QUANTUM MECHANICS MIDTERM 2

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1. Time-Independent Schrödinger Equations

1.1. **Harmonic Oscillator.**

1.1.1. *Base State.*

$$\Psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2}$$
$$E_0 = \frac{\hbar\omega}{2}$$

1.1.2. *Ladder Operators.*

$$\hat{a_+} = \frac{1}{\sqrt{2m\omega\hbar}}\left[-i\hat{p} + m\omega x\right]$$
$$\hat{a_-} = \frac{1}{\sqrt{2m\omega\hbar}}\left[i\hat{p} + m\omega x\right]$$
$$[\hat{a_-}, \hat{a_+}] = 1$$

1.1.3. *Solving.*

$$\hat{a_+}\psi_n = \sqrt{n+1}\psi_{n+1}$$
$$\hat{a_-}\psi_n = \sqrt{n}\psi_{n-1}$$
$$\psi_n = \frac{1}{\sqrt{n!}}(\hat{a_+})^n\psi_0$$
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

1.1.4. *Operators.*

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a_+} + \hat{a_-}\right)$$
$$\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}\left(\hat{a_+} - \hat{a_-}\right)$$

1.2. **Free Particle.** Here be dragons

Date: November 17, 2018.

2. Formalism

2.1. **Hilbert Space.** All wave functions live in Hilbert space. Any function in that Hilbert space can be represented as a linear combination of basis functions of that Hilbert space.

2.2. **Observables.** The $\langle O \rangle$ is always real for observables. Observables are represented by hermitian operators.

$$\langle O \rangle = \langle \Psi | \hat{O} | \Psi \rangle$$

The eigenvalues of hermitian operators is always real.

2.3. **Uncertainty Principle.**

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i}\left\langle \left[\hat{A}, \hat{B}\right] \right\rangle\right)^2$$

Observables whose operators do not commute car called incompatible observables, meaning that measuring one changes the other.

2.4. **Dirac Notation.** We can express functions as a basis vector in Hilbert space.

$$|\alpha\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} \qquad \langle\alpha| = \begin{pmatrix} \alpha_1^* \alpha_2^* \cdots \alpha_n^* \end{pmatrix}$$

The identity operator is

$$\hat{I} = \sum |\alpha\rangle\langle\alpha|$$

For some operator $\hat{O}$ and eigenvalues o_n

$$\hat{O}\,|n\rangle = o_n\,|n\rangle$$

2.4.1. *Matrix Operators.*

$$O_{nm} \equiv \langle n|\hat{O}|m\rangle$$

Hermitian if

$$O_{nm} = O^*_{mn}$$

2.4.2. *Harmonic Oscillator.*

$$\hat{a_+}\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$
$$\hat{a_-}\,|n\rangle = \sqrt{n}\,|n-1\rangle$$

2.4.3. *Wave Function.*

$$|\Psi\rangle = \sum_n A_n\,|n\rangle$$
$$A_n = \langle n|\Psi\rangle$$
$$|\Psi\rangle = \sum_n \langle n|\Psi\rangle\,|n\rangle$$