Math 215 – Fall 2017

Theory Homework 7 – Assigned October 30th, due November 6th

Note: Remember that you must show your work to get full credit for a problem.

1. Let A and B be sets. Let $\sim_A$ be an equivalence relation on A. Let $\sim_B$ be an equivalence relation on B.

   For each of the follow relations consider the properties of being reflexive, symmetric, and transitive. For each property, either prove that the property always holds, prove that the property never holds, or provide two examples for which the property holds in one but not in the other.

   (a) Let $\sim$ be a relation on $A \times B$ such that $(a_1, b_1) \sim (a_2, b_2)$ if and only if $a_1 \sim_A a_2$ **and** $b_1 \sim_B b_2$.

   (b) Let $\sim$ be a relation on $A \times B$ such that $(a_1, b_1) \sim (a_2, b_2)$ if and only if $a_1 \sim_A a_2$ **or** $b_1 \sim_B b_2$.

   (c) Let $\sim$ be a relation on $A \times B$ such that $(a_1, b_1) \sim (a_2, b_2)$ if and only if $a_1 \not\sim_A a_2$ **and** $b_1 \not\sim_B b_2$.

2. Let A be a non-empty set. let $\sim^+$ be a arbitrary relation on A. Let $\sim^-$ be a relation on A such that $a_1 \sim^- a_2$ if and only if $a_1 \not\sim^+ a_2$. Define a relation $\sim$ on A such that $a_1 \sim a_2$ if and only if $a_1 \not\sim^+ a_2$ **or** $a_1 \not\sim^- a_2$.

   (a) Prove that $\sim$ is an equivalence relation.

   (b) State how many equivalence classes $\sim$ has.

3. Let A be a nonempty set.

   In class it was stated that for any partition P of A there exists a relation $\sim$ on A such that $\sim$ is an equivalence relation, and its set of equivalence classes is exactly P. For this problem please define an equivalence relation on A that has this property. Please give a rigorous proof that the relation you define is in fact an equivalence relation. You can 'wave your hands' at the proof that the equivalence classes of the relation you define do in fact coincide with P. You are welcome to write up a proof of this last fact, but it is not required for the assignment.