Math 215
Practice 2nd Exam

September 22, 2017

Name (in block capital letters):

Instructor (tick one box): ☐ **Section 1: L. Stanhope (12:50)**

☐ **Section 2: E. Sullivan (12:50)**

Instructions: You are taking this exam under the honor system. Your signature at the bottom of this page is your promise to abide by the conditions described below. Breaking this promise violates Lewis and Clark's academic integrity policy.

1. To answer these questions, use only the knowledge in your head, and a calculator.
2. You have 90 minutes in which to take the exam. This should be one continuous time period, and not two or more periods separated by breaks. (Any breaks you take are included in the 90 minutes.) You may take the exam anywhere you wish. Start timing when you begin the exam.
3. Do not talk to any other student, whether enrolled in Math 215 or not, whether they have already taken the exam or not, about this exam until after 5 PM Friday, September 22nd. That includes: discussing specific questions, its difficulty, how long you worked on it, as well as any other exam-related topic you might imagine.
4. Be sure to carefully read the directions for each problem. Please show your work – correct answers to problems with no justification of where they come from will earn little credit.
5. Finally, do your best to think logically and clearly, and then trust your judgment. There are no "trick" questions here.

Problem	Score
1	
2	
3	
Total	

1. Please prove the following claim:

   "If n is an integer greater than zero and $4^n - 1$ is prime, then n is odd."

2. Please prove $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \binom{n+1}{2}^2$.

3. Let n be an integer. Prove that n^4 can be written as $5k$ or as $5k + 1$ for some integer k.

4. Prove that if $(a, b) = 1$ and $(a, c) = 1$, then $(a, bc) = 1$.

5. (10 points) Let

$$\begin{aligned} G &= \{n \in \mathbb{Z} : n = 2m \text{ for some } m \in \mathbb{Z}\} \\ H &= \{n \in \mathbb{Z} : n = 3k \text{ for some } k \in \mathbb{Z}\} \\ I &= \{n \in \mathbb{Z} : n^2 \text{ is odd}\} \\ J &= \{n \in \mathbb{Z} : 0 \leq n \leq 10\} \end{aligned}$$

   Find each of the following sets.

   (a) $G \cup I$

   (b) $G \cap I = \emptyset$

   (c) $G \cap H$

   (d) $J \setminus G$

   (e) $I \setminus H$

   (f) $J \cap (G \setminus H)$

6. (10 points) Find the greatest common divisor of $x^2 - 1$ and $x^3 - 4x^2 + 5x - 2$ using the Euclidean Algorithm.