Triangles

July 18, 2023

9^{th} Maths - Chapter 7

This is Problem-5 from Exercise 7.1

1. Line l is the bisector of an angle $\angle A$ and B is a point on line l. $BP = BQ$ are perpendiculars from B to the arms of $\angle A$.

    (a) $\triangle APB \cong \triangle AQB$

    (b) $BP = BQ$ or B is equidistant from the arms of $\angle A$

Construction

The input parameters for the construction are shown in Table

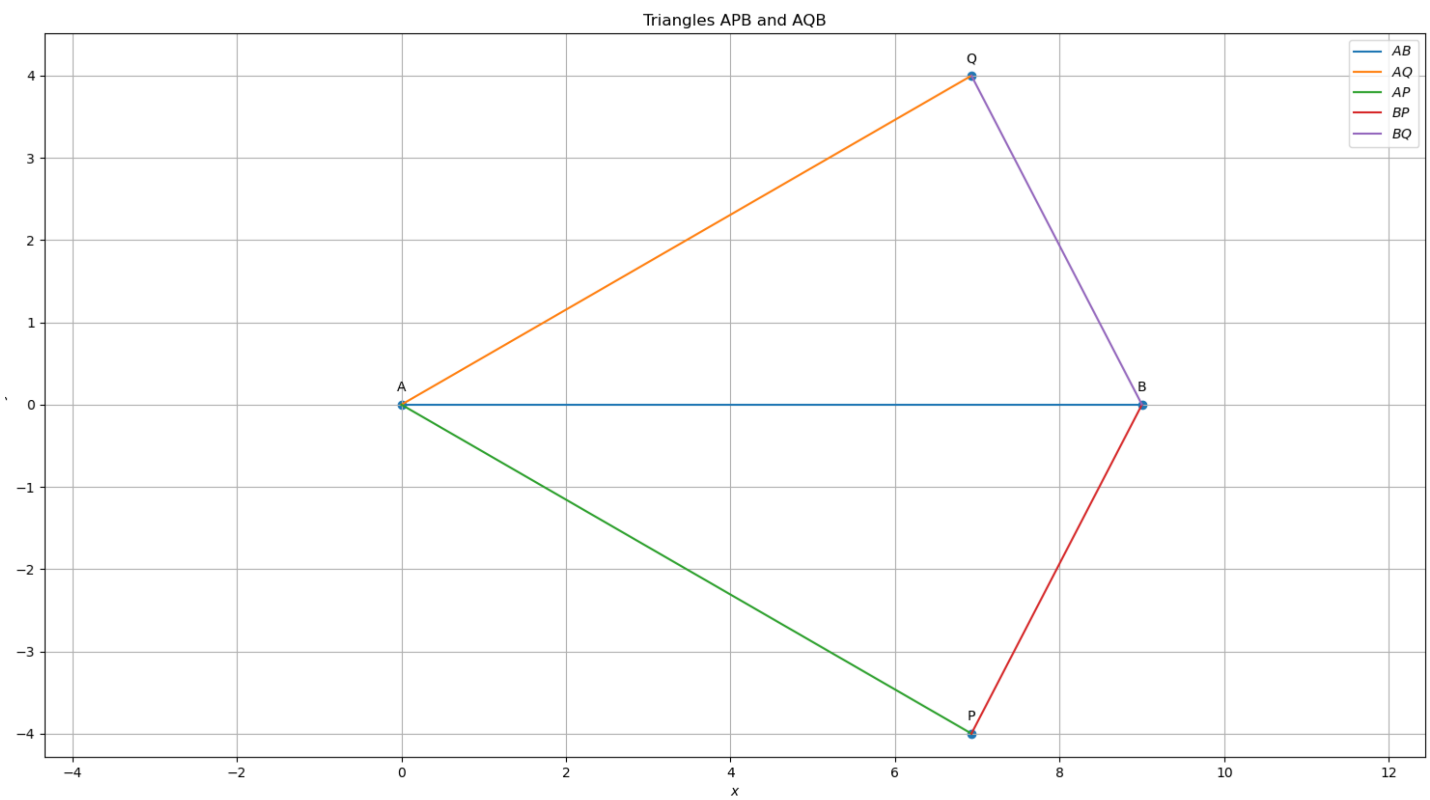

Figure 1: figure

Let $\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\mathbf{B} = a\mathbf{e_1}$, $\mathbf{Q} = \begin{pmatrix} c\cos\theta \\ c\sin\theta \end{pmatrix}$, and $\mathbf{P} = \begin{pmatrix} c\cos\theta \\ -c\sin\theta \end{pmatrix}$.

Symbol	Value	Description
θ	$30°$	$\angle BAP = \angle BAQ$
a	9	AB
c	8	AQ
$\mathbf{e}_1$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Basis vector

Table 1: Parameters

1 Solution

Given:

$$\mathbf{Q} - \mathbf{A} = \mathbf{A} - \mathbf{P} \tag{1}$$
$$\angle QAB = \angle PAB \tag{2}$$
$$\angle AQB = \angle BPA \tag{3}$$
$$AB = AB \quad \text{(common side)} \tag{4}$$

To prove:

1. $\triangle APB \cong \triangle AQB$
2. $BP = BQ$ or B is equidistant from the arms of $\angle A$

Proof:

$$\angle QAB = \angle PAB \tag{5}$$
$$\angle AQB = \angle BPA \tag{6}$$

AB is the common side of $\triangle APB$ and $\triangle AQB$. Therefore, by A-A-S rule, $\triangle APB \cong \triangle AQB$.

$$\|\mathbf{B} - \mathbf{P}\| = \left\| \begin{pmatrix} 9 \\ 0 \end{pmatrix} - \begin{pmatrix} 8\cos\theta \\ -8\sin\theta \end{pmatrix} \right\| = \left\| \begin{pmatrix} 2.07 \\ 4 \end{pmatrix} \right\| = 4.4 \tag{7}$$
$$\|\mathbf{B} - \mathbf{Q}\| = \left\| \begin{pmatrix} 9 \\ 0 \end{pmatrix} - \begin{pmatrix} 8\cos\theta \\ 8\sin\theta \end{pmatrix} \right\| = \left\| \begin{pmatrix} 2.07 \\ -4 \end{pmatrix} \right\| = 4.4 \tag{8}$$
$$\|\mathbf{B} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{Q}\| \tag{9}$$

Therefore, $BP = BQ$ or B is equidistant from the arms of $\angle A$ is proved.