Math 215 – Fall 2017

Theory Homework # 2 – Assigned September 18th, due September 22nd

Note: Remember that you must show your work to get full credit for a problem.

1. (Section 2.1 Exercise 2)

   For the following problems you will be given a set of hypotheses and a conclusion. You must create a deduction of the conclusion from the hypotheses using **ONLY** the hypotheses, tautologies, and modus ponens. When you use modus ponens, you must specify which lines you are using.

   Make sure to label every line of your deduction with how you obtained that line.

   You do not need to prove that the tautologies you use are tautologies, but you will lose points if they are not.

   (a) Consider the set of hypotheses

   $$\{\neg S \vee R, R \to P, S\},$$

   Give a deduction which concludes with P.

   (b) Consider the set of hypothesis

   $$\{A \to B, B \to C\},$$

   Give a deduction which concludes with $A \to C$.

2. For the following problems you will be given a set of hypotheses and a conclusion. You must create a deduction of the conclusion from the hypotheses using the hypotheses, tautologies, and the logical rules given below. When you use a logical rule you must state which one you are using and which lines it invokes to make its conclusion.

   Make sure to label every line of your deduction with how you obtained that line.

   You do not need to prove that the tautologies you use are tautologies, but you will lose points if they are not.

   The laws that you may use are

$\dfrac{P \to Q \quad P}{Q}$	Modus Ponens	$\dfrac{P \to Q \quad \neg Q}{\neg P}$	Modus Tollens
$\dfrac{P \quad Q}{P \wedge Q}$	Adjunction	$\dfrac{P \to Q \quad Q \to R}{P \to R}$	Hypothetical Syllogism
$\dfrac{P \wedge Q}{Q}$	Simplification	$\dfrac{P \wedge Q}{P}$	Simplification
$\dfrac{P}{P \vee Q}$	Addition	$\dfrac{Q}{P \vee Q}$	Addition
$\dfrac{P \vee Q \quad \neg P}{Q}$	Modus Tollendo Ponens	$\dfrac{P \vee Q \quad \neg Q}{P}$	Modus Tollendo Ponens

(a) Consider the set of hypotheses

$$\{(C \vee E) \to \neg M, R \to M, C\},$$

Give a deduction which concludes with $\neg R$.

(b) Consider the set of hypotheses

$$\{P \wedge Q, (P \vee Q) \to R\},$$

Give a deduction which concludes with R.

(c) Consider the set of hypotheses

$$\{\neg J \vee S, \neg L \to \neg S, J \wedge \neg L\},$$

Give a deduction which concludes with F.

(d) With the same set of hypothesis from problem 2c give a deduction which concludes with $\neg F$.

3. (Optional)

In problems 2c and 2d you started with the same set of hypotheses and were able to deduce both F and $\neg F$.

What does this tell you about the hypotheses?