Proof. $j = j_1 = j_2$, and $k_1 k_2$. To show orthogonality, we must show that the value of the inner product of the two different functions is zero.

$$\left\langle 2^{\frac{j}{2}}\Psi\left(2^j x - k_1\right), 2^{\frac{j}{2}}\Psi\left(2^j x - k_2\right)\right\rangle \tag{1}$$

$$= 2^j \int_{-\infty}^{\infty} \Psi\left(2^j x - k_1\right)\Psi\left(2^j x - k_2\right) dx \tag{2}$$

Similarly to proof **??**, we want to change the bounds of integration. We can use much of the work from proof **??**.

Case 1. $k_1 > k_2$. *We can then write an expression for* k_1 *in terms of* k_2. $k_1 = \alpha + k_2$, *where* $\alpha \geq 1$. *Taking the lower bound for* Ψ_1, *we can say*

$$\frac{1}{2^j}(\alpha + k_2) \tag{3}$$

Once again we are able to ignore the $\frac{1}{2^j}$, *Now we can notice*

$$1 + k_2 \leq \alpha + k_2 \tag{4}$$

Thus for any values of j, *and* k_1, *and* k_2, *then there will be no overlap between the functions, and so the integral will always be zero.*

Case 2. $k_2 > k_1$. *Without loss of generality we can apply the same process as in case 1, because the multiplication of two functions is commutative.*

Thus we are able to conclude that for any case where $k_1 \neq k_2$ and $j_1 = j_2$ that the two wavelets will not overlap, and thus they will be orthogonal. $\square$