Math 215 – Fall 2017

Practice Homework 11 – Assigned October 16th, due October 19th

Note: Remember that you must show your work to get full credit for a problem.

1. For each of the following statements, say whether it is true or false.

   (a) $\emptyset \in \emptyset$.

   (b) $\emptyset \subseteq \emptyset$.

   (c) $\emptyset = \{\emptyset\}$.

   (d) $\{\emptyset\} \in \emptyset$.

   (e) $\emptyset \in \{\emptyset\}$.

   (f) $\emptyset \subseteq \{\{\emptyset\}\}$.

   (g) $\emptyset \in \{\{\emptyset\}\}$.

   (h) $\{\emptyset\} \subseteq \{\{\emptyset\}\}$.

2. Find the result of the following operations.

   (a) $\mathrm{Card}\,(\emptyset)$

   (b) $\mathrm{Card}\,(\{\emptyset\})$

   (c) $\mathrm{Card}\,(\{\{\emptyset\}\})$

   (d) $\mathrm{Card}\,(\{1, 2, 3\} \cup \{3, 4, 5\})$

   (e) $\mathrm{Card}\,(\{\{1, 2, 3\}\})$

   (f) $\{a, \{b\}, c\} \cup \{\{a\}, \{b\}, c\}$

3. Describe the following sets in the form $X = \{x \mid \text{ statement about } x\}$. Feel free to use the logical symbols you have learned.

   (a) $A = \{7, 14, 21, 28, 35, \cdots\}$.

   (b) $B = \{11, 21, 31, 41, 51, \cdots\}$.

   (c) $C = \{1, 3, 9, 27, 81, \cdots\}$.

4. Describe the following sets as a list, the way the sets are given in the previous exercise.

   (a) $A = \{x \in \mathbb{N} \mid x \text{ is even and prime}\}$.

   (b) $B = \{x \in \mathbb{Q} \mid 5x \in \mathbb{Z} \text{ and } 0 \le x < 2\}$.

   (c) $C = \{x \in \mathbb{Z} \mid \sqrt{x} \in \mathbb{Z} \text{ and } x \le 30\}$.