E&M Midterm 1

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Electric Field

Gauss's Law: $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$
Point Charge: $\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$

Work

$$W = \int_{\vec{r}}^{\infty} Q\vec{E}\left(\vec{r'}\right)\vec{dl}$$

Potential

$$V = -\int_{\infty}^{\vec{r}} \vec{E}\left(\vec{r'}\right) \cdot \vec{dl}$$

Point Charge: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$
Charge Distribution: $V = \frac{1}{4\pi\epsilon_0}\int_{\Omega}\frac{\rho\left(\vec{r'}\right)}{\mathcal{R}}d\tau'$
Potential Difference: $V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E}\left(\vec{r'}\right)\vec{dl}$

$$\vec{E} = \nabla V$$

Conductors

A conductor is a material with an "infinite" number of free charges that can move freely without resistance.

(1) Electric field inside a conductor is zero. $\vec{E} = 0$
(2) $\rho = 0$ everywhere inside the conductor.
(3) Any excess charge must be on surface of conductor.
(4) The entire conductor has the same potential.

(5) Electric fields from excess charges are always perpendicular to the surface.

Laplace's Equation: $\vec{\nabla}^2 V \equiv 0$

The potential in a space Ω is uniquely determined if:

(1) The charge density of $\rho\left(\vec{r}\right)$ is specified.
(2) The value of V is specified on the boundaries.

Differential Equations

$$\frac{\partial^2 \mathbf{X}}{\partial x^2} = k^2 \mathbf{X} \implies \mathbf{X} = Ae^{kx} + Be^{-kx}$$
$$\frac{\partial^2 \mathbf{X}}{\partial x^2} = -k^2 \mathbf{X} \implies \mathbf{X} = Asin(kx) + Bcos(ky)$$

Apply all boundary conditions, then solve for coefficients through dot product.