



$$f_1(x_1) = w_1 \cdot x_1 \quad \Rightarrow \quad \frac{df_1}{dx_1} = w_1, \quad \frac{df_1}{dw_1} = x_1$$

$$f_2(x_2) = w_2 x_2 \quad \Rightarrow \quad \frac{df_2}{dx_2} = w_2$$

$$f_3(x_3) = w_3 x_3 \quad \Rightarrow \quad \frac{df_3}{dx_3} = u_3, \quad \frac{df_3}{du_3} = x_3$$

$$f_4(f_1) = \text{sigmoid}(f_1) \quad \Rightarrow \quad \frac{df_4}{df_1} = \cancel{1} \cdot s(f_1) \cdot (1 - s(f_1))$$

$$f_5(f_2) = s(f_2) \quad \Rightarrow \quad \frac{df_5}{df_2} = \cancel{1} \cdot s(f_2) \cdot (1 - s(f_2))$$

$$f_6(f_4) = u_1 \cdot f_4 \quad \Rightarrow \quad \frac{df_6}{df_4} = u_1, \quad \frac{df_6}{du_1} = f_4$$

$$f_7(f_5) = u_2 \cdot f_5 \quad \Rightarrow \quad \frac{df_7}{df_5} = u_2, \quad \frac{df_7}{du_2} = f_5$$

$$f_8(f_6, f_7, f_3) = f_3 + f_6 + f_7 \quad \Rightarrow \quad \frac{df_8}{df_3} = 1, \quad \frac{df_8}{df_6} = 1, \quad \frac{df_8}{df_7} = 1$$

$$y(f_8) = s(f_8) \quad \Rightarrow \quad \frac{dy}{df_8} = \cancel{1} \cdot s(f_8) \cdot (1 - s(f_8))$$

$$E = \frac{1}{2} \cdot (t - y)^2 \quad \frac{dE}{dy} = (y - t) \quad \text{Chain Rules:}$$

$$\frac{dE}{dw_1} = \frac{dE}{dy} \cdot \frac{dy}{dw_1} = \frac{dE}{dy} \cdot \frac{dy}{df_8} \cdot \frac{df_8}{df_6} \cdot \frac{df_6}{df_4} \cdot \frac{df_4}{df_1} \cdot \frac{df_1}{dw_1}$$

$$\frac{dE}{du_1} = \frac{dE}{dy} \cdot \frac{dy}{du_1} = \frac{dE}{dy} \cdot \frac{dy}{df_8} \cdot \frac{df_8}{df_6} \cdot \frac{df_6}{du_1}$$

$$\frac{dE}{du_2} = \frac{dE}{dy} \cdot \frac{dy}{du_2} = \frac{dE}{dy} \cdot \frac{dy}{df_8} \cdot \frac{df_8}{df_7} \cdot \frac{df_7}{du_2}$$

$$\frac{dy}{du_3} = \frac{dE}{dy} \cdot \frac{dy}{df_8} \cdot \frac{df_8}{df_3} \cdot \frac{df_3}{du_3}$$

Results:

$$\frac{dE}{dw_1} = (y - t) \cdot [s(f_8) \cdot (1 - s(f_8))] \cdot 1 \cdot u_1 \cdot [s(f_1) \cdot (1 - s(f_1))] \cdot x_1$$

$$\frac{dE}{du_1} = (y - t) \cdot [s(f_8) \cdot (1 - s(f_8))] \cdot 1 \cdot f_4$$

$$\frac{dE}{du_2} = (y - t) \cdot [s(f_8) \cdot (1 - s(f_8))] \cdot 1 \cdot f_5$$

where:

$$\frac{dE}{du_3} = (y - t) \cdot [s(f_8) \cdot (1 - s(f_8))] \cdot 1 \cdot x_3$$

$$f_8 = u_1 \cdot s(w_1 \cdot x_1) + u_2 \cdot s(w_2 \cdot x_2) + u_3 \cdot x_3$$

$$f_1 = w_1 \cdot x_1$$

$$f_4 = s(w_1 \cdot x_1)$$

$$f_5 = s(w_2 \cdot x_2)$$

$$y = s(f_8)$$

Finally: w_1', u_2', u_3' - new weights

$$w_1' = w_1 - \alpha \cdot \frac{dE}{dw_1}$$

$$u_2' = u_2 - \alpha \cdot \frac{dE}{du_2}$$

$$u_3' = u_3 - \alpha \cdot \frac{dE}{du_3}$$

where α - learning rate (e.g. 0.1, 0.05, 0.2)