Chapter 1

Counting Sheep, Among Other Things

1.1 Learning to Count

The Addition Principle

Suppose there is a task, T, to be completed and there are numerous different methods, $M_1, M_2, \ldots, M_k$ which can be used to complete T. If method M_1 can be accomplished in a_1 ways, method M_2 in a_2 ways,, and method M_k in a_k ways, and $M_1, M_2, \ldots, M_k$ are mutually exclusive, then T can be compleated in $a_1 + a_2 + \cdots + a_k$ ways.

The Multiplication Principle

Suppose there is a task, T, to be completed, but now the task can be completed in stages. That is, T can be broken down into subtasks, $t_1, t_2, \ldots, t_m$ so that T is accomplished only after all of the sub tasks have been completed. If t_1 can be done b_1 ways, t_2 in b_2 ways, , and $t)m$ in $b)m$ ways. Then T can be completed in $b_1 \cdot b_2 \cdot \cdots \cdot b_m$ ways.

1.2 Permutations

Definition 1.2.1. *The notation $n!$, which we read as n factorial, is defined as $n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 3 \cdot 1 \cdot 1$.*

Definition 1.2.2. *An arrangment of n objects is called a permutation of the objects.*

Definition 1.2.3. *An arrangement of r distinct objects out of a collection of n distinct objects is called an r-permutation of the n objects.*

Theorem 1.2.1. *The number of r-permutations of n objects is $\frac{n!}{(n-r)!}$. This is often denoted $P(n,r)$ or ${}_nP_r$.*

1.3 Combinations

Definition 1.3.1. *A collection of r objects chosen from n distinct objects without regard to the order of the objects is called an r-combination of the n objects.*

Theorem 1.3.1. *The number of r-combinations of n objects is $\frac{n!}{(n-r)!\cdot r!}$. This number is denoted $C(n,r)$ or ${}_nC_r$ or $\left(\begin{array}{c} n \\ r \end{array}\right)$.*