Review Suggestions for Midterm 2
Math 215, Fall 2017

Please use this study guide to identify what to study for Midterm 2 on Monday, and to practice some new problems. Once you feel well-prepared use the practice test to simulate the exam experience. (This means working at a desk in a quiet space without the aid of notes or the textbook, and without any distracting technology.) The practice test will be posted on Moodle by Tuesday evening. General suggestions for taking timed math tests are posted on Professor Stanhope's website: `https://sites.google.com/a/lclark.edu/stanhope/study-strategies`.

In order to allow more time for Midterm 2 it will be given as a timed, outside of class exam. To do this first identify a 90 minute period of time during your day on Monday. Note that our class will be canceled that day. At the beginning of this 90 minute period please stop by Professor Stanhope or Professor Sullivan's office (students from both sections can go to either professor) to pick up the exam. After signing the rules and honor statement for the exam, you will take the exam to a quiet place (Watzek works well, for example) and take the exam in 90 minutes. When you are done you will drop the exam off at Professor Stanhope or Professor Sullivan's office (again, students from both sections can go to either professor). A sample of the rules and honor statement is posted on our Moodle page.

---

1 Direct and Indirect Proofs in Number Theory

You will be expected to be able to construct direct and indirect (usually contradiction) proofs of statements from number theory. Please review notes and homework from Sections 2.0, 2.1 and 2.2 in the text.
To review for Direct & Indirect Proofs in Number Theory please:

- Review the definition of $a|b$
- Review the the definition of odd ($2n+1$ with $n \in \mathbb{Z}$, or not even – your choice) and even ($2n$ with $n \in \mathbb{Z}$) integers.
- Review the definition of prime, composite and irreducible integers.
- Recall the relationship between prime and irreducible integers. (prime implies irreducible)
- Know the precise statement of the Division Algorithm. What does it mean to say that the Division Algorithm is an "existence and uniqueness" result?

Practice Problems:

1.1 Is it true that for every natural number n, 3 never divides n^2+1? Please prove your answer.

1.2 Prove the following statement: If $a \mid b$ and $a \mid c$, then $a \mid (bx+cy)$

1.3 Prove the following statement: If $n \nmid (n-1)!$, then $n \leq 4$ or n is prime.

---

2 Induction

You will be expected to be able to use the Principle of Finite Induction (I also call this Proof by Mathematical Induction, PMI), Theorem 2.3.4, to prove a statement. Note that I will not tell you when to use induction. So please be sure to review *when* to use induction in addition to *how* to use induction. The Second Principle of Finite Induction (I call this Proof by Strong Induction, PSI), Theorem 2.3.6 will not be tested on this test.
To review for PMI please:

- Go over the feedback you received on all PMI homework problems and take a look at the solution keys. Can you redo these problems without hesitation and without looking at your notes?
- Review the PMI examples in the text.
- Practice a couple of the practice problems below.
- NO BACKWARDS PROOFS.
- Also be sure to include all of the structure of a PMI. (Base Case first, Inductive Hypothesis that says "Assume the statement is true for $k \geq BaseCaseNumber$". Inductive Step that says "We now use the IH to prove the statement for $k+1$," pointing out where the IH is used.)

Practice Problems:

2.1 Prove that $1 + 3 + 5 + \cdots + (2n-1) = n^2$.

2.2 Prove that $1 + 2n \leq 3^n$ for all $n \in \mathbb{N}$.

2.3 Prove that for all $n \geq 1$, $\sum_{i=1}^{n} \frac{1}{3^i} = \frac{3^n - 1}{2 \cdot 3^n}$.

---

3 Greatest Common Divisors

Please go over the feedback you received on all Section 2.4 and 2.5 homework problems. Can you redo these problems without hesitation and without looking at your notes?
Please know:

- The definition of the GCD of two integers.
- How to use the Euclidean Algorithm to find the GCD of two integers and of two polynomials with real coefficients.
- How to use the (extended) Euclidean Algorithm to express the GCD of two integers or polynomials as a linear combination of the two integers.
- Please know the statement of Theorem 2.4.3.
- Please know know what it means for two integers to be relatively prime.

- Please know the statement of Theorem 2.4.5. Please know how to prove this Theorem, citing Theorem 2.4.3 for one of the directions.
- Practice the proof of Theorem 2.4.6.

Also try some of the practice problems below.

3.1 Find the prime number decomposition of 2730 and 3150. Then find $\gcd(2730, 3150)$ using the Division Algorithm. Check your answer using the prime number decomposition. Now express the GCD as a linear combination of 2730 and 3150. Finally find the LCM of these two numbers.

3.2 Without using the Fundamental Theorem of Arithmetic, show that if $d = gcd(a, b)$ then $\frac{a}{d}$ and $\frac{b}{d}$ are relatively prime.

3.3 (This is a problem from the 2000 Putnum exam.) Prove that the following number is an integer:

$$\frac{\gcd(m, n)}{n}\binom{n}{m}$$

---

4 The Fundamental Theorem of Arithmetic

Please know the statement of this Theorem. What does it mean to say that this is an "existence and uniqueness" result? Also try some of the practice problems below.

4.1 Suppose you have the prime factorization of two integers. Using the prime factorization, give an expression for the GCD of those integers.

4.2 Integer m is the *least common multiple* (LCM) of two integers a and b if $a \mid m$ and $b \mid m$, and for any n such that $a \mid n$ and $b \mid n$, we have $m \leq n$. Using the prime factorization of integers a and b, give an expression for the least common multiple of those integers.

4.3 What can be said about the product of the GCD and the LCM of two integers?

---

5 Set Theory Basics

Please be familiar with what a set is, how to use set builder notation, what is meant by the intersection and union of two sets, and what is meant by the cardinality of a finite set. Exercises 1 through 4 in Section 4.1, Exercise 1 in Section 4.2, and Exercise 1abcd and 3ab in Section 4.3, are good practice problems.