

# Straight Lines

## 11<sup>th</sup> Maths - Chapter 10

This is Problem-12 from Exercise 10.3

- Two lines passing through point  $\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  intersect each other at an angle of  $60^\circ$ . If the slope of one line is 2, find the equation of the other line.

### 1 Solution

Let  $\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  be the given point, and the slope of one line 2. Let the slope of the other line be  $m$ , and the angle between them be  $60^\circ$ .

**Input data:**

$$\text{Direction vector } \mathbf{m}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (1)$$

$$\text{Direction vector } \mathbf{m}_2 = \begin{pmatrix} 1 \\ m \end{pmatrix} \quad (2)$$

$$\cos \theta = \frac{1}{2} \quad (3)$$

The angle between two vectors is then expressed as:

$$\cos \theta = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (4)$$

$$\frac{1}{2} = \frac{\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ m \end{pmatrix} \right\|} \quad (5)$$

$$\frac{1}{2} = \frac{2m + 1}{\sqrt{5} \sqrt{m^2 + 1}} \quad (6)$$

$$\frac{1}{4} = \frac{4m^2 + 4m + 1}{5m^2 + 5} \quad (7)$$

$$11m^2 + 16m - 1 = 0 \quad (8)$$

From the quadratic equation, the roots can be found as:

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (9)$$

$$= \frac{-16 \pm \sqrt{16^2 - 4(11)(-1)}}{2(11)} \quad (10)$$

$$= \frac{-16 \pm \sqrt{300}}{22} \quad (11)$$

$$= \frac{-8 - 5\sqrt{3}}{11} \quad (12)$$

$$\text{or, } = \frac{-8 + 5\sqrt{3}}{11} \quad (13)$$

Therefore, the equation of the other line can be determined using these values.

1. Line passing through point  $\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  with slope  $m = \frac{-8-5\sqrt{3}}{11}$

$$\mathbf{n}^\top \{\mathbf{x} - \mathbf{P}\} = 0 \quad (14)$$

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \quad (15)$$

$$\begin{pmatrix} \frac{-8-5\sqrt{3}}{11} & -1 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\} = 0 \quad (16)$$

2. Line passing through point  $\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$  with slope  $m = \frac{-8+5\sqrt{3}}{11}$

$$\mathbf{n}^\top \{\mathbf{x} - \mathbf{P}\} = 0 \quad (17)$$

$$\mathbf{n} = \begin{pmatrix} m \\ -1 \end{pmatrix} \quad (18)$$

$$\begin{pmatrix} \frac{-8+5\sqrt{3}}{11} & -1 \end{pmatrix} \left\{ \mathbf{x} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\} = 0 \quad (19)$$

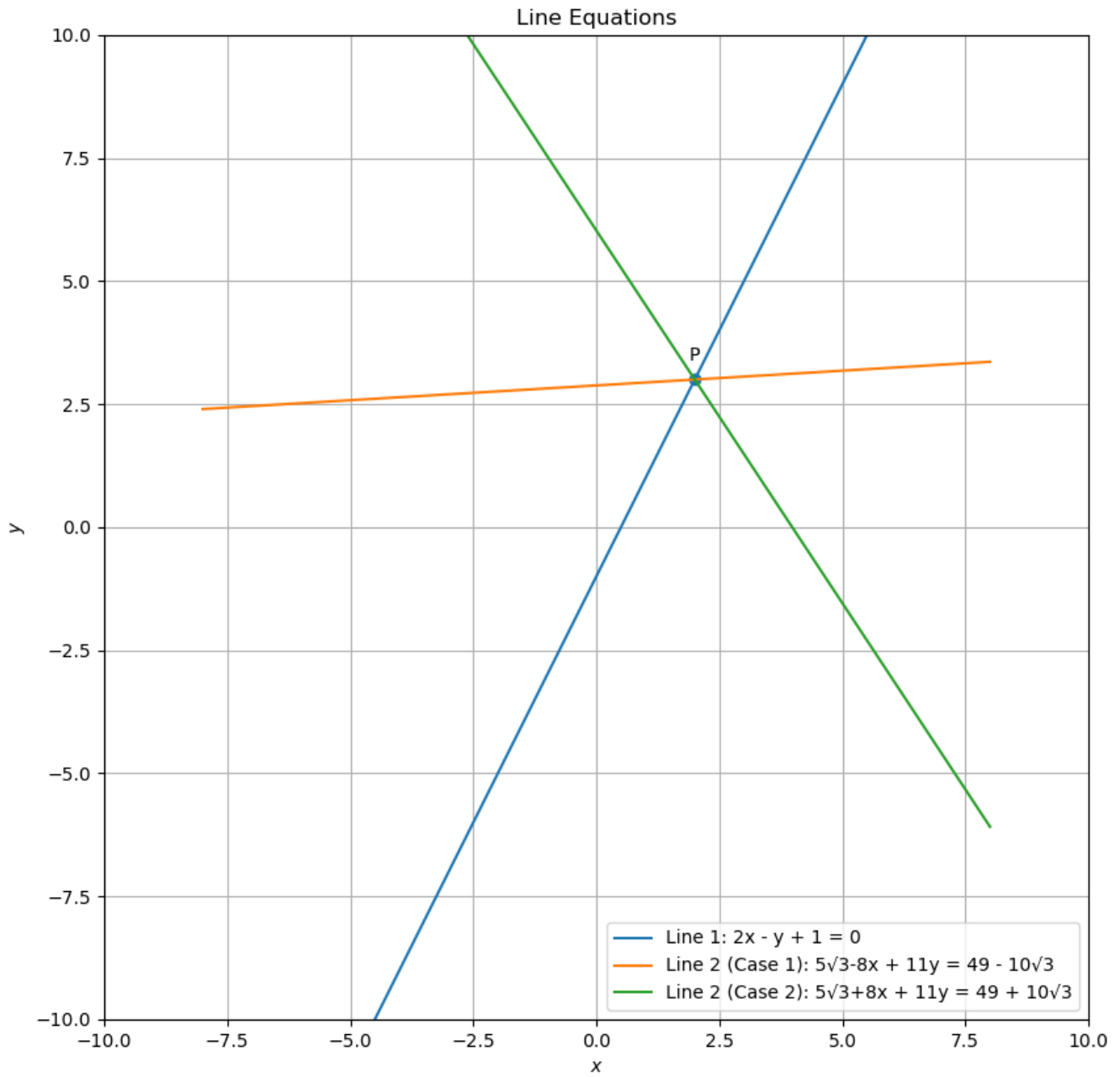


Figure 1: straight line