Math 215 – Fall 2017

Practice Homework 16 – Assigned November 9th, due November 13th

Note: Remember that you must show your work to get full credit for a problem.

1. Consider the following false proof that $\text{Card}(\mathbb{N}) \neq \text{Card}(\mathbb{Q})$.

   Proof. [**FALSE**]

   we proceed by contradiction. Suppose there is a bijection $f : \mathbb{N} \to \mathbb{Q}$.

   Then we create a number z as follows. We will define $z = .z_1 z_2 z_3 \cdots$ where z_n is a single digit number and represents the n^{th} number after the decimal point.

   We define z_n to be the smallest single digit number that is not equal the the n^{th} decimal place of $f(n)$.

   Then by construction z is not in the image of f and thus f is not a bijection.

   Thus no bijection exists from $\mathbb{N}$ to $\mathbb{Q}$.

   Thus $\text{Card}(\mathbb{N}) \neq \text{Card}(\mathbb{Q})$.

   □

   State the logical error(s) in the prove and explain why the proof fails.

2. Informally show that $\mathbb{Z}$ and $\mathbb{Z} \times \mathbb{Z}$ have the same cardinaity.

3. A local school offers basketball, baseball, and football as sports activities.

   You know that 12 students play basketball (and possible other activities), that 8 students play baseball (and possible other activities), 13 students play football (and possible other activities), 5 students play basketball and football (and possible other activities), 4 students play basketball and baseball (and possible other activities), 6 students play baseball and football (and possible other activities), and 1 students plays basketball, football, and baseball.

   How many students play in at least one sport?

4. A local school offers basketball, baseball, and football as sports activities.

   You know that 15 students play basketball (and possible other activities), that 12 students play baseball (and possible other activities), 14 students play football (and possible other activities), 8 students play basketball and football (and possible other activities), 7 students play basketball and baseball (and possible other activities), 5 students play baseball and football (and possible other activities).

   Additionally who know that there are 25 student who play at least one sport, how many student play in all three?