Diagonalizable Matrices

- A matrix R is *diagonalizable* when it can be written as a product shown below
- P is a non-singular matrix
- Called a *similarity transformation*
  - How an operator changes under a change of basis

$$R = P^{-1}AP \tag{1}$$

Define Polynomials of Matrices

- Polynomial Algebra of matrices:
  - 1 corresponds to identity
  - scalar multiplication
  - powers are products of matrix with itself or iterated compositions
- Cayley-Hamilton theorem: any matrix satisfies its own characteristic polynomial

$$f(A) = a_n A^n + \cdots + a_1 A + a_0 I \tag{2}$$

Characteristic Polynomial

- The Characteristic Polynomial of a matrix is computed by means of a special determinant
- The roots of characteristic polynomial are the *eigenvalues*
- Eigenvectors are vectors satisfying the expression

$$\Delta(\lambda) = |(\lambda I - A)| \tag{3}$$

$$Rv = \lambda v \tag{4}$$

$$0 = (\lambda I - R)v \tag{5}$$

Diagonal Factorization, Spectral Decomposition

- Reversing the process we can write a matrix as a similarity transformation of a diagonal matrix
- If A is real-symmetric then the roots of its characteristic polynomial are guaranteed to be real.
- Eigenvectors belonging to distinct eigenvalues are orthogonal to one another $u \cdot v = 0$
- In general, not every matrix can be diagonalized

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \tag{6}$$

Principal Component Analysis in Two Dimensions

Principal Components Analysis PCA (Basic Idea)

Project data onto the eigenvectors of the sample covariance matrix