

Thermo Physics Review

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1 3D-Ideal Gas Law

$$PV = NkT \quad (1)$$

$$Nk = nR \quad (2)$$

1.1 Equipartition Theorem

Only true if f is quadratic. E.g. If kinetic energy and potential energy are given by quadratic functions

$$y = nf \frac{1}{2} kT \quad (3)$$

$$f = 3 \quad \text{Monatomic} \quad (4)$$

$$f = 5 \quad \text{Diatomic} \quad (5)$$

2 1-st Law

$$U = Q + W \quad (6)$$

Values are (+) if work or heat is added or done *on* the system. Values are (−) if work or heat is removed from or done *by* the system.

$$dW = -PdV \quad (7)$$

$$W = - \int_{V_i}^{V_f} PdV \quad (8)$$

2.1 Cycles

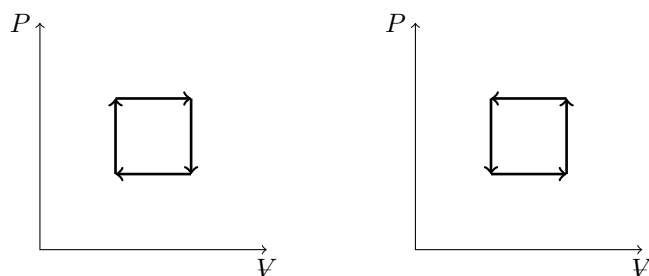


Figure 1: Left is Heat engine cycle. Right is Refrigerator cycle.

Efficiency is given by benefit divided by cost. Maximum efficiency is only found in the Caront Cycle.

$$Eff_{max} = 1 - \frac{T_C}{T_H} \quad (9)$$

3 Gas Expansion

Isothermal Temperature is constant $T = \text{const.}$

Adiabatic No heat is transferred $Q = 0$, $PV^{\frac{2+f}{f}} = \text{const.}$, $\Delta S = 0$.

$$C_V = \frac{\partial U}{\partial T} \quad C_P = \frac{\partial U}{\partial T} + P \left(\frac{\partial V}{\partial T} \right)_P \quad (10)$$

$$C_V = \frac{1}{2}fNK \quad C_P = Nk \left(\frac{f+2}{2} \right) \quad \text{Only for ideal gasses} \quad (11)$$

4 Multiplicity

Einstein Solid

$$\Omega = \frac{(N+q-1)!}{q!(N-1)!} \approx \left(\frac{eq}{N} \right)^N$$

Two Level System

$$\Omega = \frac{N!}{N_{\uparrow}!N_{\downarrow}!} = \frac{N!}{N_{\uparrow}!(N-N_{\uparrow})!}$$

Ideal Gas

$$\Omega = \frac{V^N}{N!h^{3N}} \left[\frac{2\pi^{\frac{3N}{2}}}{\left(\frac{3N}{2}-1\right)!} \cdot (2mU)^{\frac{3N-1}{2}} \right]$$

5 Entropy and Heat

Heat Capacities

$$C_V \equiv \left(\frac{\partial U}{\partial T} \right)_{N,V} \quad (12)$$

$$(13)$$

For an Einstein solid with $q \gg N$

$$C_V = \frac{\partial}{\partial T} (NkT) = Nk \quad (14)$$

For a monatomic ideal gas

$$C_V = \frac{\partial}{\partial T} \left(\frac{3}{2}NkT \right) = \frac{3}{2}Nk \quad (15)$$

Measuring Entropies

$$dS = \frac{dU}{T} = \frac{Q}{T} \quad (16)$$

$$dS = \frac{C_V dT}{T} \quad (17)$$

$$\Delta S = S_f - S_i = \int_{T_i}^{T_f} \frac{C_V}{T} dT \quad (18)$$

$$C_V \rightarrow 0 \quad \text{as} \quad T \rightarrow 0 \quad (19)$$

6 Thermo-Dynamic Potentials

$$S = S(N, V, U) \quad (20)$$

$$dS = \frac{\partial S}{\partial N} dN + \frac{\partial S}{\partial V} dV + \frac{\partial S}{\partial U} dU \quad (21)$$

$$= -\frac{\mu}{T} dN + \frac{P}{T} dV + \frac{1}{T} dU \quad (22)$$

$$dU = T dS + \mu dN \quad (23)$$

$$\frac{\partial U}{\partial S} = T \quad \frac{\partial U}{\partial N} = \mu \quad \frac{\partial U}{\partial V} = -P \quad (24)$$

Enthalpy $H \equiv U + PV \quad T = 0$

Helmholts Free Energy $F \equiv U - TS \quad P = 0$

Gibbs Free Energy $G \equiv U - TS + PV$

7 Boltzmann Statistics

7.1 Partition Function

$$Z \equiv \sum_s e^{-\frac{E(s)}{kT}} \quad (25)$$

Where s are the possible states of a single particle. Z cannot be measured but it contains all the information relevant to a system.

7.2 Boltzmann Distribution

$$P(s) = \frac{e^{-\frac{E(s)}{kT}}}{Z} \quad (26)$$

This gives the probability of a particle being in any given state s . Using this and the partition function, anything about a system can be derived.