent of that in another domain so that no net magnetic field is produced. When the material is magnetized, the domains of a ferromagnetic material are aligned, producing a very strong contribution to the magnetic field. This alignment can persist in magnetically hard materials, even when the external field is removed, thus leading to permanent magnets.

7.11 Diamagnetism

Diamagnetic materials are those materials in which the magnetic moments of all electrons in each atom cancel, leaving each atom with a zero magnetic moment in the absence of an external field. In an applied magnetic field, a very small magnetic moment induced that tends to weaken the field. This effect is independent of temperature. Superconductors are diamagnetic with a magnetic susceptibility equal to -1.

8 Chapter 28: Magnetic Induction

8.1 Magnetic Flux ϕ_m

General definition

$$\phi_m = \int_s \vec{B} \cdot \hat{n} dA \tag{105}$$

Uniform field, flat surface bounded by coil of N turns

$$\phi_m = NBA \cos\theta \tag{106}$$

where A is the area of the flat surface bound by a single turn.

Units

$$1Wb = 1T \cdot m^2 \tag{107}$$

Due to current in a circuit

$$\phi_m = LI \tag{108}$$

Due to current in two circuits

$$\phi_{m1} = L_1 I_1 + M I_2 \tag{109}$$

$$\phi_{m2} =:_2 I_2 + M I_1 \tag{110}$$

8.2 EMF

Faraday's law (includes both induction and motion emf)

$$\mathcal{E} = -\frac{d\phi_m}{dt} \tag{111}$$

Induction (time-varying magnetic field, C stationary)

$$\mathcal{E} = \oint_C \vec{E} \cdot d\vec{l} \tag{112}$$

Rod moving perpendicular to both its length and $\vec{B}$

$$|\mathcal{E}| = vBl \tag{113}$$

Self-induced (back emf)

$$\mathcal{E} = -L\frac{dI}{dt} \tag{114}$$

8.3 Faraday's Law

$$\mathcal{E} = -\frac{d\phi_m}{dt} \tag{115}$$

8.4 Lenz's Law

The induced emf and induced current are in such a direction as to oppose, or tend to oppose, the change that produces them.

Alternative statement

When a magnetic flux through a surface changes, the magnetic field due to any induced current produces a flux of its own — through the same surface and opposite sign to the change in flux.

8.5 Inductance

Self-inductance

$$L = \frac{\phi_m}{I} \tag{116}$$

Self-inductance of a solenoid

$$L = \mu_0 n^2 Al \tag{117}$$

Mutual inductance

$$M = \frac{\phi_{m21}}{I_1} = \frac{\phi_{m12}}{I_2} \tag{118}$$

Units and constants

$$1H = 1Wb/A = 1T \cdot m^2/A \tag{119}$$

$$\mu_0 = 4\pi \times 10^{-7} H/m \tag{120}$$

8.6 Magnetic Energy

Energy stored in an inductor

$$U_m = \frac{1}{2} L I^2 \tag{121}$$

Energy density in a magnetic field

$$u_m = \frac{B^2}{2\mu_0} \tag{122}$$

9 Chapter 30: Maxwell's Equations and Electromagneteic Waves

9.1 Maxwell's Displacement Current

Ampère's law can be generalized to apply to currents that are not steady (and not continuous) if the current I is replaced by $I + I_d$, where I_d is Maxwell's displacement current:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} \tag{123}$$

Generalized form of Ampère's law

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \left(I + I_d \right) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\phi_e}{dt} \tag{124}$$

9.2 Maxwell's Equations

The laws of electricity and magnetism are summariezed by Maxwell's equations.

Gauss's law

$$\oint_S E_n dA = \frac{1}{\epsilon_0} Q_{\text{inside}} \tag{125}$$

Gauss's law for magnetism

$$\oint_S B_n dA = 0 \tag{126}$$

Faraday's law

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S B_n dA = -\int_S \frac{\partial B_n}{\partial t} dA \tag{127}$$

Ampère's law modified

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \left(I + I_d\right) = \mu_0 I + \mu_0 \epsilon_0 \int_S \frac{\partial E_n}{\partial t} dA \tag{128}$$