Math 215 – Fall 2017

Theory Homework 8 – Assigned November 6th, due November 13th

Note: Remember that you must show your work to get full credit for a problem.

1. In each of the parts below we are given sets B_k for $k \in \mathbb{Z}^+$. Please find $\bigcup_{k=1}^{\infty} B_k$ and $\bigcap_{k=1}^{\infty} B_k$. Please prove your assertions.

   (a) $B_k = \{0, 1, 2, 3, \ldots, 2k\}$

   (b) $B_k = \{10 + k\} \cup \{x \in \mathbb{R} \; : \; \frac{3}{k} \leq x < \frac{5k+2}{k}\}$

2. In each of the parts below please define a family of sets $E_1, E_2, E_3, \ldots$ where $E_i \subset \mathbb{R}$ for each $i \in \mathbb{N}$ where no two sets E_i are equal to each other and such that the given conditions hold. After you define your families of sets, please prove the set equalities in each of the given conditions.

   (a) $\bigcup_{i=1}^{\infty} E_i = [0, \infty)$ and $\bigcap_{i=1}^{\infty} E_1 = [0, 1]$.

   (b) $\bigcup_{i=1}^{\infty} E_i = \mathbb{R}$ and $\bigcap_{i=1}^{\infty} E_i = \{3\}$.

3. Suppose $f : A \to B$, and that $X_1, X_2, X_3, \ldots$ is an infinite family of subsets of A, and that $Y_1, Y_2, Y_3, \ldots$ is an infinite family of subsets of B.

   (a) Please use element chasing to show that

   $$f^{-1}\left(\bigcup_{k=1}^{\infty} Y_k\right) = \bigcup_{k=1}^{\infty} f^{-1}(Y_k)$$

   (b) The statement below is false. In particular one direction of the set equality does not work. Please identify the direction that fails and give a counterexample showing how it fails. Then prove the direction that works using element chasing.

   $$f\left(\bigcap_{k=1}^{\infty} X_k\right) = \bigcap_{k=1}^{\infty} f(X_k)$$