Homework 2: One dimensional integrals and applications
(**Due Friday September 28**
Total points= 110)

September 19, 2018

1 WARM UP **(20 POINTS)**

Do Exercise 5.4

2 THE POTENTIAL AND ELECTRIC FIELD OF A LINEAR CHARGE DISTRIBUTION **(60 POINTS)**

The electric potential from a charge distribution is given by

$$V(x,y,z) = \frac{1}{4\pi\epsilon}\int dx'dy'dz' \frac{\rho(\vec{r}')}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}, \tag{2.1}$$

while the electric field is $\vec{E} = -\nabla V$:

$$\vec{E} = \frac{1}{4\pi\epsilon}\int dx'dy'dz'\rho(\vec{r}') \frac{\vec{r}-\vec{r}'}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}}, \tag{2.2}$$

where ρ is the charge distribution. In these expressions $\vec{r}$ is the position vector of the observer and $\vec{r}'$ is the position vector of the charge distribution. In the case our charges are distributed on a line, we can replace the volume integral by a line integral.

Now, assume you have a uniform linear charge density $\rho = 1$ Coulomb/m distributed over an ellipse with minor and major axis 4 and 9 m, respectively. Put the major axis of the ellipse on the x-axis and the minor one on the y-axis.

1. **Write** an expression of V and E in a form of an integral over a single variable. **Hint: you can parametrize your ellipse as** $x' = 3\cos\theta'$, $y' = 2\sin\theta'$**, where** $0 \leq \theta' < 2\pi$**.**

2. **Write** a code that enable you to obtain the value of the potential and electric field at a general point (x, y, z). **Use** the Simpson's rule in your code. Your code should also give an estimation of the error. **Use** your code to find the potential and electric field at $(1, 4, 7)$ **using** $h = .1$.**s**et $\epsilon = 1$

3. **Write** a code that uses the adaptive trapezoidal method to give you an accuracy to 6 digits. **Use** your code to find the potential and electric field at $(1, 2, 5)$ to 6 digits.

4. **Use the density plot** in order to visualize $V(x, y, z = 1)$.

3 PLAYING WITH RAW DATA **(30 POINTS)**

My former student Ben Kolligs and I have been working on Monte Carlo simulations to calculate what is known as the Reny mutual information in XY spin systems. You are given three different kinds of energy as raw data E_1, E_2, E_3 as a function of temperature T, where we use $\Delta T = h = 0.1$. We use these data to compute Reny mutual information as

$$RMI(T) = \int_T^{T_{max}} \frac{2E_1(T) - E_2(T) - 2E_3(T)}{T^2} dT. \tag{3.1}$$

You can find what is T_{max} and T_{min} from the data file in the folder HW2.

1. **Use** the data files you are given to compute the integral $RMI(T)$ for any T using the Simpson's rule. **Plot** $RMI(T)$ as a function of T.

2. **Estimate** the error of your calculations.

3. **Bonus 20 points** It took us about 14days to generate this set of data, and hence, it is not a cheap or easy process. Therefore, we are trying to find the best method to perform the integration such that error is minimized. Given that there is an uncertainty of $\pm.1$ in E_1, E_2, E_3, then **Suggest** a method that can help us achieve our goal.