PRACTICE EXAM 3 – VERSION 2

Theory. State and prove the Superposition Principle applicable to non-homogeneous linear equations of the form

$$f(t)\frac{d^2y}{dt^2} + g(t)\frac{dy}{dt} + h(t)y = k(t).$$

Non-homogeneous linear equations. Find the general solution of each of the following equations:

(1) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 1$;

(2) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 1$;

(3) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 5y = 1$.

(Near-)Resonance. Find a forced response of the undamped oscillator with mass 1kg and spring constant $16\frac{\text{kg}}{\text{s}^2}$ which is exposed to the periodic force of $\cos(4t)\,\text{kg}\,\frac{\text{m}}{\text{s}^2}$. Draw a relatively detailed graph (in the ty-plane) of the forced response you found.

Utilizing Conserved Quantities. Consider the system

$$\begin{cases} \dfrac{dx}{dt} = ye^{xy} \\ \\ \dfrac{dy}{dt} = -xe^{xy}. \end{cases}$$

(1) Argue that $E(x, y) = x^2 + y^2$ is a conserved quantity for this system.

(2) Draw the phase portrait of the system.

(3) If initially we have $x(0) = y(0) = 1$ and if at some time t we have $x(t) = 0$, what can you say about $y(t)$ at that time?