Mathematic Formulation of the Double-Precision Orbit Determination Program

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Abstract

This documents the complete mathematical model for the Double-Precision Orbit Determination Program (DPODP), a third-generation program which has recently been completed at the Jet Propulsion Laboratory. The DPODP processes earth-based doppler, range, and angular observable of the spacecraft to determine values of the parameters that specify the spacecraft trajectory for lunar and planetary missions.

Contents

1 Introduction

This report documents the mathematical model for the Double-Precision Orbit DeteDetermination Program (DPODP). The DPODP will be used to determine values of the parameters that specify the spacecraft trajectory for lunar and planetaryplanetary missions; it will be used for both real-time and post-flight reduction of tracking data. The DPODP differentally corrects *a priori* estimaes of injection parameters, physical constants, maneuver parameters, and station locations to minimize the sum of weighted squares of residual errors betbetween observed and computed quantities.

The basic limitations on the *accuracy of computed observables* are the inaccuracies in the troposphere and ionosphere corrections. Before these corrections are added, the computed values of the doppler and range observales have accuracies of $10^{-5} m/s$ and $0.1m$, respectivly. The parameters whose values may be estimated by the DPODP are:

1. Injection parameters. Rectangular components of the spacecraft position and velocity vectors at the injection epoch.

2. Reference parameters. Parameters that affect the relative position and velocity of the sun, palents, and the moon:

   A_E The number of kilometers per astronomical unit (AU). This parameter converts the precomputed eliocentric ephemrides of eight planeplanets and the earth-moon barycenter from astronomical units to kilometers.

   R_E Scaling factor for lunar ephemeris, km/fictitious earth radius. This factor converts the precomputed geopcentric lunar ephemeris from fictitious earth radii to kilometers.

   E Osculation orbital elements for the precomputed ephemeris of a planet, earth-moon barycenter, or the moon, The estimated correction ΔE is used to differentially correct position and velocity obtobtained from the precomputed ephemeris.

   μ_E, μ_M Gravitational constants for the earth and moon, km^3/s^2. These parameters affect the location of the earth-moon barycenter.

3. Gravitational constats. The constant μ_i is the gravitational constant for body i, such as the sun, a planet, or the moon. (Note thathat μ_E and μ_M are also listed under reference parameterparameters.)

4. Harmonic coefficient. The harmonic coefficients j_n, C_{nm}, S_{nm}, along with the gravitational constant μ, describe the gravitational field of a planet or moon.

5. Parameters affecting the acceleration of the spacecraft due to solar radiation pressure.

6. Coefficient of quadratic for small acceleration acting along each spacecraft axis. These quadratics are used to represent gas leaks and small forces arising from operation of the attitude control system.

7. Parameters affecting spacecraft motor burns.

8. Parameters affecting the transformation from universal time to ephemeris time.

9. Coefficient of quadratics which represent the departure of atomic time at each tracking station from broadcast UTC time.

10. Station parameters.

    (a) Radius

    (b) Latitude

    (c) Longitude

    or

    (a) Distance from spin axis

    (b) Height above equator

    (c) Longitude for each tracking station and a land spacecraft on a plaplanet or the moon.

    For a tracking ship:

    (a) Spherical coordinates at an epoch

    (b) velocity

    (c) azimuth

11. Speed of light. And adopted constant which defines the light-second as the basic length unit; it is not normally included in the solution vector.

12. Constant bias for range observables.

13. Spacecraft transmitter frequency for one-way doppler.

14. Biases affecting observed angles.

15. Relativity parameter γ. This parameter will be added to the program. It is equal to $(1+\omega)/(2+\omega)$ where ω is the coupling constant of the scalar field, a free parameter of the Brans-Dike theory of gravitation.

Given the *a priori* estimate of the parameter vector q, the program integrates the spacecraft acceleration using the second-sum numerical integration method to give position and velocity at any desired time. Using the spacecraft ephemeris along with the precomputed ephemerides for the other bodies within the solar system, and the parameter vector q, the program computes values for each observed quantity (normally doppler, range, or angles) and forms the *observed* minus *computed* (O-C) residuals.

In addition to integrating the acceleration of the spacecraft to obtaini the spacecraft ephemeris, the program integrates the partial derivative fo the spaspacecraft acceleration with respect to (wrt) the parameter vector q using the second-sum numerical integration procedure to give the partial derivderivative of the spacecraft state vector X (position and velocity components) wrt the parameter vector q, $\partial X/\partial q$. Using $\partial X/\partial q$, thee program computes the partial derivative of each computed observalbe quantity z wrt q, $\partial z/\partial q$. Given the O $-$ C residuals, $\partial z/\partial q$, and the weights applied to each residual along with the *a priori* parameter vector andand its convariance matrix, the program computes the differential correction Δq to the parameter vector. Starting with $q + \Delta q$, the program computess a new spacecraft ephemeris, residuals, and the part-partial derivatives and obtains a second differential correction Δq. This process is repeated until convergence is obtained and the sum of weighted squares of residual errors between observed and computed quantities is minimized.

The DPODP formulation was heavily influenced by the general theory of relativity. 2 gives the equations from general relativity, which are the basis of the DPODP formulation, and also the principle relativistic equaitons containd in the formulation.

The time transformations used throught the program and formulation for computing thre relative position, velocity, acceleration, and jerk for any two celestial bodies are described in **??**.

2 Relativistic Terms of DPODP Formulation

In the general theory of relativity is basically a geometrical theory. The geometry is embodied in the components of the symmetrical metrict tensor g_{pq}:

$$g_{pq} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \quad (1)$$

The subscripts 1, 2, 3, and 4 correspond to the space-time coordinates x^1, x^2, x^3, and x^4, which are associated with a particular space-time frame of reference.