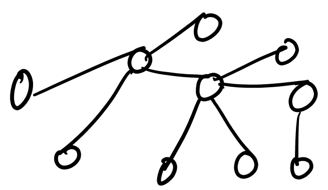
Epidemics on N/wks.



1927

To model spread of diseases
spread of rumours
" News
Advts.

Disease

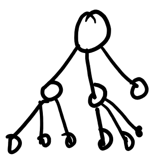
Simple model of contagion

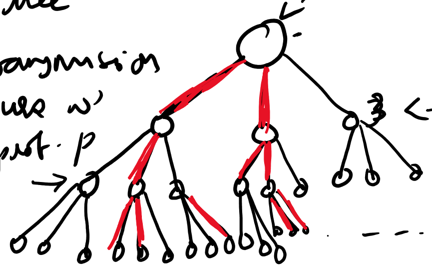
1st wave : One person infected. He enters the population.
Disease is transmitted to each person he meets w' probability p.
If he meets $\langle k \rangle$ people, then total prob. of infection $\rightarrow p.\langle k \rangle$

2nd wave : Each infected person from 1st wave meets $\langle k \rangle$ new people and independently transmits infec^n w' prob. p.

3rd wave :

Let's organize as a tree





On an average, every person infects more than 1 person else transmits may stop.

$\langle k \rangle$ ppl are met

$p.\langle k \rangle$ - average no. of secondary infections from 1 node

If p is very less or $\langle k \rangle$ is very less, then transmission will be very less.

$R_0 = p.\langle k \rangle$ - avg no. of new infected ppl/nodes on every step

On n^{th} step $R_0^n = (p.\langle k \rangle)^n$ → geometrical growth

On n^{th} step $\quad R_0 = (p \cdot \langle k \rangle)$ growth

$R_0 > 1$, avg grows geometrically as R_0^{n}

$R_0 < 1$, avg shrinks $\quad$ "

$n \to t$, geometric growth $\to$ exponential growth

if continuous time

$R_0 = 1$

Mathematical Epidemiology

Popula^n is divided into various classes/compartments

(S, I, R)

$S(t)$: Susceptible. Not yet infected w/ disease at time t.

$I(t)$: Infected, No. of individuals who have been infected. So these pple are capable of spreading disease

$R(t)$: Recovered, Can't infect again or transmit disease to others.

1) SI $\quad$ 2) SIS $\quad$ 3) SIR $\quad$ 4) SIRS . . .

SI model

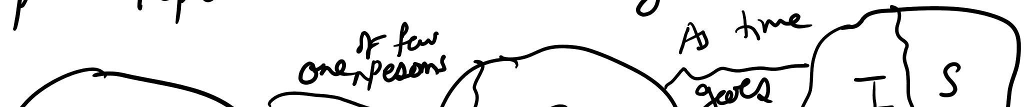
In this model, the popula^n is divided into 2 classes.

$$S \longrightarrow I$$

$I(t)$ – infected ppl in the popula^n at time t.

Assump^n : Population does not change.

Initially



Initially (S) $\xrightarrow[\text{gets infected}]{\text{one / few persons}}$ (I | S) $\xrightarrow{\text{goes}}$ (I | S) $\xrightarrow{\text{eventually}}$ (I)

all ppl are infected

If N is the total no. of pple in the popula^n.

At time t $\quad S(t) + I(t) = N$

At time 0 $\quad S_0 + \underbrace{I_0}_{\text{very small no}} = N$

Let $\beta \to$ transmission rate ; ie per unit time, how many are getting infected.

$i = \frac{I}{N} \to$ frac^n of infected pple

$s = \frac{S}{N} \to$ frac^n of susceptible pple

Model SI as ODE

Infection Equa^n

ppl w/ new infec^n during δt

$$I(t + \delta t) = I(t) + \beta \frac{S(t)}{N} I(t)\, \delta t$$

How / What is the rate of change of I. How fast it grows ??

$$\frac{\partial I(t)}{\partial t} = \beta \frac{S(t)}{N} \cdot I(t).$$

In terms of Fraction $\quad i(t) = \frac{I(t)}{N}\ ; \quad s(t) = \frac{S(t)}{N}$

$$\frac{\partial i(t)}{\partial t} = \beta\, s(t) \cdot i(t).$$

No. of $\cancel{s}$ $S(t) + I(t) = N$

$s(t) + i(t) \ldots \frac{S(t) + I(t)}{N} = \frac{N}{N} = 1$

No. of susceptible ppl → $S(t) + I(t) = N$

$$s(t) + i(t) = \frac{S(t)}{N} + \frac{I(t)}{N} = \frac{S(t)+I(t)}{N} = \frac{N}{N} = 1$$

Thus, $s(t) + i(t) = 1 \Rightarrow s(t) = 1 - i(t)$

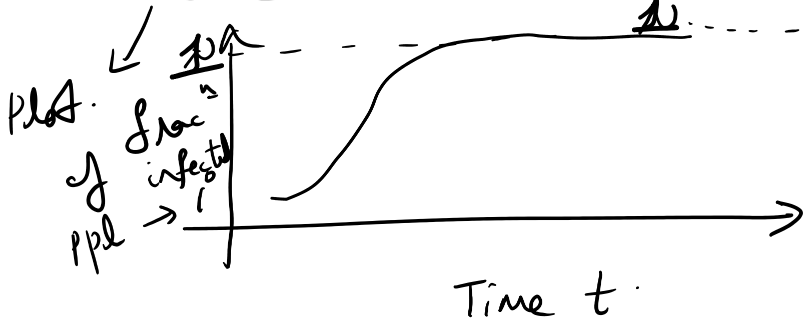
frac^n of susceptible ppl

$$i(0) = i(t=0) = i_0$$

$$\frac{di(t)}{dt} = \beta \cdot (1 - i(t)) \cdot i(t)$$

$$i(t) = \boxed{\frac{i_0}{i_0 + (1-i_0)e^{-\beta t}}}$$ ← logistic eq^n

Plot.



$\lim_{t \to \infty} I(t) = N$ and $\lim_{t \to \infty} s(t) = 0$

→ Eventually all are infected