Calibration of Discrete Random Walk (DRW) Model via G.I Taylor's Dispersion Theory

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Discrete Random Walk DRW Model

- **DRW model** predicts particle trajectories and their relative velocities in a turbulent flow, in combination with a RANS-based simulation of the background flow.
- Particle path is predicted via integration of the force balance equation :

$$\frac{du_p}{dt} = F_D(u-u_p) + \frac{g_x(\rho_p-\rho)}{\rho_p} + F_x$$

$$u = \bar{u} + u'(t) \text{ where } u' = \zeta\sqrt{\overline{u'^2}} = \sqrt{2k/3}$$

where " ζ " is a zero mean, unit variance and normally distributed random number.

- Value of the turbulent fluctuations ($\boldsymbol{u'}$) and/or value of the random variable $\boldsymbol{\zeta}$ is a function of the Eddy-Particle interaction time scale $\boldsymbol{T}$ defined as:

$$T = min(T_L, t_{cross})$$

Eddy Life Time Scale
(Particle interacts with eddy till it diffuses out)

$$T_L \approx C_L \frac{k}{\epsilon}$$

Crossing Time Scale
(Particle cross an eddy and starts to interact with another eddy)

$$t_{cross} = -\tau ln[1-(\frac{L_e}{\tau|u-u_p|})]$$

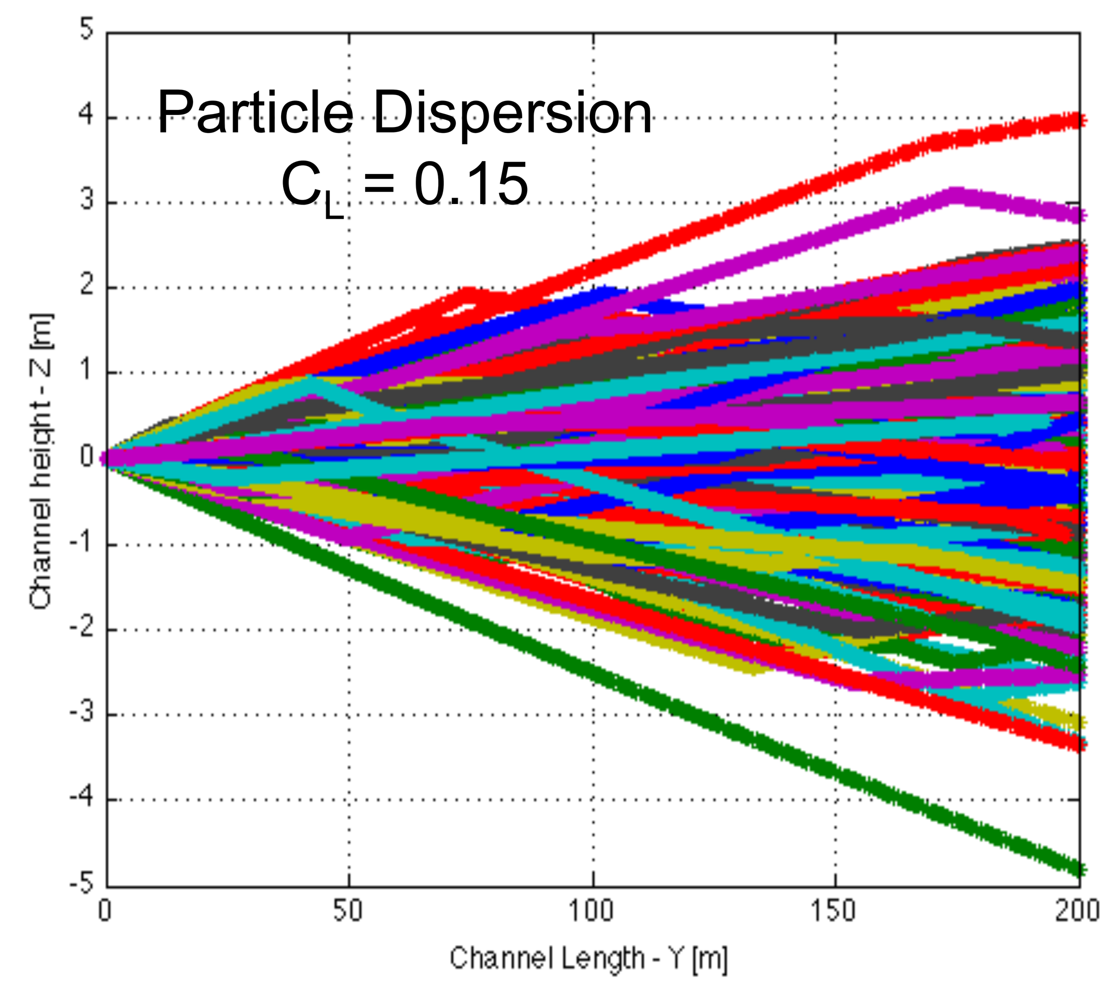

Fig. 1: Dispersion of 250 particle realizations in a water channel (V=2 m/s) with the default value of C_L (C_L=0.15). Dispersion is being overestimated.

The time scale coefficient, $\boldsymbol{C_L}$, is dependent on the nature of the turbulence in the flow field.

C_L Appropriate Value → Accurate time scale $\boldsymbol{T}$

DRW Calibration via G.I Taylor Theory

- **G.I Taylor dispersion theory** predicts particle dispersion based on the characteristics of a homogeneous, isotropic turbulent flow:

$$\sqrt{[X^2]} = \sqrt{(2IT[u^2])}$$

$\sqrt{[X^2]}$: RMS of particle position in one direction

I : The turbulent time scale, defined based on velocity correlation coefficient.

T : Particle residence time

$\sqrt{[u^2]}$: RMS of particle velocity

Steps for C_L calibration:

I. Compute the average particle dispersion for different $\boldsymbol{C_L}$ values using the DRW model

II. Estimate particle dispersion using G.I Taylor theory.

III. Match results of steps **I** and **II** to calibrate the value of $\boldsymbol{C_L}$ for the desired turbulent flow .

Results & Application

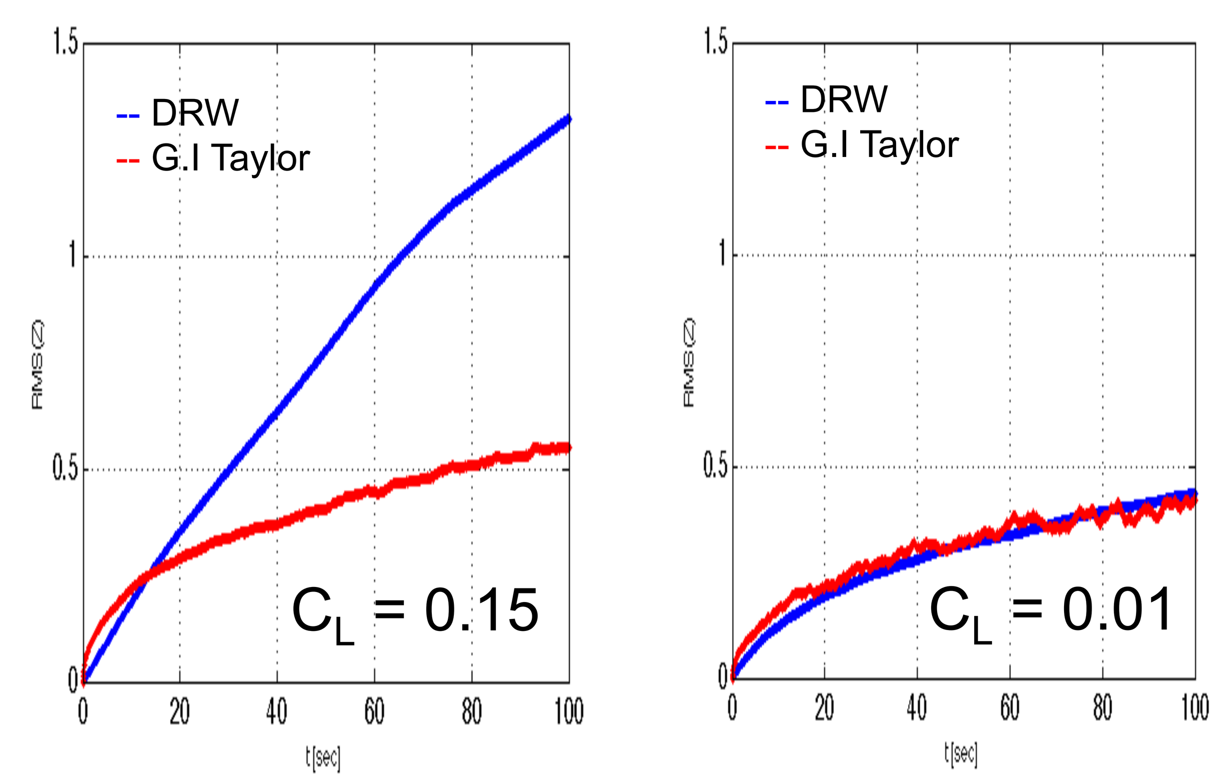

Fig. 2: Comparison between modeled dispersion via DRW model (blue) and G.I Taylor theory (red) with different values of C_L.

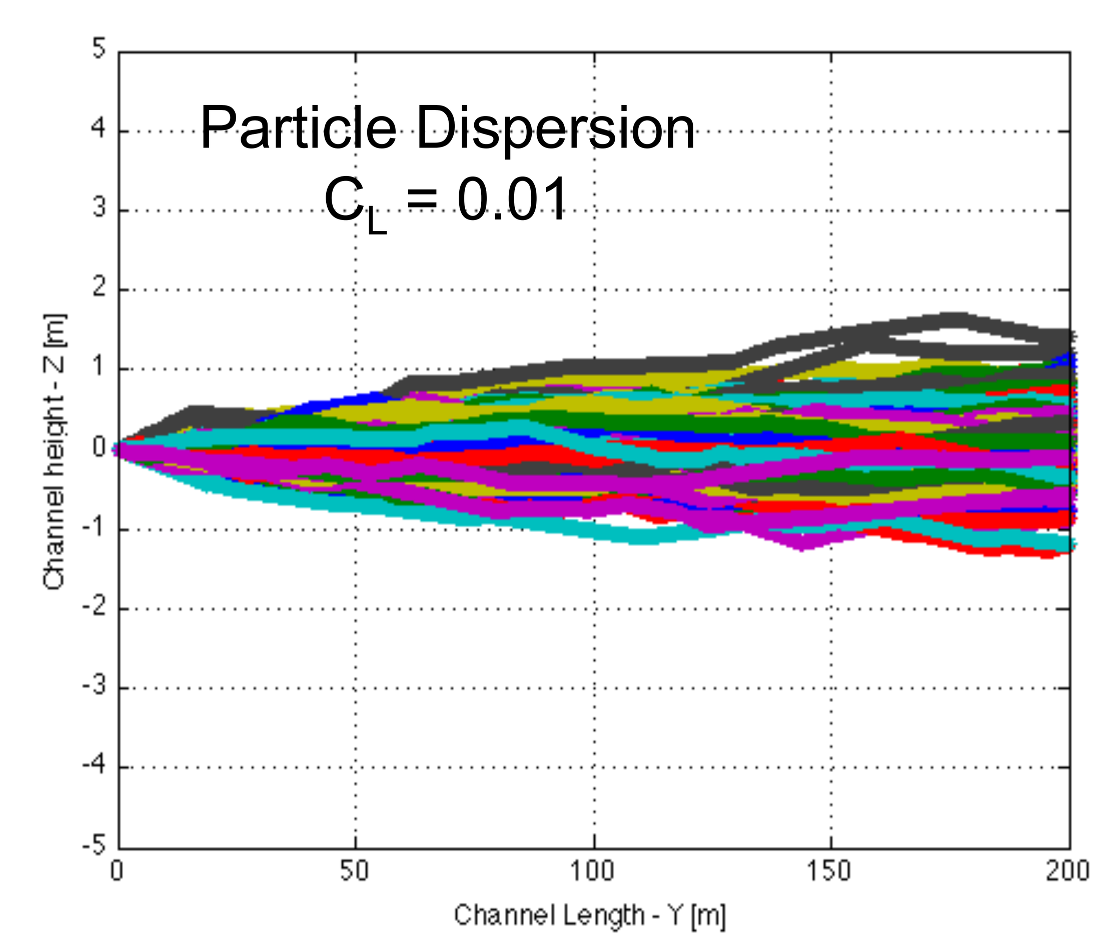

Fig. 3: Dispersion of 250 particle realizations for in a water channel with calibrated C_L value of 0.01 (C_L=0.01). Dispersion is modeled more reasonable in compare to results in fig. 1 (C_L=0.15) .

- An applications of this calibrated model is investigation of the a tidal turbine wake interaction with suspended sedimentations in a tidal channel.

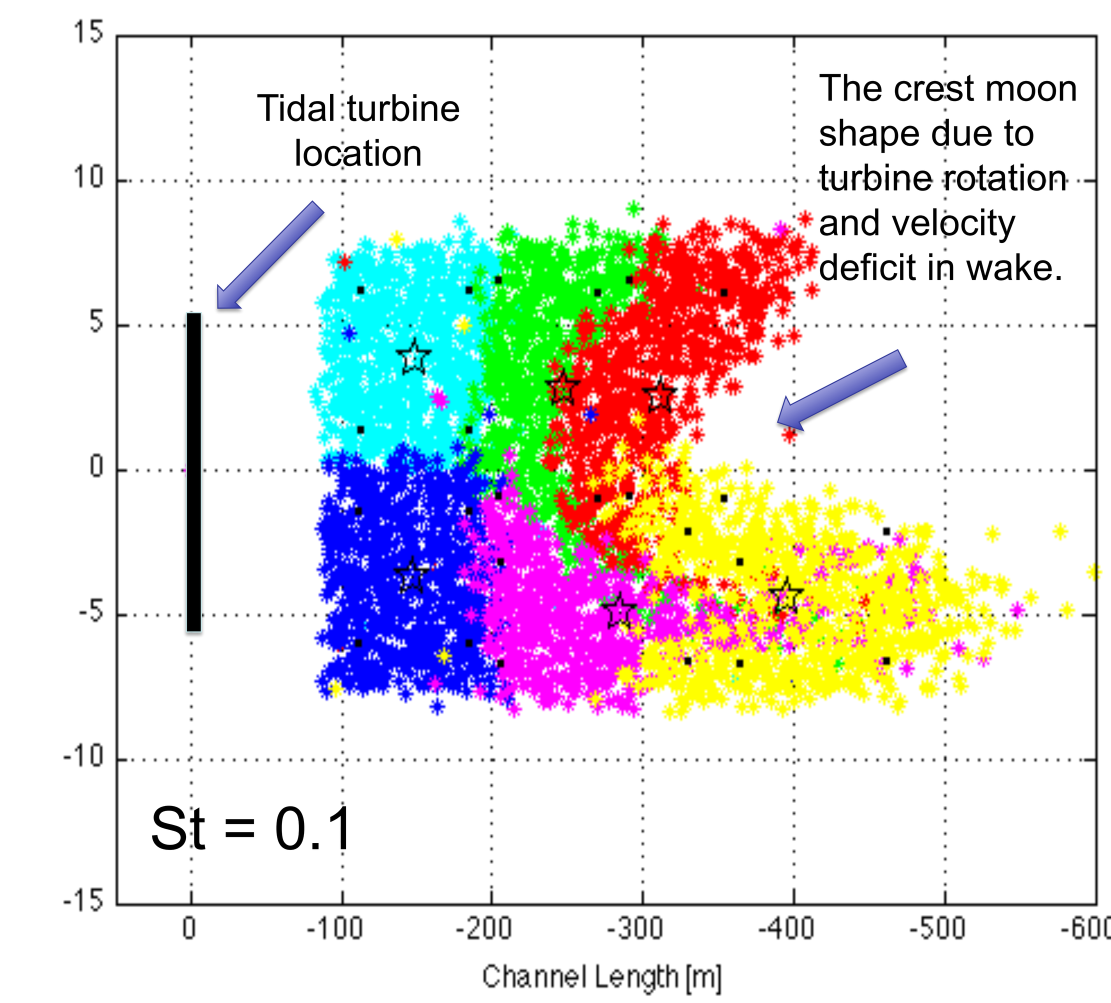

Fig. 4: Sedimentation of particles (St=0.1) after interaction with a tidal turbine turbulent wake. Modeled with the calibrated DRW model (C_L = 0.01).

Conclusions

- In DRW model, particle transport depends critically in Eddy-Particle time scale, $\boldsymbol{T}$. The default value of time scale coefficient constant ($\boldsymbol{C_L}$ = **0.15,** ANSYS FLUENT) overestimates dispersion.
- The Eddy-Particle time scale needs to be calibrated based on characteristics of the specific turbulent flow modeled.
- This work presented a general methodology to calibrated the DRW model via G.I Taylor dispersion theory.
- The calibrated model is applied toward an environmental fluid mechanics topic: sedimentation of particles in the wake of a tidal turbine.