

# Differential Equations (Exam #1)

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# 1 Summary

- Mixing Problems
  - Setup
  - Analyze
- Find equilibrium solutions and perform qualitative analysis.
- Find solution using analytic methods
- Fundamental Theorem
  - Solution curves don't cross unless RHS turns *ugly* on you.
- Euler method
  - Within reason
- Slope field
  - Not much of number crunching
- Some discussion about a parameter

## 2 Reading & Writing Differential Equations

The form of a differential equation must be the rate equals some equation that incorporates the equation.

$$\frac{dy}{dt} = f(t, y(t)) \quad (1)$$

Where  $f(t, y(t))$  is some function that involves  $y(t)$  and optional involves  $t$ .

### 2.1 Exponential Model

**Idea:** Relative growth rate is some constant:  $k$ .

$$\frac{dy}{dt} = k \cdot y(t) \quad (2)$$

### 2.2 Logistic Model

**Idea:** The relative growth rate may not be constant, and may depend on circumstances. It may depend on  $y(t)$  itself. Where  $M$  is the maximum limit for  $y(t)$ .

$$\frac{dy}{dt} = ky(t) \left( 1 - \frac{y(t)}{M} \right) \quad (3)$$

## 3 Qualitative Analysis

**Basic Idea:** Do careful accounting of slopes and connect the points.

## 4 Analytic Methods

### 4.1 Stream Plots/Slope Fields

## 5 Numeric Methods