# *Lab* 12

## **Unification, Recursion, First Order Logic, Expert Systems:**

#### **Unification:**

In Prolog, two terms unify:

- if they are the same term, or
- if they contain variables that can be uniformly instantiated with terms in such a way that the resulting terms are equal

When trying to unify terms, Prolog scans the knowledge base top down.

#### **Recursive Definitions**

Predicates can be defined recursively. Roughly speaking, a predicate is recursively defined if one or more rules in its definition refers to itself.

Let's consider an example. Suppose we have a knowledge base recording facts about the child relation:

```
child(bridget, caroline).
child(caroline, donna).
```

That is, Caroline is a child of Bridget, and Donna is a child of Caroline. Now suppose we wished to define the descendant relation; that is, the relation of being a child of, or a child of a child of, and so on. Here's a first attempt to do this. We could add the following two non -recursive rules to the knowledge base:

```
\begin{array}{cccc} \operatorname{descend}\left(X,Y\right) & :- & \operatorname{child}\left(X,Y\right). \\ \\ \operatorname{descend}\left(X,Y\right) & :- & \operatorname{child}\left(X,Z\right), \\ \\ & & \operatorname{child}\left(Z,Y\right). \end{array}
```

Now, fairly obviously these definitions work up to a point, but they are clearly limited: they only define the concept of descendant-of for two generations or less. That's ok for the above knowledge base, but suppose we get some more information about the child-of relation and we expand our list of child-of facts to this:

```
child(anne,bridget).
child(bridget,caroline).
child(caroline,donna).
child(donna,emily).
```

Now our two rules are inadequate. For example, if we pose the queries

```
?- descend(anne,donna).
or
?- descend(bridget,emily).
```

we get the answer no, which is not what we want. Sure, we could 'fix' this by adding the following two rules:

```
\label{eq:descend} \begin{array}{ll} \text{descend}\left(\textbf{X}, \textbf{Y}\right) & :- & \text{child}\left(\textbf{X}, \textbf{Z}\_1\right), \\ & & \text{child}\left(\textbf{Z}\_1, \textbf{Z}\_2\right), \end{array}
```

But, let's face it, this is clumsy and hard to read. Moreover, if we add further child-of facts, we could easily find ourselves having to add more and more rules as our list of child-of facts grow, rules like:

```
\label{eq:child} \begin{split} \text{descend}(X,Y) &:- & \text{child}(X,Z\_1)\,, \\ & & \text{child}(Z\_1,Z\_2)\,, \\ & & \text{child}(Z\_2,Z\_3)\,, \\ \\ & & \cdot \\ & & \cdot \\ & & \cdot \\ & & \cdot \\ & & \text{child}(Z\_17,Z\_18)\,. \\ & & \text{child}(Z\_18,Z\_19)\,. \\ & & \text{child}(Z\_19,Y)\,. \end{split}
```

This is not a particularly pleasant (or sensible) way to go!

But we don't need to do this at all. We can avoid having to use ever longer rules entirely. The following recursive predicate definition fixes everything exactly the way we want:

```
\begin{array}{cccc} \operatorname{descend}\left(X,Y\right) & :- & \operatorname{child}\left(X,Y\right) \,. \\ \\ \operatorname{descend}\left(X,Y\right) & :- & \operatorname{child}\left(X,Z\right), \\ \\ & & \operatorname{descend}\left(Z,Y\right) \,. \end{array}
```

What does this say? The declarative meaning of the base clause is: if Y is a child of X, then Y is a descendant of X. Obviously sensible. So what about the recursive clause? Its declarative meaning is: if Z is a child of X, and Y is a descendant of Z, then Y is a descendant of X. Again, this is obviously true.

So let's now look at the procedural meaning of this recursive predicate, by stepping through an example. What happens when we pose the query:

```
descend (anne, donna)
```

Prolog first tries the first rule. The variable X in the head of the rule is unified with anne and Y with donna and the next goal Prolog tries to prove is

```
child (anne, donna)
```

This attempt fails, however, since the knowledge base neither contains the fact child(anne,donna) nor any rules that would allow to infer it. So Prolog backtracks and looks for an alternative way of proving descend(anne,donna). It finds the second rule in the knowledge base and now has the following subgoals:

```
child(anne,_633),
descend( 633,donna).
```

Prolog takes the first subgoal and tries to unify it with something in the knowledge base. It finds the fact child(anne,bridget) and the variable \_633 gets instantiated to bridget. Now that the first subgoal is satisfied, Prolog moves to the second subgoal. It has to prove

```
descend (bridget, donna)
```

This is the first recursive call of the predicate descend/2. As before, Prolog starts with the first rule, but fails, because the goal

```
child(bridget,donna)
```

cannot be proved. Backtracking, Prolog finds that there is a second possibility to be checked for descend(bridget,donna), namely the second rule, which again gives Prolog two new subgoals:

```
child(bridget,_1785),
descend(_1785,donna).
```

The first one can be unified with the fact child(bridget, caroline) of the knowledge base, so that the variable 1785 is instantiated with caroline. Next Prolog tries to prove

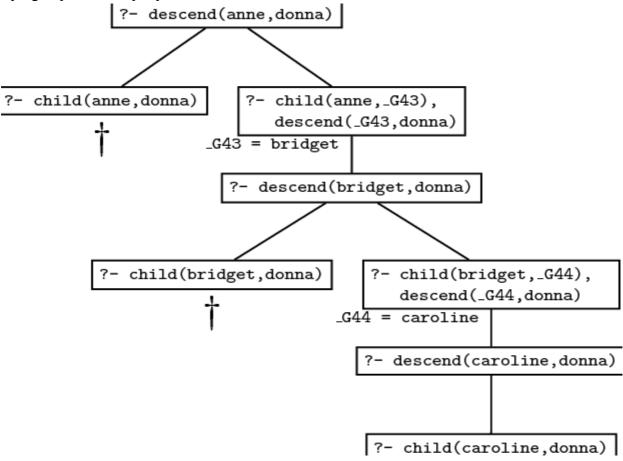
```
descend (caroline, donna) .
```

This is the second recursive call of predicate descend/2. As before, it tries the first rule first, obtaining the following new goal:

```
child(caroline, donna)
```

This time Prolog succeeds, since child(caroline,donna) is a fact in the database. Prolog has found a proof for the goal descend(caroline,donna) (the second recursive call). But this means that descend(bridget,donna) (the first recursive call) is also true, which means that our original query descend(anne,donna) is true as well.

Here is the search tree for the query descend(anne,donna). Make sure that you understand how it relates to the discussion in the text; that is, how Prolog traverses this search tree when trying to prove this query.



It should be obvious from this example that no matter how many generations of children we add, we will always be able to work out the descendant relation. That is, the recursive definition is both general and compact: it contains all the information in the non-recursive rules, and much more besides. The non-recursive rules only defined the descendant concept up to some fixed number of generations: we would need to write down infinitely many non-recursive rules if we wanted to capture this concept fully, and of course that's impossible. But, in effect, that's what the recursive rule does for us: it bundles up the information needed to cope with arbitrary numbers of generations into just three lines of code.

Recursive rules are really important. They enable to pack an enormous amount of information into a compact form and to define predicates in a natural way. Most of the work you will do as a Prolog programmer will involve writing recursive rules.

### Lab Task:

- i) medical diagnostic system
- ii) university admission and scholarship knowledge base