CALCULUS III REVIEW

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Contents

1 Geometry

1.1 Cartesian Coordinates

Commonly sketching an equation and explaining it in words. Try to sketch out the graph, checking several point. Then attempt to classify the shape.

1.2 Vector Algebra

Things that appear commonly with vector algebra questions.

- Sketching

  Simply drawing the vector from base point.

- Transformation to vector from u, v

  Given $\vec{a}$ and $\vec{b}$.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + u \left\langle \begin{matrix} \vec{a_x} \\ \vec{a_y} \\ \vec{a_z} \end{matrix} \right\rangle + v \left\langle \begin{matrix} \vec{b_x} \\ \vec{b_y} \\ \vec{b_z} \end{matrix} \right\rangle$$

$$T(u, v) = \begin{cases} x = x_0 + u\vec{a_x} + v\vec{b_x} \\ y = y_0 + u\vec{a_y} + v\vec{b_y} \\ z = z_0 + u\vec{a_z} + v\vec{b_z} \end{cases}$$

- Dot Product $(\cdot)$

$$a_x b_x + a_y b_y + a_z b_z = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos(\gamma)$$

- Cross product

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

- Length of a vector

$$L = \sqrt{\vec{a} \cdot \vec{b}}$$

- Area of parallelogram

$$A^2 = \det \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$$

- Volume of parallelotope

$$V^2 = \det \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{a} \cdot \vec{c} & \vec{b} \cdot \vec{c} & \vec{c} \cdot \vec{c} \end{vmatrix}$$

1.3 Curvilinear Coordinates

Determining transformations, and parameterization of equations, computing ∂_u and ∂_v, computing area element(dA), and normal vector field($\vec{N}$).

$$Parameterization : T(u, v)$$

- ∂_u / ∂_v

$$\partial_u = \left\langle \begin{array}{cc} \frac{\partial_x}{\partial_u} & \frac{\partial_y}{\partial_u} \end{array} \right\rangle$$
$$\partial_v = \left\langle \begin{array}{cc} \frac{\partial_x}{\partial_v} & \frac{\partial_y}{\partial_v} \end{array} \right\rangle$$

- Area element(dA)

$$dA = \sqrt{\det \begin{vmatrix} \partial_u \cdot \partial_u & \partial_u \cdot \partial_v \\ \partial_u \cdot \partial_v & \partial_v \cdot \partial_v \end{vmatrix}}$$

- Normal Vector Field($\vec{N}$)

$$\vec{N} = \partial_u \times \partial_v$$

1.3.1 Polar Coordinates

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$(r, \theta)$$
$$dA = rdrd\theta$$

1.3.2 Cylindrical Coordinates

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$z = z$$
$$(r, \theta, z)$$
$$dA = rdrd\theta dz$$

1.3.3 Spherical Coordinates

$$x = r\sin(\phi)\cos(\theta)$$
$$y = r\sin(\phi)\sin(\theta)$$
$$z = r\cos(\phi)$$
$$(r, \theta, \phi)$$
$$dA = r^2\sin(\phi)drd\theta d\phi$$

2 Multi-Variable Integration

Setting up integrals and evaluating them.

2.1 1 Dimensional

$$\int \rho \; dS$$
$$\int \rho(u) \; du$$

2.2 2 Dimensional

$$\iint \rho \; dA$$
$$\iint \rho(u,v) \cdot Stretch \; dudv$$

2.3 3 Dimensional

$$\iiint \rho \; dV$$
$$\iiint \rho(u,v,w) \cdot Stretch \; dudvdw$$

3 Differential Calculus

3.1 Linearizion/Taylor Approximation

$$Jacobian:$$
$$DT = \begin{bmatrix} \partial u_1 & \partial v_1 \\ \partial u_2 & \partial v_2 \end{bmatrix}$$
$$Linearization:$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \approx f(x_0,y_0) + \begin{bmatrix} \frac{dx}{du}|_{(x_0,y_0)} & \frac{dx}{dv}|_{(x_0,y_0)} \\ \frac{dy}{du}|_{(x_0,y_0)} & \frac{dy}{dv}|_{(x_0,y_0)} \end{bmatrix} \cdot \begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix}$$
$$x \approx x_0 + \begin{bmatrix} \frac{dx}{du}|_{(x_0,y_0)} & \frac{dx}{dv}|_{(x_0,y_0)} \end{bmatrix} \cdot \begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix}$$
$$y \approx y_0 + \begin{bmatrix} \frac{dy}{du}|_{(x_0,y_0)} & \frac{dy}{dv}|_{(x_0,y_0)} \end{bmatrix} \cdot \begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix}$$

$$QuadraticApproximation:$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \approx f(x_0, y_0) + \begin{bmatrix} \frac{dx}{du}|_{(x_0,y_0)} & \frac{dx}{dv}|_{(x_0,y_0)} \\ \frac{dy}{du}|_{(x_0,y_0)} & \frac{dy}{dv}|_{(x_0,y_0)} \end{bmatrix} \cdot \begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix}$$
$$+ \frac{1}{2} \cdot \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \cdot \begin{bmatrix} \frac{d^2x}{dy^2}|_{(x_0,y_0)} & \frac{d^2x}{dudv}|_{(x_0,y_0)} \\ \frac{d^2y}{dudv}|_{(x_0,y_0)} & \frac{d^2y}{dv^2}|_{(x_0,y_0)} \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

3.2 Optimization(With or Without Constraints

Determine when the *Jacobian* is 0. That is a critical point, then using *Hessian* and quadratic approximation to determine shape(*Bowl*, *Saddle*, *etc.*). HELP!

3.3 Chain Rule

Derivative of outside evaluated at (x_0, y_0) of the inside times the derivative of the inside evaluated at the base point.

$$(x, y) = T_1(u, v)$$
$$(z, w) = T_2(x, y)$$
$$DT_1 = Jacobian\ for\ T_1$$
$$DT_2 = Jacobian\ for\ T_2$$
$$D(T_2 \circ T_1) = DT_2|_{(x_0,y_0)} \cdot DT_1|_{(u_0,v_0)}$$
$$DT_2(T_1(u, v)) \cdot DT_1(u, v)$$
$$\begin{bmatrix} \frac{dz}{du} & \frac{dz}{dv} \\ \frac{dw}{du} & \frac{dw}{dv} \end{bmatrix} = \begin{bmatrix} \frac{dz}{dx} & \frac{dz}{dy} \\ \frac{dw}{dx} & \frac{dw}{dy} \end{bmatrix} \cdot \begin{bmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{bmatrix}$$

4 Vector Calculus

4.1 What are (div/curl/grad/potential)

4.1.1 Divergence

1. Extent that a vector field exits a bound area.
2. Extent that a vector field goes away from a point.

$$div(\vec{V}) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

4.1.2 Circulation

1. Extent that a vector field follows the boundary of a bound area.
2. Extent that a vector field goes around a point.

$$\overrightarrow{curl}(\vec{V}) = \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle$$

4.1.3 Gradient

grad is the rate of change in $f(x, y)$ as x and y change.

$$\overrightarrow{grad}(f) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

4.1.4 Potential

HELP!

1. Create a system of equations equaling $\overrightarrow{grad}(f)$ to $\vec{V}$.
2. Solve the first equation for f through integration, resulting with C in terms of the other variables.
3. Derive f in terms of y and z with a $\frac{\partial C}{\partial y}$ and a $\frac{\partial C}{\partial z}$.
4. Equal the other equations to the just derived values for $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$.
5. Solve the second equation for C through integration, resulting with a D in terms of the final variable.
6. Derive C in terms of the final variable with a $\frac{\partial D}{\partial z}$.
7. Equal the final equation to the just derived value for $\frac{\partial C}{\partial z}$.
8. Solve the final equation for D through integration, resulting with an E that is just a number.
9. Combine all solved functions into one final f.

$$\begin{cases} \frac{\partial f}{\partial x} = \vec{V}_x \\ \frac{\partial f}{\partial y} = \vec{V}_y \\ \frac{\partial f}{\partial z} = \vec{V}_z \end{cases}$$

$$\Downarrow$$

$$f = f(x, y, z) + C(y, z) + D(z) + E$$

4.2 State the Fundamental Theorems

HELP!

- Gradient Theorem:

$$\int_a^b \overrightarrow{grad}(f) = f(b) - f(a)$$

- Stokes' Theorem:

$$\iint_S \overrightarrow{curl}(\vec{V}) \cdot \vec{N} dA = \int_C \vec{V} \cdot \vec{T} ds$$

- Divergence Theorem:

$$\iiint_{\Omega} div(\vec{V})dA = \iint_{S} \vec{V} \cdot \vec{N} ds$$

- Green's Theorem(Circulation):

$$\iint_{S} curl(\vec{V})dA = \int_{C} \vec{V} \cdot \vec{T} ds$$

- Green's Theorem(Divergence):

$$\iint_{S} div(\vec{V})dA = \int_{C} \vec{V} \cdot \vec{N} ds$$

4.3 Compute Using Fundamental Theorems

Enter in equation, and use theorems when possible.