SIS Model

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$$\begin{tikzcd} S \arrow[r] & I \arrow[l, bend right] \end{tikzcd}$$

$$S \to I$$

$$S \to I \to R$$

Let $S \to$ no. of susceptible individuals

$I \to$ no. of infected individuals

If the total popula^n $= N$

$$S(t) + I(t) = N \quad \left(\because \text{Assump}^n : \text{No death, No birth} \right)$$

$$\text{(A)} \to \frac{dS}{dt} = -\beta \frac{SI}{N} + \alpha I \quad \xrightarrow{\quad} \text{recovery rate}$$

$$\text{(B)} \to \frac{dI}{dt} = \beta \frac{SI}{N} - \alpha I$$

$\underline{\beta}$: Rate of transmission of disease.

$$\beta S \frac{I}{N}$$

βSI : An avg. infected person makes contact sufficient to infect $\underline{\beta S}$ other people per unit time

$$\frac{S}{N}$$

αI : $\to$ α is the frac^n of infected ppl who recover and reenter S class per unit time

$$\frac{dS}{dt} + \frac{dI}{dt} = 0 \quad \left(\because S + I = N \text{ is constant} \right)$$

$$S + I = N$$

$$I = N - S$$

$$S = N - I$$

$$\frac{dI}{dt} = \beta SI - \alpha I$$

$$S = N - I$$

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \alpha I$$

$$= \beta \frac{I}{N}(N - I) - \alpha I$$

$$= \beta N I - \beta I^2 - \alpha I.$$

$$= I\left(\frac{\beta \cancel{N}}{\cancel{N}} - \alpha\right) - \beta I^2.$$

$$\frac{dI}{dt} = 0.$$

$$I(\beta N - \alpha) - \beta I^2 = 0.$$

$$I(\beta N - \alpha - \beta I) = 0.$$

$I = 0$; or $\beta N - \alpha - \beta I = 0$

$$\beta I = \beta N - \alpha$$

$$I = \underline{N - \alpha/\beta}$$

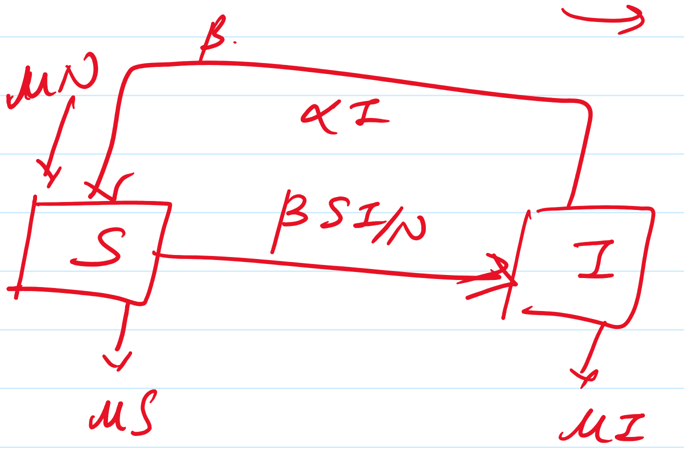
Reproductive no. in SIS $\rightarrow$ $\frac{\beta}{\alpha}$

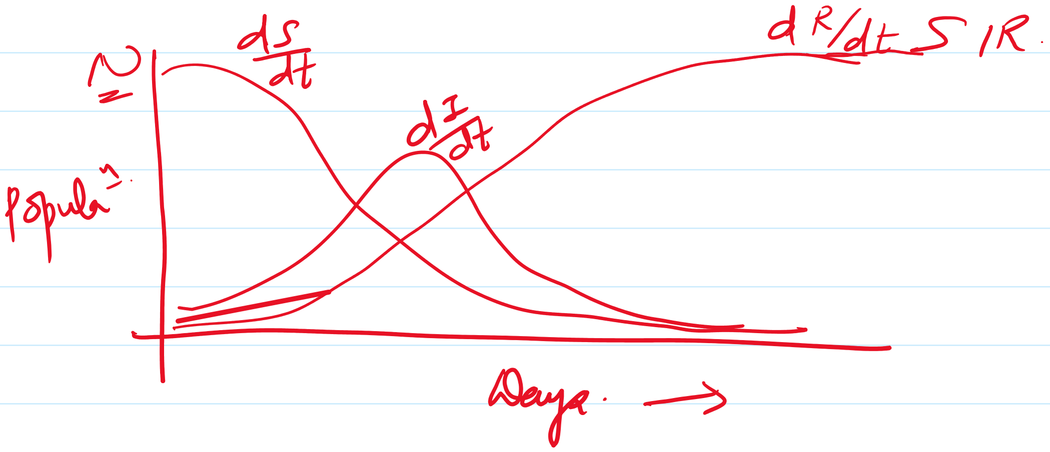
$R_0 < 1 \Rightarrow I = 0$ is stable $\rightarrow$ No more pple get infected.

$R_0 > 1 \Rightarrow I = \underline{N - \alpha}$ is stable $\rightarrow$ Infection

$s(t) = \frac{S}{N}.$

$i(t) = \frac{I}{N}$




N
Popula^n.
dS/dt
dI/dt
dR/dt SIR.
Days. →