

Math 215

3rd Practice Exam

November 21, 2017

Name (in block capital letters):

Instructor (tick one box): ☐ Section 1: L. Stanhope (12:50)

☐ Section 2: E. Sullivan (12:50)

Instructions: You are taking this exam under the honor system. Your signature at the bottom of this page is your promise to abide by the conditions described below. Breaking this promise violates Lewis and Clark's academic integrity policy.

1. To answer these questions, use only the knowledge in your head, and a calculator.
2. You have 90 minutes in which to take the exam. This should be one continuous time period, and not two or more periods separated by breaks. (Any breaks you take are included in the 90 minutes.) You may take the exam anywhere you wish. Start timing when you begin the exam.
3. Do not talk to any other student, whether enrolled in Math 215 or not, whether they have already taken the exam or not, about this exam until after 5 PM Friday, September 22nd. That includes: discussing specific questions, its difficulty, how long you worked on it, as well as any other exam-related topic you might imagine.
4. Be sure to carefully read the directions for each problem. Please show your work – correct answers to problems with no justification of where they come from will earn little credit.
5. Finally, do your best to think logically and clearly, and then trust your judgment. There are no “trick” questions here.

Problem	Score
1	
2	
3	
4	
5	
6	
Total	

1. Define $t : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $t(x, y) = x + y$.
 - i. What is the range of t ?
 - ii. Is t onto? Why/Why not?
 - iii. What is the pre-image of zero?
 - iv. What is the pre-image of 1?
 - v. Is t one-to-one? Why/Why not?
 - vi. If t is not one-to-one, please change the domain of t to a subset of \mathbb{R}^2 so that t restricted to this new subset is one-to-one AND has the same range as t .
2. Use set builder notation to write the set of all natural numbers that leave a remainder of 3 when divided by 5.
3. Is the following statement true? Please prove or give a counterexample.

$$"A \subseteq C \text{ if and only if } A \cup (B \cap C) = (A \cup B) \cap C"$$

4. Let $f : A \rightarrow B$ be a function, and let $C \subseteq D \subseteq B$. Use element chasing to show that $f^{-1}(C) \subseteq f^{-1}(D)$. Please recall that the pre-image, or inverse image, $f^{-1}(E)$ of some subset $E \subseteq B$ is defined by

$$f^{-1}(E) = \{x \in A \mid f(x) \in E\}$$

5. Let A be a nonempty set. Suppose $g : A \rightarrow \mathbb{R}$ is an injective function. Define a relation \sim on A in the following way. For $a, b \in A$,

$$a \sim b \text{ if and only if } g(a) \leq g(b)$$

- a. Please prove, or disprove by giving a counterexample, that \sim is reflexive.
 - b. Please prove, or disprove by giving a counterexample, that \sim is symmetric.
 - c. Please prove, or disprove by giving a counterexample, that \sim is transitive.
6. Let $F : \mathbb{N} \rightarrow \mathcal{P}(\mathbb{N})$ be a function given by $F(n) = \{m \in \mathbb{N} : m|n\}$.
 - a. Please compute $F(10)$.
 - b. Please determine whether or not F is one-to-one. Justify your conclusion with a proof or counterexample.
 - c. Please determine whether or not F is onto. Justify your conclusion with a proof or counterexample.