PRACTICE EXAM 3 – VERSION 1

Theory. A huge component of our course deals with oscillations through second order linear equations

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = f.$$

(1) What do the parameters m, b, k and f represent?

(2) In what units should the quantities y, t, m, b, k and f be measured in?

(3) The following four pictures show graphs of solutions of differential equations of the form

$$\frac{d^2y}{dt^2} + b(t)\frac{dy}{dt} + y = 0.$$

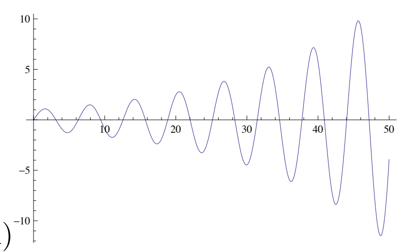
Match the graphs to the following conditions:

(a) The parameter b is a positive constant.

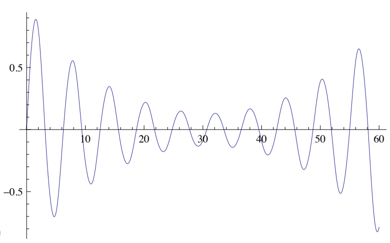
(b) The parameter b is positive, but changes in time. Occasionally, it is so big that it "stops all motion".

(c) The parameter b is a negative constant.

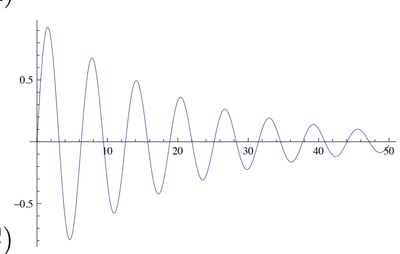
(d) The parameter b is at times positive and at times negative.



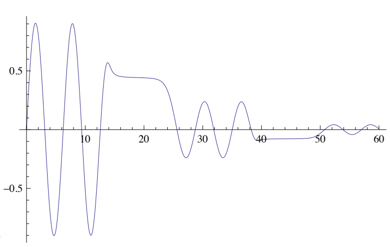
(A)



(B)



(C)



(D)

Non-homogeneous linear equations. Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} + y = t + 3e^t.$$

(Near-)Resonance. Let $\omega \approx 1$. Consider the following IVP:

$$\tfrac{d^2y}{dt^2} + y = \cos(\omega t), \quad y(0) = 0, \tfrac{dy}{dt}(0) = 0.$$

(1) Find the solution of this IVP.

(2) The solution you just found should feature beats. Utilize the trigonometric identity

$$\cos(A) - \cos(B) = 2\sin\left(\frac{B-A}{2}\right)\sin\left(\frac{B+A}{2}\right)$$

to determine the period and the amplitude of the beats.

(3) Take the limit of your solution as $\omega \to 1$.

(4) Verify that the function you just discovered is a solution of the resonant IVP

$$\tfrac{d^2y}{dt^2} + y = \cos(t), \quad y(0) = 0, \tfrac{dy}{dt}(0) = 0.$$

Utilizing conserved quantities. Use conserved quantities to draw the phase portrait of the system

$$\frac{d^2y}{dt^2} + 5y^4 = 5.$$