Math 215 – Fall 2017

Theory Homework – Assigned September 11th, due September 18th

Note: Remember that you must show your work to get full credit for a problem.

1. Consider the combinatorial identity

$$\binom{n}{k} = \binom{n}{n-k},$$

where $n \geq 0$ and $0 \leq k \leq n$. Prove it in the following ways:

   (a) Prove it by using algebra.

   (b) Prove it using a counting argument.

2. Let $S(n, k)$ denote the number of ways to place n labeled balls into k unlabeled boxes such that every box has at least one ball. (these are called Stirling numbers of the second kind).

   Then

$$S(n+1, k) = kS(n, k) + S(n, k-1),$$

for $n \geq 1$ and $k \geq 1$.

   (a) Verify that the identity holds for $n = 3$ and $k = 2$ by computing all parts of the identity and showing that they are equal.

   (b) Prove the above identity by using a counting argument.

3. Consider the combinatorial identity

$$\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i},$$

for $n \geq 0$. Prove it by using a counting argument.