Straight Lines

11^{th} Maths - Chapter 10

This is Problem-12 from Exercise 10.3

1. Two lines passing through point $\begin{pmatrix} A \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ intersect each other at an angle of $60°$. If the slope of one line is 2, find the equation of the other line.

1 Solution

Let $\begin{pmatrix} A \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ be the given point, and the slope of one line $m_1 = 2$. Let the slope of the other line be m, and the angle between them be $60°$.

Input data:

$$\text{Direction vector } \begin{pmatrix} m_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{1}$$

$$\text{Direction vector } \begin{pmatrix} m_2 \end{pmatrix} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{2}$$

$$\cos\theta = \frac{1}{2} \tag{3}$$

The angle between two vectors is then expressed as:

$$\cos\theta = \frac{\begin{pmatrix} m_1 \end{pmatrix}^\top \begin{pmatrix} m_2 \end{pmatrix}}{\| \begin{pmatrix} m_1 \end{pmatrix} \| \| \begin{pmatrix} m_2 \end{pmatrix} \|} \tag{4}$$

$$\frac{1}{2} = \frac{\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ m \end{pmatrix} \right\|} \tag{5}$$

$$\frac{1}{2} = \frac{2m + 1}{\sqrt{5}\sqrt{m^2 + 1}} \tag{6}$$

$$\frac{1}{4} = \frac{4m^2 + 4m + 1}{5m^2 + 5} \tag{7}$$

$$11m^2 + 16m - 1 = 0 \tag{8}$$

From the quadratic equation, the roots can be found as:

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{9}$$

$$m = \frac{-16 \pm \sqrt{16^2 - 4(11)(-1)}}{2(11)} \tag{10}$$

$$m = \frac{-16 \pm \sqrt{300}}{22} \tag{11}$$

$$m = \frac{-8 - 5\sqrt{3}}{11} \tag{12}$$

$$or \tag{13}$$

$$m = \frac{-8 + 5\sqrt{3}}{11} \tag{14}$$

Therefore, the equation of the other line can be determined using these values.

case 1: Line passing through point $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ with slope $m = \frac{-8-5\sqrt{3}}{11}$

$$\mathbf{n}^\top \left\{ \mathbf{x} - \mathbf{P} \right\} = 0 \tag{15}$$

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \frac{-8-5\sqrt{3}}{11} & -1 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\} = 0 \tag{17}$$

then the equation for $m = \frac{-8-5\sqrt{3}}{11}$ is $(5\sqrt{3}+8)x + 11y = 49 + 10\sqrt{3}$

case 2 : Line passing through point $\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ with slope $m = \frac{-8+5\sqrt{3}}{11}$

$$\mathbf{n}^\top \left\{ \mathbf{x} - \mathbf{P} \right\} = 0 \tag{18}$$

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \tag{19}$$

$$\begin{pmatrix} \frac{-8+5\sqrt{3}}{11} & -1 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\} = 0 \tag{20}$$

Therefore then the equation for $m = \frac{-8+5\sqrt{3}}{11}$ is $(5\sqrt{3}-8)x + 11y = 49 - 10\sqrt{3}$

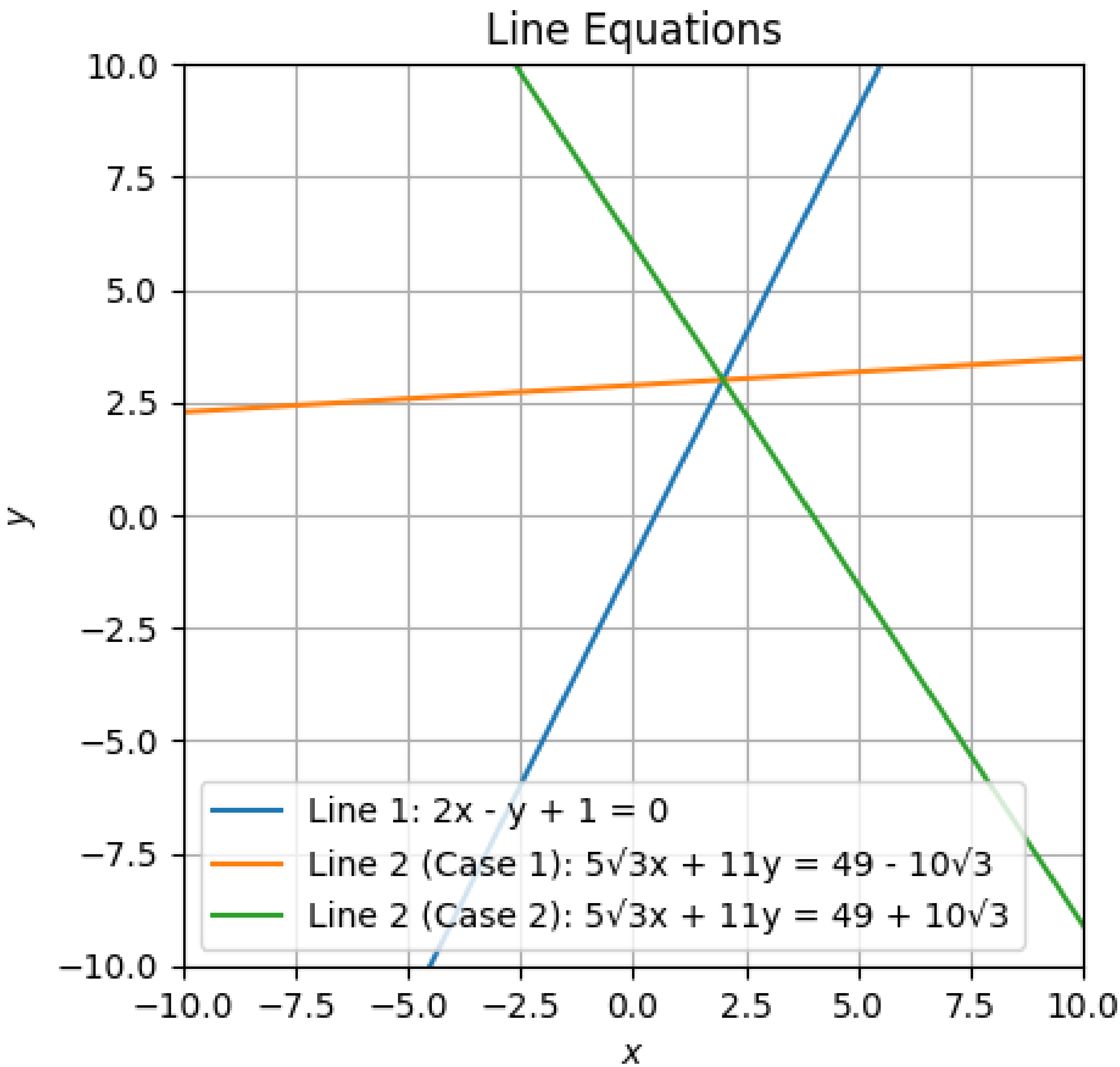

Figure 1: straight line