Math 215 – Fall 2017

Theory Homework 3 – Assigned September 25th, due October 1st

Note: Remember that you must show your work to get full credit for a problem.

1. Let r be a rational number and n be an irrational number. Prove that $r + n$ is a irrational number.

2. Using a counting argument prove that

$$\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}.$$

   Please remember the three parts of writing up a counting argument: (a) a clear statement of the counting task being undertaken, (b) a counting argument that obtains the expression on the left hand side of the equation, and (c) a counting argument that obtains the expression on the right hand side of the equation.

3. We define a positive integer p to be **prime** if

$$\forall a \in \mathbb{Z}^+ \forall b \in \mathbb{Z}^+ \left(p \mid ab \longrightarrow p \mid a \text{ or } p \mid b\right).$$

   We define a positive integer p to be **irreducible** if

$$\forall a \in \mathbb{Z}^+ \, \forall b \in \mathbb{Z}^+ \left(p = ab \longrightarrow a = 1 \text{ or } b = 1\right).$$

   Prove that $\forall p \in \mathbb{Z}^+$, if p is prime, then it is irreducible.