Math 215 – Fall 2017

Practice Homework 8 – Assigned October 5th, due October 9th

Note: Remember that you must show your work to get full credit for a problem.

1. Prove that for all positive integers n that

$$\sum_{i=1}^{n} i^2 = 1 + 4 + 9 + \cdots + (n-1)^2 + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

2. Prove that for all non-negative integers n that

$$\sum_{i=0}^{n} 3^i = 1 + 3 + 9 + \cdots + 3^{n-1} + 3^n = \frac{3^{n+1} - 1}{2}.$$

3. Prove that the sum of the fist n odd numbers is n^2. That is, prove

$$\sum_{i=1}^{n} (2i - 1) = 1 + 3 + 5 + \cdots + 2n - 1 = n^2.$$

4. Let $f_1,\ f_2,\ f_3, \cdots, f_n$ be functions. Let $g_0(x) = x$, $g_1 = f_1$, and $g_i = f_i \circ g_{i-1}$ for all integers $i \geq 2$. Thus $g_3(x) = f_3(f_2(f_1(x)))$. Prove that for all integers $n \geq 1$,

$$g_n'(x) = \prod_{i=1}^{n} f_i'(g_{i-1}(x)).$$