

Straight Lines

11th Maths - Chapter 10

This is Problem-12 from Exercise 10.3

1. Two lines passing through point $\vec{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ intersect each other at an angle of 60° . If the slope of one line is 2, find the equation of the other line.

1 Solution

Let $\vec{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ be the given point, and the slope of one line $m_1 = 2$. Let the slope of the other line be m , and the angle between them be 60° .

Input data:

$$\text{Direction vector } \vec{m}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\text{Direction vector } \vec{m}_2 = \begin{pmatrix} 1 \\ m \end{pmatrix}$$

$$\cos \theta = \frac{1}{2}$$

The angle between two vectors is then expressed as:

$$\begin{aligned}\cos \theta &= \frac{\vec{m}_1^\top \vec{m}_2}{\|\vec{m}_1\| \|\vec{m}_2\|} \\ \frac{1}{2} &= \frac{\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ m \end{pmatrix} \right\|} \\ \frac{1}{2} &= \frac{2m + 1}{\sqrt{5} \sqrt{m^2 + 1}} \\ \frac{1}{4} &= \frac{4m^2 + 4m + 1}{5m^2 + 5} \\ 11m^2 + 16m - 1 &= 0\end{aligned}$$

From the quadratic equation, the roots can be found as:

$$\begin{aligned}m &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ m &= \frac{-16 \pm \sqrt{16^2 - 4(11)(-1)}}{2(11)} \\ m &= 0.06 \quad \text{or} \quad m = -1.514\end{aligned}$$

Therefore, the equation of the other line can be determined using these values.

1. Line passing through point $\vec{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ with slope $m = 0.06$

$$\vec{n}^\top (\vec{x} - \vec{P}) = 0 \tag{1}$$

$$\vec{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \tag{2}$$

$$(0.06 \quad -1) \left(\vec{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) = 0 \tag{3}$$

then the equation for $m=0.06$ is $y = 0.06x + 2.88$

2. Line passing through point $\vec{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ with slope $m = -1.514$

$$\vec{n}^\top (\vec{x} - \vec{P}) = 0 \quad (4)$$

$$\vec{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \quad (5)$$

$$(-1.514 \quad -1) \left(\vec{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) = 0 \quad (6)$$

Therefore then the equation is $1.514x + y = 0.02$

