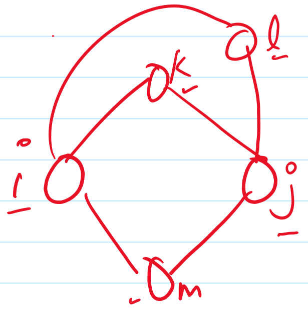
Similarity

10 April 2023 07:38



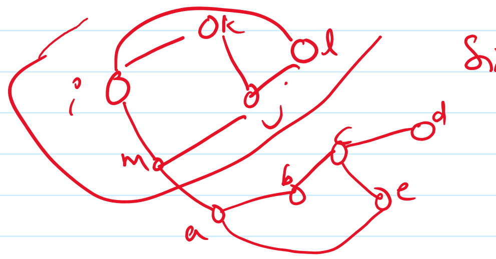
1. Structural Equivalence ✓
2. Automorphic "
3. Regular Equivalence ✓
4. Probabi/ Stochastic

Look at the nbrs. → Structural Eq.
for sim(i, j)

sim(i, j) = $\underline{n_{ij}}$ nbrs. common to i, j

→ $\text{sim}(i,j) = \dfrac{n_{ij}}{n}$

n ← no. of vertices



$\text{Sim}(i,j) = \dfrac{3}{10}$

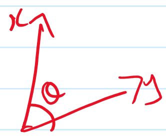
$= 0.3$

nbrs(i) = (k, m, l)

nbrs(j) = (m, l, k)

$n_{ij} = 3$

0 & 1



Cosine measure

$$\cos\theta = \frac{\vec{x}\cdot\vec{y}}{|x|\cdot|y|}$$

$$\begin{array}{c|cccccc} & i & j & k & l & m & \\ & a & & & b & c & \\ i & 0 & 0 & 0 & 1 & 1 \\ j & 0 & 0 & 1 & 1 & 1 \\ k & 1 & 1 & 0 & 0 & 0 \\ l & 1 & 1 & 0 & 0 & 0 \\ m & 1 & 1 & 0 & 0 & 0 \end{array}$$

$A = A^T$ in undirec

$$\text{sim}(i,j) = \frac{i \cdot j}{\sqrt{\ }} = \frac{\sum_k A_{ik} A_{kj}}{\sqrt{\sum A_{ik}^2 \sum A_{kj}^2}}$$

norm $(\sqrt{0^2+0^2+1^2+1^2+1^2}$

Salton's Cosine

$\sum A_{ik}^2 = 3 = k_i = \deg(i)$

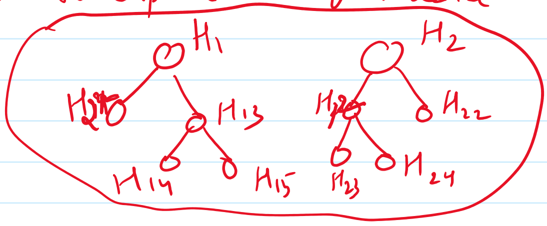
$\sum A_{kj}^2 = 3 \qquad \deg(j)$

$$\text{sim}(i,j) = \frac{\sum_k A_{ik} \cdot A_{kj}}{\sqrt{\sum A_{ik}^2 \sum A_{kj}^2}} = \frac{\ }{\sqrt{k_i k_j}} = \frac{n_{ij}}{\sqrt{\cancel{\deg(i).\deg}(j)}}$$

$$\sigma_{i,j} = \frac{n_{ij}}{\sqrt{\underbrace{\deg(i)}_{k_i}.\underbrace{\deg(j)}_{k_j}}}$$

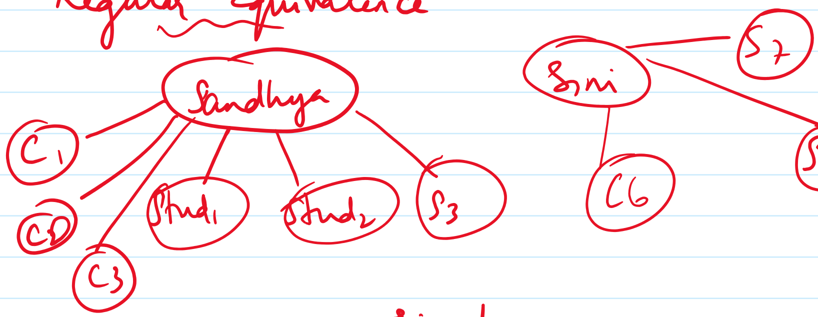
$$\sqrt{\underbrace{deg(i)}_{K_i} \cdot \underbrace{deg(j)}_{K_j}}$$

2. Automorphic Equivalence (i, j)



Isomorphic

3. Regular Equivalence

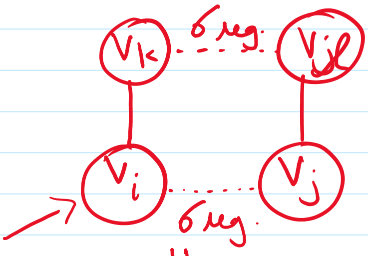


(Sandhya, Srini)

$\overrightarrow{sim}$(Sandhya, Sandhya)

should be ↑

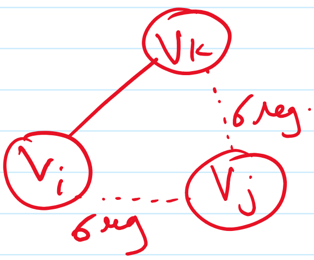
v_i & v_j are $\overset{\text{similar}}{\text{reg. equ.}}$ their nbrs. v_k, v_l are similar



$$\sigma_{ij} = \alpha \sum_{kl} A_{ik} A_{jl} \sigma_{kl}$$

In Matrix form

$$\sigma = \alpha A \sigma A$$



$$\sigma_{ij} = \alpha \sum_{k} A_{ik} \sigma_{kj}$$

$$\sigma = \alpha A \sigma$$

$\sigma_{ii} = 0$ if formula

To make σ_{ii} high value

$$\rightarrow \sigma = \alpha A \sigma + I$$

$$\sigma = (I - \alpha A)^{-1}$$

$$\frac{\sigma}{\sigma} = \alpha A + \frac{I}{\sigma}$$

$$I = \alpha A + \frac{I}{\sigma}$$

$$(I - \alpha A) = \frac{I}{\sigma} \Rightarrow \sigma = I (I - \alpha A)^{-1}$$

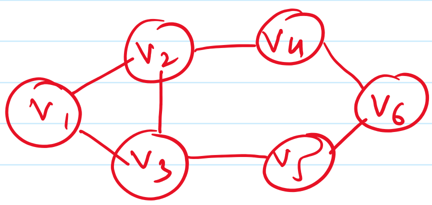
$$(I - \alpha A)^{-1}$$

$$(I-\alpha A) = \frac{I}{\sigma} \Rightarrow \sigma = \frac{I}{(I-\alpha A)} = (I-\alpha A)^{-1}$$

$$\sigma = (I-\alpha A)^{-1}$$

related to eigen value.

$\alpha < \frac{1}{\lambda_{max}}$ for making matrix invertible



$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ & \vdots & & & & \end{bmatrix}$$

$\lambda_{max} = 2.43$

$\alpha < 1/\lambda_{max} = 0.3$

$$(I - 0.3A)^{-1} = \begin{array}{c} \\ v1 \rightarrow \\ \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccccc} & 2 & & & & \end{array} \\ \begin{bmatrix} 1.43 & 0.73 & & & & \\ & 1.63 & 0.80 & & & \\ & & 1.63 & & & \\ & & & 1.31 & & \\ & & & & 1.31 & 0 \\ & & & & & 1.27 \end{bmatrix} \end{array}$$

Identity matrix

σ_{ii} to be high $\Rightarrow$ 0.80

$\sigma_{23} = 0.80$ as per reg. equivalence.

In gen.

$\sigma = \alpha A \sigma A$ (gen. formula)

Path measure.

→ longer paths have less wtg.
Shorter paths have high wtg.

→ Katz similarity