

Straight Lines

11th Maths - Chapter 10

This is Problem-12 from Exercise 10.3

1. Two lines passing through point $\mathbf{P} = (2, 3)$ intersect each other at an angle of 60° . If the slope of one line is 2, find the equation of the other line.

1 Solution

Let $\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ be the given point, and the slope of one line 2. Let the slope of the other line be m , and the angle between them be 60° .

Input data:

$$\text{Direction vector } \mathbf{m}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (1)$$

$$\text{Direction vector } \mathbf{m}_2 = \begin{pmatrix} 1 \\ m \end{pmatrix} \quad (2)$$

$$\cos \theta = \frac{1}{2} \quad (3)$$

The angle between two vectors is then expressed as:

$$\cos \theta = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (4)$$

$$\frac{1}{2} = \frac{\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ m \end{pmatrix} \right\|} \quad (5)$$

$$\frac{1}{2} = \frac{2m + 1}{\sqrt{5}\sqrt{m^2 + 1}} \quad (6)$$

$$\frac{1}{4} = \frac{4m^2 + 4m + 1}{5m^2 + 5} \quad (7)$$

$$11m^2 + 16m - 1 = 0 \quad (8)$$

From the quadratic equation, the roots can be found as:

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (9)$$

$$m = \frac{-16 \pm \sqrt{16^2 - 4(11)(-1)}}{2(11)} \quad (10)$$

$$m = \frac{-16 \pm \sqrt{300}}{22} \quad (11)$$

$$m = \frac{-8 - 5\sqrt{3}}{11} \quad (12)$$

$$\text{or} \quad (13)$$

$$m = \frac{-8 + 5\sqrt{3}}{11} \quad (14)$$

Therefore, the equation of the other line can be determined using these values.

case 1: Line passing through point \mathbf{P} with slope $m = \frac{-8-5\sqrt{3}}{11}$

$$\mathbf{n}^\top \{\mathbf{x} - \mathbf{P}\} = 0 \quad (15)$$

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \quad (16)$$

$$\left(\frac{-8-5\sqrt{3}}{11} \quad -1 \right) \left\{ \mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\} = 0 \quad (17)$$

Then the equation for $m = \frac{-8-5\sqrt{3}}{11}$ is $(5\sqrt{3} + 8)x + 11y = 49 + 10\sqrt{3}$

case 2 : Line passing through point \mathbf{P} with slope $m = \frac{-8+5\sqrt{3}}{11}$

$$\mathbf{n}^\top \{\mathbf{x} - \mathbf{P}\} = 0 \quad (18)$$

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \quad (19)$$

$$\left(\frac{-8+5\sqrt{3}}{11} \quad -1 \right) \left\{ \mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\} = 0 \quad (20)$$

Therefore then the equation for $m = \frac{-8+5\sqrt{3}}{11}$ is $(5\sqrt{3} - 8)x + 11y = 49 - 10\sqrt{3}$

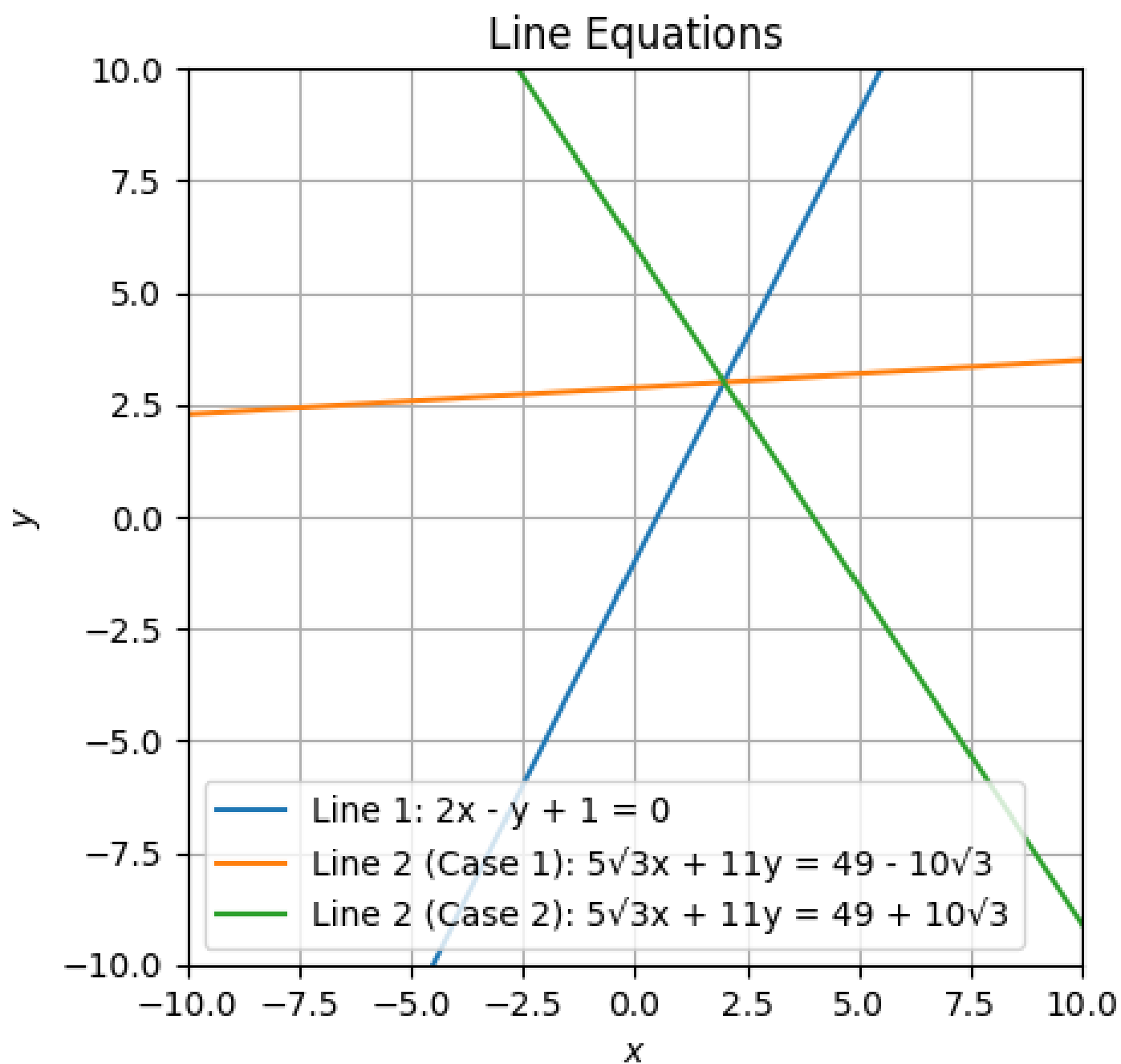


Figure 1: straight line