Math 215 – Fall 2017

Theory Homework 10 – Assigned November 27th, due December 4th

Note: Remember that you must show your work to get full credit for a problem.

1. This problem concerns permutations of the numbers in the set $\{1, 2, 3, \cdots, n\}$. Recall that early in the semester we wrote permutations as ordered lists such as 31542 (here taking $n = 5$). Informally we say that a permutation has a pair of consecutive numbers if somewhere in the list you can find two consecutive numbers in increasing order. Again taking $n = 5$, the example 31542 has no consecutive numbers while the permutation 31452 has a pair of consecutive numbers, the 45 that appears in the third and fourth spots.

   Later we learned that a permutation of a set is in fact a bijection from the set to itself. Using this formal perspective, we say that a permutation p of $\{1, 2, 3, \cdots, n\}$ has consecutive numbers if there exists $k, \ell \in \{1, 2, 3, \cdots, n-1\}$ such that $p(k) = \ell$ and $p(k+1) = \ell + 1$, that is it sends a pair of consecutive numbers to a pair of consecutive numbers.

   (a) First consider all permutations of the first 6 positive integers $\{1, 2, 3, 4, 5, 6\}$. How many such permutations have no consecutive numbers?

   (b) Now consider all permutations of the first n positive integers $\{1, 2, 3, \cdots, n\}$. Let $C(n)$ denote the number of permutations on the first n positive integers that have no consecutive numbers. Find a formula for $C(n)$.