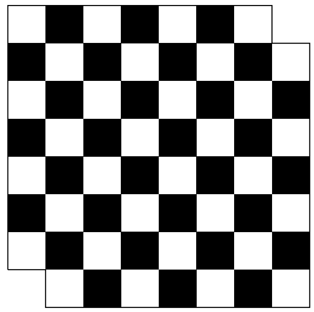
Math 215 – Fall 2017

Theory Homework 5 – Assigned October 9th, due October 16th

Note: Remember that you must show your work to get full credit for a problem.

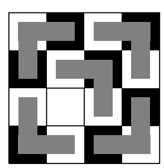
1. Consider an 8 by 8 chessboard with the bottom left and top right corners removed.



   Show that either there exists a way to tile the chessboard with dominoes such that none of the dominoes overlap or go off the board, or show that no such tiling exists.

2. Consider a arbitrary 2^n by 2^n chessboard with exactly one square missing. Show that for all $n \in \mathbb{Z}^+$ that every 2^n by 2^n chessboard with exactly one square missing can by titled by the following triomino:

   Here is an example of a B_2 board being tiled by the above triomino.



3. Let F_n denote the number of ways to tile a 2 by n chessboard with dominoes. Prove that for $n \geq 3$ that $F_n = F_{n-1} + F_{n-2}$.

   Example: there are 3 ways to tile a 2 by 3 chess board, so $F_3 = 3$.