SIR

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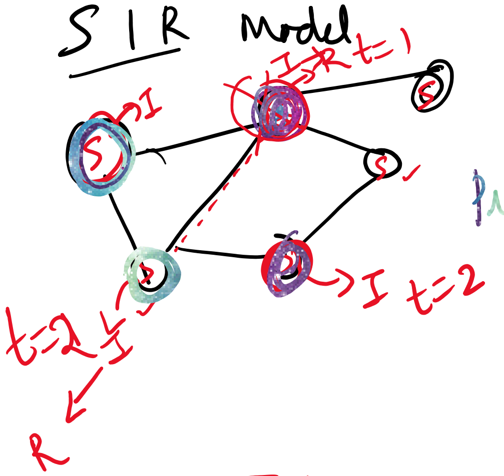
SIR Model



Process steps

S : People who may get infected
$S(t)$: at time t

I : People who are currently infected
$I(t)$: at time t

R : People recovered and will never be again susceptible
$R(t)$ at time t

β : Transmission rate of infection

γ : Recovery rate

$$S_{100} \xrightarrow{\beta S \frac{I}{N}} I \xrightarrow{\gamma I} R$$

Total popula^n : N : $S+I+R=N$

$$S(t)+I(t)+R(t)=N$$

Assump^n : No birth / No death

$\frac{I}{N}$: Frac^n of individuals who are infected.

$S\frac{I}{N}$: rate of individuals going from $S \to I$

$$\frac{dS}{dt} = -\beta \frac{SI}{N}$$

$$\frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

(Proportion of susceptible ppl getting infected with change in time)

$\frac{dy}{dx}$

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0 \quad (S+I+R=N)$$

Non-linear system of eq^{ns}

Coupled system of eq^{ns}

$$\frac{dI}{dt} > 0 \rightsquigarrow \text{Infec}^n \text{ keeps on } \uparrow$$

ie Disease will be epidemic

$$\therefore \beta \frac{SI}{} - \gamma I > 0$$

$$\therefore \frac{\beta S I}{N} - \gamma I > 0$$

$$\frac{\beta S \cancel{I}}{N} > \gamma \cancel{I}$$

$$\frac{\beta S \cancel{I}}{\gamma N} > 1$$

Definitely $\frac{S}{N} < 1$

$\frac{\beta}{\gamma} > 1$ to really establish that disease is epidemic

$\beta \rightarrow$ rate of transmission of infection

$\gamma \rightarrow$ recovery rate.

Reproduction no. $\rightarrow \frac{\beta}{\gamma}$; if $R_0 > 1$, transmission $\uparrow$

$$\frac{dI}{dt} = \frac{\beta S I}{N} - \gamma I \implies \frac{dI}{dt} = I\left(\frac{\beta S}{N} - \gamma\right)$$

$$I(t)^{=0} = e^{\left(\frac{\beta S}{N} - \gamma\right)}$$

Is this < 0 !

ie Is $\frac{\beta S I}{N} - \gamma I < 0$

$$I\left(\frac{\beta S}{N} - \gamma\right) < 0$$

$$\frac{\beta S}{N} - \gamma < 0$$

$$\frac{\beta S}{N} < \gamma$$

$$\frac{\beta S}{\gamma N} < 1$$

$\beta \rightarrow$ transmission rate

$\gamma \rightarrow$ recovery rate