Professor Kate Ashley
MISM 6202

Foundations of Data Analysis for Business
PROBLEM SET 2 – Ardalan Mahdavieh

A company manufactures and sells two models of lamps, the Allister (A) and the Zainab (Z). To manufacture each lamp, the manual work involved is 20 minutes for the Allister and 30 minutes for the Zainab. The mechanical (machine) work involved is 20 minutes for the Allister and 10 minutes for the Zainab. The manual work available per week is 100 hours and the machine is limited to only 80 hours per week. Knowing that the profit per unit is $15 for the Allister and $10 for the Zainab, respectively, determine the quantities of each lamp that should be manufactured each week to obtain the maximum profit. *You may ignore integrality requirements for this problem.*

1. Write an expression for the objective function of this problem, using the specified variable names. *(5 pts)*

   a = Allister
   z = Zainab
   Maximize: Profit = 15a + 10z

2. Write a mathematical expression for the constraint on available manual work. *(5 pts)*

   Manual Work: 20a + 30z =< 100

3. Write a mathematical expression for the constraint on available machine work. *(5 pts)*

   Machine Work: 20a + 10z =< 80

Suppose you solved the lamp production linear program in Excel and obtained an optimal production mix of 210 Allister lamps and 60 Zainab lamps. Answer the following questions based on the Sensitivity Report shown below.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C6	# Manufactured Allister	210	0	15	5	8.333333333
D6	# Manufactured Zainab	60	0	10	12.5	2.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E9	Manual work	100	7.5	100	140	20
E10	Machine work	80	37.5	80	20	46.66666667

4. What is the optimal profit per week for this company? *(5 pts)*

   Profit = 15(210) + 10(60), Profit = $3,750

5. Suppose the company wants to hire an additional full-time employee to work 40 hours per week on lamp production. What is the maximum amount this employee could be paid per week without reducing the company's profits? *(5 pts)*

   Max pay/week = 7.5*20 = $150

6. The company expects available manual labor to be 10 hours lower than usual next week, due to scheduled vacation time. What will be the new optimal profit level? *(5 pts)*

   Profit = 3,750 – 7.5(10) = $3,000

7. Suppose the company is considering purchasing an additional machine, which would double the available machine time per week. What can you say about the optimal profit with an additional machine? *(5 pts)*

   With the additional machine, the optimal profit should increase given that there are no additional variable or fixed cost. However, the current optimal profit is no longer valid and has to be re-calculated. This is because the allowable increase on the machine constraint is only 20 minutes per week and we are trying to increase it by 80 minutes.

8. If the margin on the Allister lamp drops to $12 due to higher material costs, what can you say about the new optimal production mix? *(5 pts)*

   The allowable decrease for Allister lamp is $8.33 and in this case, the price was only reduced by $3. Since the reduced price is within the allowable decrease value, our optimal production mix should remain the same.

Refer to the following description for Q9-10:

A client of an investment management firm wishes to invest $750,000 in the bonds shown in the table below. No more than 25% of the client's money can be invested in any given company, and at least 50% should be invested in long-term bonds (maturing in 10+ years). No more than 35% can be invested in DynaStar, Eagle Vision, and OptiPro combined.

Company	Return	Years to Maturity	Rating
Acme Chemical	8.65%	11	1 - Excellent
DynaStar	9.50%	10	3 - Good

EagleVision	10.00%	6	4 - Fair
Micro Modeling	8.75%	10	1 - Excellent
OptiPro	9.25%	7	3 - Good
Sabre Systems	9.00%	13	2 - Very Good

9. Formulate the linear program for the investment problem, i.e., state the decision variables and write mathematical expressions for the objective function and constraints. *(15 pts)*

Decision variables:

- a = Acme Chemical investment
- d = DynaStar investment
- e = EagleVision investment
- m = Micro Modeling investment
- = OptiPro investment
- s = Sabre Systems investment

Objective function:

- Maximize: ROI = 0.0865a + 0.095d + 0.1e + 0.0875m + 0.0925o + 0.09s

Constraints:

- a + d + e + m + o + s = 750,000
- a =< 187,500
- d =< 187,500
- e =< 187,500
- m =< 187,500
- o =< 187,500
- s =< 187,500
- a + d + m + s >= 375,000
- d + e + o =< 262,500
- a,d,e,m,o,s >= 0

10. Solve the linear program using Excel or R. Submit your spreadsheet or code along with this assignment and report the optimal values of the decision variables and the objective function on your write-up. *(15 pts)*

Optimal Value: $68,887.50

Objective function:

- Maximize: ROI = 0.0865(112500) + 0.095(75000) + 0.1(187500) + 0.0875(187500) + 0.0925(0) + 0.09(187500)

WRITE YOUR OWN LP. *(30 pts)*

11. Write your own linear programming problem with at least three decision variables and at least three constraints, at least one of which is binding. Submit the text of your problem description and an Excel spreadsheet in which you set up and solve the LP. *The more creative and realistic your problem is, the better!*

Arddie Inc, produces organic peanut butter. The company owns two production factories in Kittery, ME and Scarsdale, NY. The following are their production costs and production capacity for each month:

Production Factory	Cost/box	Capacity (boxes)
Kittery, ME	$18.50	300
Scarsdale, NY	$16.00	200

The peanut butter jars are packed in boxes and shipped to warehouses in Durham, NH and Patterson, NY for storage purposes. The warehouses then have to ship all of the boxes to distributors in NYC, Boston and New Haven.

For the upcoming month, the distributors have requested the following number of boxes based on their demand;

- NYC: 200 boxes
- Boston: 150 boxes
- New Have: 100 boxes

Based on the following shipping schedule that contains the shipping cost and capacity, formulate a linear program to determine how many boxes of peanut butter should be produced and how many boxes should be shipped to retailers to meet their demands at the lowest cost.

From	To	Cost/box	Capacity (boxes)
Kittery	Durham	$1.50	200
Kittery	Patterson	$4.00	200
Scarsdale	Durham	$5.00	200
Scarsdale	Patterson	$2.00	200
Durham	Boston	$2.50	200
Durham	NYC	$6.00	100
Patterson	NYC	$2.00	100
Patterson	New Haven	$3.50	200

Decision variables:

- x1= Shipping from Kittery to Durham
- x2 = Shipping from Kittery to Patterson
- x3 = Shipping from Scarsdale to Durham
- x4 = Shipping from Scarsdale to Patterson
- x5 = Shipping from Durham to Boston
- x6 = Shipping from Durham to NYC
- x7 = Shipping from Patterson to NYC
- x8 = Shipping from Patterson to New Haven
- x9 = Kittery production
- x10= Scarsdale production

Objective function:

- Minimize: Cost of production and distribution = $1.5x_1 + 4x_2 + 5x_3 + 2x_4 + 2.5x_5 + 6x_6 + 2x_7 + 3.5x_8 + 18.5x_9 + 16x_{10}$

Constraints:

- Kittery capacity: x1 + x2 <= 300
- Scarsdale capacity: x3 + x4 <= 200
- Boston demand: x5 = 150
- NYC demand: x6 + x7 = 200
- New Haven demand: x8 = 100
- Durham shipping and receiving: x1 + x3 – x5 – x6 = 0
- Patterson shipping and receiving: x2 + x4 – x7 – x8 = 0
- Truck 1 capacity: x1 <= 200
- Truck 2 capacity: x2 <= 200
- Truck 3 capacity: x3 <= 200
- Truck 4 capacity: x4 <= 200
- Truck 5 capacity: x5 <= 200
- Truck 6 capacity: x6 <= 100
- Truck 7 capacity: x7 <= 100
- Truck 8 capacity: x8 <= 200
- Kittery Production: -x1 - x2 + x9 = 0
- Scarsdale production: -x3 – x4 + x10 = 10
- Non negativity: $X_{ij} >= 0$