# Complex Variables (Exam #1)

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# Contents

# 1 Complex Plane

## 1.1 How do we represent complex number geometrically?

Complex numbers are represented as two dimensional coordinates in a plane, with the $x$ axis as the real part of the complex number and the $y$ is the imaginary part of the complex number.

## 1.2 What is meant under the following terms?

$Re(z)$ This represents only the real portion of the complex number. For example if $z = x+iy$ then $Re(z) = x$.

$Im(z)$ This represents only the imaginary portion of the complex number. For example if $z = x + iy$ then $Im(z) = y$.

$|z|$ This is the **modulus** of $z$, or the length. It is calculated as follows: if $z = x + iy$, then $|z| = \sqrt{x^2 + y^2}$.

$\bar{z}$ This is the **conjugate** of $z$. It is found by flipping the sign of the imaginary part of $z$. So if $z = x + iy$, then $\bar{z} = x - iy$.

$\arg(z)$ This is the **argument** of $z$. It is the set of all angles that represent the position of the complex variable. Since every multiple of $2\pi$ is equal, then this is the set of all angles offset by some multiple of $2\pi$.

$\mathrm{Arg}(z)$ This is the **principle value** of the argument of $z$. It is a single value in the range $-\pi$ to $\pi$ which represents the angle that the complex number is at in polar coordinates.

## 1.3 Properties of modulus and conjugate

# 2 Basic Elementary Functions

# 3 Visualization of Functions

# 4 Riemann surfaces and branches of multivalued functions

# 5 Functions of complex variable defined through series

# 6 Holomorphic functions and Cauchy-Riemann equations