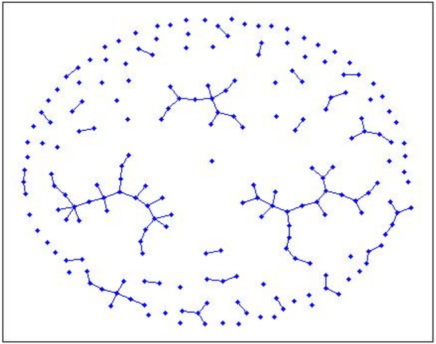
Applications and Types of Graphs

- Mathematical abstraction for modeling the connections between things
- Social Networks: Nodes are people and edges are some kind of relationship (friend, contact, etc.)
  - Online or Real World
- Different types of Networks: Electrical Circuits, Gene Regulatory Networks, Hyperlink Relationships of Web Pages
- Theory of Computing: Deterministic and Non-Deterministic Finite Automata

Graphs

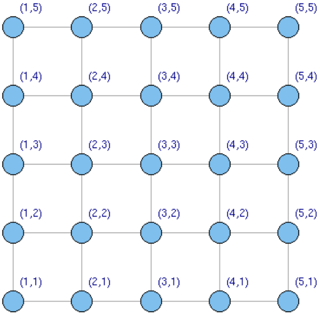
- Powerful mathematical abstraction
- Structure consisting of nodes and connections between the nodes (edges)
- $G = (V, E)$
- $|V|$ number of nodes, *order* of the graph
- $|E|$ number of edges, *size* of the graph
- *degree* number of incoming edges to a node

How do graphs look in general?



- Erdos-Renyi model constructs a random graph where each edge exists independently with probability p
- Reasonable sampling of graphs in general

Networks based upon regular structures



- We might also construct a network corresponding to our intuition, or regular structure, such as a rectangular or horizontal axis

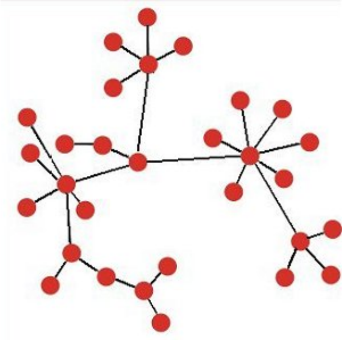
Degree Distribution of a Graph

- Degree is the number of connections of a given node
- The *Degree Distribution* is the probability $P(k)$ that any node in the graph has degree k
- Probability distribution over the integers
- Random graphs obey a Binomial Distribution
- It was discovered that social networks and other "real world" networks often had a different type of distribution
- Scale-Free Networks: Watts and Strogatz, Barabasi and Albert

Power law distributions

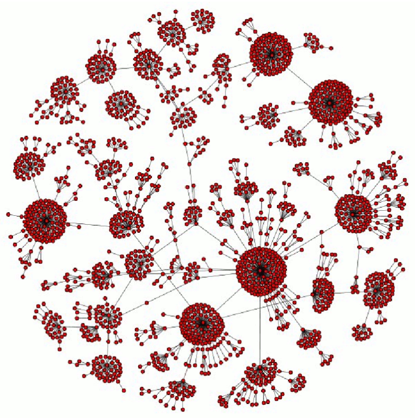
- $P(k) \propto k^{-\gamma}$
- Power law distribution have larger distributions than typical exponential family distributions such as the normal or exponential distribution

Networks with a Power-Law Distribution



- Majority of nodes have a small number of connections, but a few have a large number of connections
- The highly connected nodes serve as hubs providing connectivity for the entire network

Larger Scale Free Networks



- Larger scale free networks might look as above

Applications

- Identify vulnerable nodes in a network
- Stop the spread of a contagion in a network
- Maximize viral marketing campaign

networkx package in python

- networks is a package for general graph computations

```
import networkx as nx
import matplotlib.pyplot as plt

F = nx.powerlaw_cluster_graph(9, 1, 0.4)
G = nx.barabasi_albert_graph(100, 2)
H = nx.grid_2d_graph(10, 10)
I = nx.complete_graph(10)
J = nx.cycle_graph(10)
K = nx.erdos_renyi_graph(100, 0.01)
L = nx.pappus_graph()
```

Pappus Graph

- `nx.degree_histogram(G)` is the degree distribution of the graph G
- For the Pappus graph the function returns `[0, 0, 0, 18]`
- What does it mean?
- What is the size of the graph, what is the order of the graph?

In-Class Exercise

- Use the graph generators in from the `networkx` package given in the previous slide
- For each type of graph first reason about how the degree distribution should look
- For each type of graph produce a drawing of the graph as well as a plot of the degree distribution
- Drawing can be accomplished with `nx.draw(G); plt.show()` for a graph G
- Degree distribution can be accomplished using `nx.degree_histogram(G)`