Problem 1. In our analysis of standing waves we showed that the Dirichlet boundary condition implies that the constant λ appearing in both the space and time ODEs is negative. Here we investigate the implications of λ being negative by comparing to the situation where λ is not negative.

1. Suppose $\lambda = 0$. Describe the behavior of solutions to $\frac{d^2A}{dt^2} = \lambda A$.
2. Suppose $\lambda > 0$. Describe the behavior of solutions to $\frac{d^2A}{dt^2} = \lambda A$.
3. Are either of the cases above consistent with oscillatory behavior? What is the connection between the sign of the eigenvalue λ and oscillatory behavior?

Problem 2. This problem applies the separation of variables method from class to find solutions $u(t,x)$ of the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

under the assumption of Dirichlet boundary conditions:

$$u(t,0) = 0 \quad \text{and} \quad u(t,L) = 0.$$

Note that we are assuming $0 \leq x \leq L$. Here we are modeling heat flow in a metal rod of length L with circular cross-section. We assume that the rod is insulated so that heat can only enter or leave at the ends. The Dirichlet boundary conditions amount to holding the ends of the rod at temperature zero. Given this problem please complete the following.

a Write down the eigenvalues of the 1-dimensional Laplace operator that you obtain as you complete this problem. Please show the work that you do to get these eigenvalues.

b. Write down the eigenfunctions, also called standing waveforms, corresponding to each of the eigenvalues you found in part (a). Please show the work that you do to get these eigenfunctions.

c. Use a graph to argue why a few of the standing waveforms found in part (b) appear to be correct. Please turn in both the graph and the argument why the graph implies your standing waveforms appear to be correct.

d. Write down all of the standing wave solutions of this problem. Please show the work that you do to get these standing wave solutions.