THEORETICAL DYNAMICS REVIEW

ARDEN RASMUSSEN

1. Rockets

Variable mass

$$m\frac{d\vec{v}}{dt} = \overrightarrow{F_{ext}} + \underbrace{\overrightarrow{U_{e,r}}\frac{dm}{dt}}_{T=\text{thrust}}$$

For rocket fired upwards $\overrightarrow{F_{ext}} = -mg$.

1.1. Tsiolkovsky's Rocket Equations.

$$V_f - V_i = U_{e,r}\ln\left(\frac{m_i}{m_f}\right)$$

1.2. Center of Mass.

$$\overrightarrow{R} \equiv \frac{\sum m_k \overrightarrow{r}_k}{M_{tot}}$$

$$K = \underbrace{\frac{1}{2}M_{tot}V_{cm}^2}_{\text{translating}} + \underbrace{\frac{1}{2}I_{cm}\omega^2}_{\text{rotating}}$$

2. Rolling down incline

Conserve of E

$$Mgh = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 \quad I_{cm} = fMR^2$$

No rotational $V_{cm} = \sqrt{2gh}$ With rotational $V_{cm} = \sqrt{\frac{2gh}{1+f}}$

2.1. Rolling Without Slipping.

$$V_{cm} = \omega r$$

3. Collisions

3.1. Inelastic.

p_{tot} conserved, E_{mech} **not** conserved. Use total mass for the final mas, as they stick together.

3.2. Elastic.

p_{tot} and E_{mech} are both conserved. Things collide and don't stick. $p_{1f} \perp p_{2f}$. Use $p_{ix} = p_{fx}$, and same with y, then use the relation found there in the expression for E.

Objects end on trajectories perpendicular to one another.

Date: May 7, 2019.

4. Central Force Motion

Angular momentum $\overrightarrow{L}$ conserved, use polar coordinates, motion in plane $\perp \overrightarrow{L}$.

$$\overrightarrow{p} = m\overrightarrow{v}$$
$$\overrightarrow{L} = I\omega = fmr^2\dot{\theta}$$

$$\overrightarrow{N} = \frac{d\overrightarrow{L}}{t}$$

Torque is time rate of change of angular momentum.

$$V_{eff}(r) = V(r) + \frac{L^2}{2mr^2}$$

- Circular
- Elliptical
- Parabolic
- Hyperbolic

$$E(r) = \frac{1}{2}m\dot{r}^2 + V_{eff}(r)$$

$$T = \frac{2\pi}{\dot{\theta}} \quad f = \frac{\dot{\theta}}{2\pi}$$

4.1. Small Oscillations.

a is the radius of circular motion. Find this by taking the first derivative and setting it equal t zero.

$$k = \frac{d^2V}{dx^2}\Big|_{x_min} \implies k_r = \frac{d^2V_{eff}}{dr^2}\Big|_a$$
$$\omega_r = \sqrt{\frac{k_r}{m}}$$

5. Hamiltonian

$$H = \sum_{k=1} p_k \dot{q}_k - L$$

$$p_k \equiv \frac{\partial L}{\partial \dot{q}_k}$$

$$\frac{\partial H}{\partial p_k} = \dot{q}_k$$
$$\frac{\partial H}{\partial q_k} = -\dot{p_k} + Q_k$$
$$\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

$\frac{dH}{dt} =$ if $Q_k = 0$, which means all forces are derivable form a potential, and $\frac{\partial H}{\partial t} = 0$, which means that H is not explicit function of time (does not have t directly in it).

6. Lagrangians

$$L = K - V$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$

7. Coupled Oscillators

$$m\ddot{x}_1 = -k(x_1 - l) + k_{12}(x_2 - x_1 - l_{12})$$
$$m\ddot{x}_2 = -k_{12}(x_2 - x_1 - l_{12}) - k(x_2 - (l + l_12))$$

$$u_1 \equiv x_1 - l \quad u_2 \equiv x_2 - (L + l_{12})$$

$$m\ddot{u}_1 = -ku_1 + k_{12}(u_2 - u_1)$$
$$m\ddot{u}_2 = -ku_2 - k_{12}(u_2 - u_1)$$

Since there is no dampening, we guess

$$u_1 = \beta_1 \cos(\omega t + \theta)$$
$$u_2 = \beta_2 \cos(\omega t + \theta)$$

If there was dampening, one would guess with complex numbers. We assume that ω, and θ are the same for both masses.

Now plug this into the previous equations, and construct system of equations. For this it will be

$$\underbrace{\begin{pmatrix} -m\omega^2 + k + k_{12} & -k_{12} \\ -k_{12} & -m\omega^2 + k + k_{12} \end{pmatrix}}_{A} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

To have non-trivial solutions, we need $\det(A) = 0$. So take the determinant, and set it equal to zero. Use that to solve for ω, you will probably need to use quadratic formula, but also okay to cheat a bit.