

Algorithm Design

Contents

- Chapter 1
- A First Problem: Stable Matching
 - The Question
 - Formulating the Problem

Chapter 1

A First Problem: Stable Matching

The Question

The Stable Matching Problem originated in 1962 from Davis Gale and Lloyd Shapley. Gale and Shapley asked: Given a set of preferences among employers and applicants, can we assign applicants to employers so that for every employer E , and every applicant A who is not scheduled to work for E , at least one of the following two things in the case?

1. E prefers everyone of its accepted applicants to A ; or
2. A prefers her current situation over working for employer E .

If this hold, the outcome is stable: individual self-interest will prevent any applicant/employer deal from being made behind the scenes.

Formulating the Problem

A “bare-bones” version of the problem can be useful for a basic solution: each of n applicants applies to each of n companies, and each company wants to accept a *single* applicant. This preserves the fundamental issues of the original problem.

Gale and Shapley, observed that this problem can be viewed as devising a system that n men and n women can end up getting married, and everyone is seeking to be paired with exactly one individual of the opposite gender.

So consider a set $M = m_1, \dots, m_n$ of n men, and a set $W = w_1, \dots, w_n$ of n women. Let $M \times W$ denote the set of all possible ordered pairs of the form (m, w) , where $m \in M$ and $w \in W$. a *matching* S is a *set* of ordered pairs, each of $M \times W$, with the property that each member of M and each member of W appears in at most one pair in S . A *perfect matching* S' is a matching with the property that each member of M and each member of W appears in *exactly* one pair in S' .