This handout provides a complete listing of Number Theory Tools up to and including Section 2.1 of the text. Note that the tools in the text are listed below, but some other tools have been added. You are welcome to use any of the tools below as you complete problems. The added tools are included to clarify the meaning of even and odd, establish a key property of an odd number, and throw in a few more useful properties of the "divides" relation. An asterisk on a Definition, Theorem or Corollary indicates that it does not appear in the text.

Informal Definition*. *The integers, denoted as* $\mathbb{Z}$ *consist of the numbers*

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

Definition 2.1.2. *Let* n *and* k *be two integers. If there is an integer* q *such that*

$$n = k \cdot q$$

then we say that k *divides* n *(or alternatively that* n *is divisible by* k*), and we write* $k|n$*.*

Definition 2.1.5*. *Let* n *be an integer. If there is some integer* q *such that* $n = 2q$*, then we say that* n *is* even*. Otherwise we say that* n *is* odd*.*

Theorem 2.1.1, corrected to be Theorem 2.2.1 (The Division Algorithm). *Let* m *and* n *be integers with* $m > 0$*. Then there is a unique pair of integers* q *and* r *such that* $n = mq + r$*, where* $0 \leq r < m$*.*

Corollary 2.1.6*. *An integer* n *is odd if and only if there is an integer* q *such that* $n = 2q + 1$*.*

Theorem 2.1.3. *If* a*,* b *and* c *are integers such that* $a|b$ *and* $b|c$ *then* $a|c$*.*

Theorem 2.1.7*. *If* a*,* b *and* c *are integers such that* $a|c$ *then* $a|(bc)$*.*

Theorem 2.1.4. *If* a*,* b *and* c *are integers such that* $a|b$ *and* $a|c$ *then* $a|(b+c)$*.*

Theorem 2.1.8*. *If* a*,* b*,* c*,* m *and* n *are integers such that* $a|b$ *and* $a|c$ *then* $a|(mb+nc)$*.*

Theorem 2.1.3. But really a corollary to Theorem 2.1.4. *If* a*,* b *and* c *are integers such that* $a|b$ *and* $a|c$ *then* $a|(b-c)$*.*