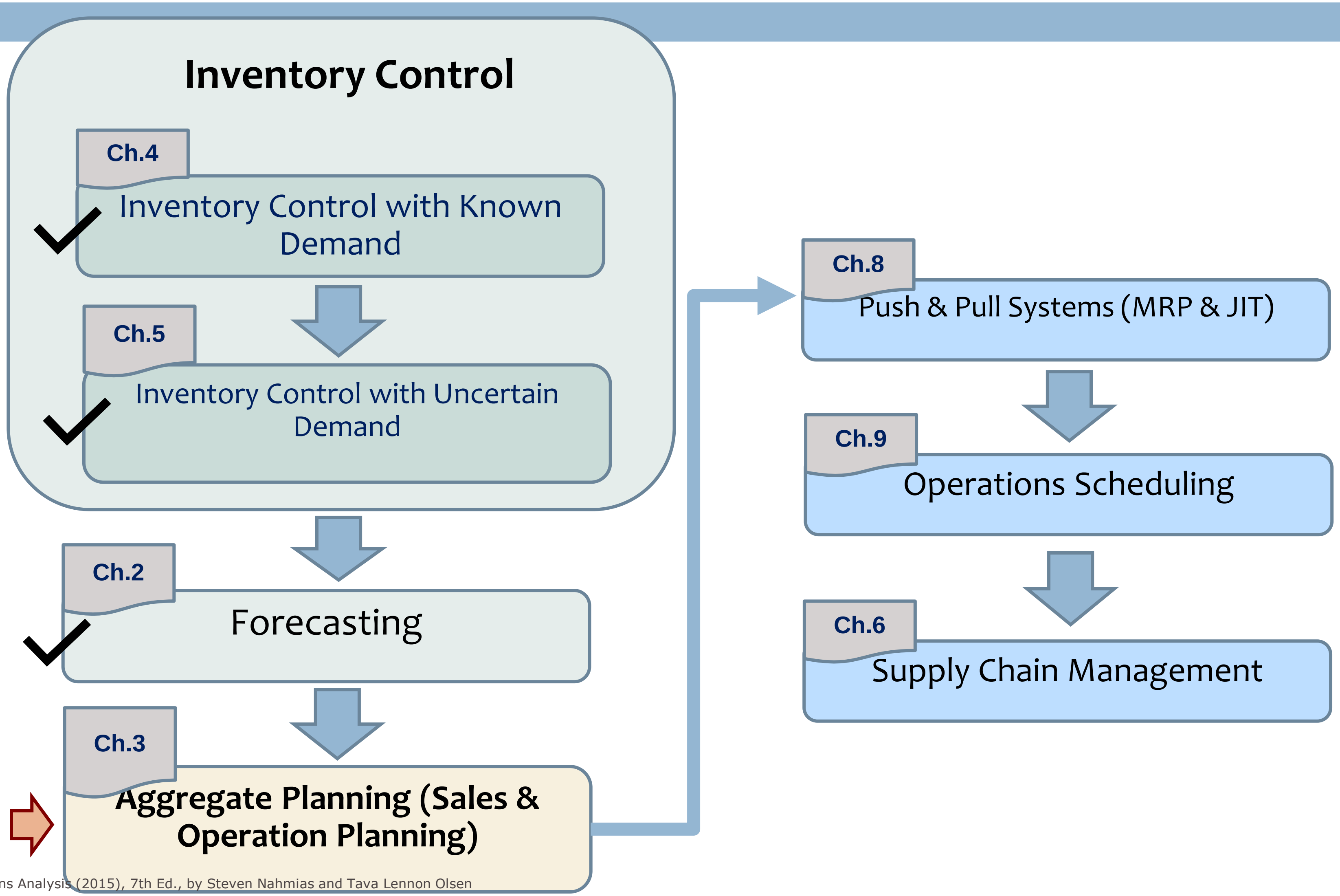
LECTURE 4:
AGGREGATE PLANNING

Production and Operations Planning

Where are we now on the course map?



Agenda

- Introduction
- Aggregate Planning of Production and Capacity
  - Aggregate unit
  - Hierarchical Production Planning
  - Relevant Costs
  - Competing Objectives
  - Aggregate Planning Strategies
  - A prototype Example
- Aggregate Planning Problem: A Linear Programing Model
- Extensions

Introduction to Aggregate Planning

- ***Aggregate Planning*** or ***Sales and Operations Planning*** or ***Macro Planning***
  - Developing a top-down sales and operations plan for the entire firm
  - Or for a product line
  - An aggregate plan that all levels of firm as well as suppliers can work to

Introduction to Aggregate Planning

- **Goal:** To plan gross work force levels and set firm-wide production plans.
  - The number of employees the firm should retain,
  - The quantity and mix of products to be produced,
  - The amount of inventory to be kept, over a predetermined planning horizon. Typically, 6-24 months

- The information to be processed is aggregated by consolidating:
  - Similar items or services into product groups,
  - Different machines into machine centers,
  - Customers into market regions.

Aggregate Planning of Production and Capacity

- Suppose that $D_1, D_2, \ldots, D_T$ are the forecasts of demand for aggregate units over the planning horizon (T periods).

- The problem is to determine both work force levels (W_t) and production levels (P_t) to minimize total costs over the T period planning horizon.

Aggregate Planning of Prod. & Capacity:
Aggregate Unit

- Concept is predicated on the idea of an "*aggregate unit*" of production.
  - actual units
  - weight (tons of steel)
  - volume (gallons of gasoline)
  - time (worker-hours)
  - dollars of sales

- **Example:** Consider a beer company producing nonalcoholic, light, regular, and strong types of beer in draft, can, and bottle. Aggregate demand for the company can be expressed in terms of **liters**!

Aggregate Planning of Prod. & Capacity:
Hierarchical Production Planning (HPP)

- ***Items***
  - The final products to be delivered to the customer.
  - An item is often referred to as an SKU (stock-keeping unit) and represents the finest level of detail in the product categories.
- ***Families***
  - A group of items that share a common manufacturing setup cost.
- ***Types***
  - Groups of families with production quantities that are determined by a single aggregate production plan.

Aggregate Planning of Prod. & Capacity:
Important Issues

- ***Smoothing.*** Refers to the costs and disruptions that result from making changes from one period to the next.
- ***Bottleneck Planning.*** Problem of meeting peak demand because of capacity restrictions.
- ***Planning Horizon.*** Assumed given (T), but what is "right" value? Rolling horizons and end-of-horizon effect are both important issues.
- ***Treatment of Demand.*** Assume demand is known. Ignore uncertainty to focus on the predictable/systematic variations in demand, such as seasonality.

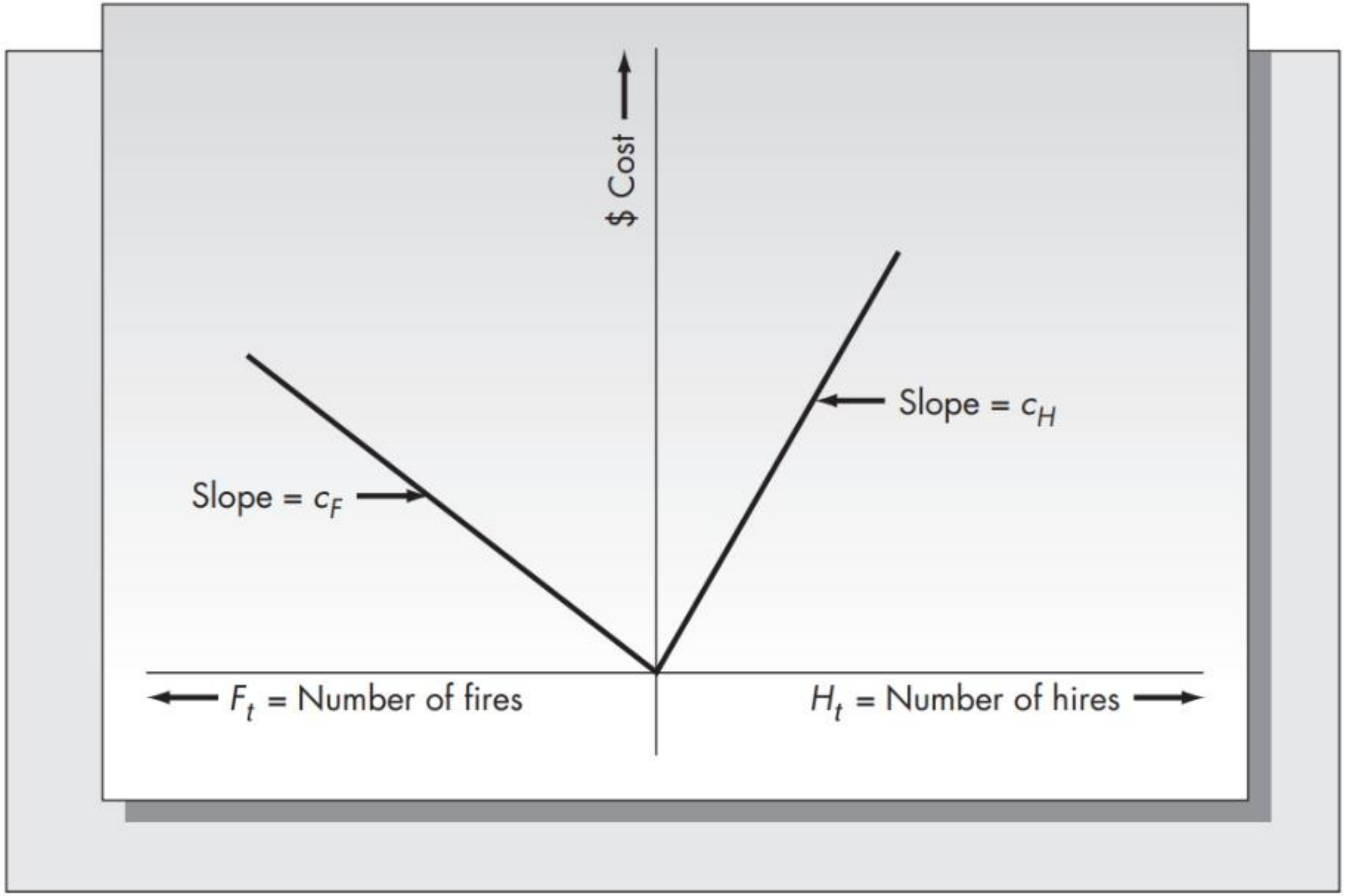
Aggregate Planning of Prod. & Capacity:
Relevant Costs

- ***Smoothing Costs***
  - The costs that accrue as a result of change in workforce or production level from one period to the next. For example, cost of changing the size of workforce (hiring or lay off)
  - For hiring new workers: costs related to:
    - Advertising positions,
    - Interviewing candidates,
    - Training new hires,
  - In case of firing: Severance pays, etc.
  - In reality, these costs can be asymmetric and non-linear because
    - The cost of increasing the production rate may be different from that of decreasing it;
    - The marginal costs may increase with the magnitude of the changes.
  - However, we will use a linear function of the number of employees that are hired or fired.
  - A typical cost function for change in the workforce size in Figure 3-4.

Aggregate Planning of Prod. & Capacity:

Relevant Costs

FIGURE 3–4
Cost of changing the size of the workforce



Aggregate Planning of Prod. &Capacity:
Relevant Costs

- ***Holding Costs***
  - The cost of the capital tied up in inventory plus costs due to insurance, storage, etc.
  - Linear in the number of units being held at a particular point in time, defined as "$/unit/period".
  - It is charged at the end of the period.

- ***Shortage Costs***
  - Cost of demand exceeding stock on hand.
  - These costs arise from opportunity costs of lost sales or customer goodwill.
  - At the Aggregate Planning level, usual assumption is that backordering does not occur. If does occur, parameter usually defined as "$/unit backordered/period".

Aggregate Planning of Prod. & Capacity:
Relevant Costs

- ***Regular -Time Costs***
  - Involve the production cost during the usual working hours. They include:
    - The actual payroll costs of regular employees working on regular time;
    - The direct and indirect cost of materials;
    - Other manufacturing expenses like overheads and employee benefits.

  - Regular payroll costs are a “sunk cost” when all production is carried out on regular time.
  - If there is no overtime or worker idle time then regular payroll costs will not be considered in comparison of different strategies.

Aggregate Planning of Prod. & Capacity:
Relevant Costs

- ***Overtime and Subcontracting Costs***
  - As the production rate increases beyond a certain point, the operation must be run at overtime premium rates.
  - Costs per unit rise sharply because of premiums and reduced efficiency when overtime is used.
  - Subcontracting refers to the production of items by an outside supplier.
  - These costs are generally assumed be linear.

- ***Other Costs***: idle time cost, etc.

Aggregate Planning of Prod. & Capacity:
Competing objectives

- **To react quickly to anticipated changes in demand**. This would require making frequent and potentially large changes in the size of the workforce. It may be cost effective.

- **To adopt the objective of retaining a stable workforce**. This usually results in large buildups of inventory during the periods of low demand.

- **To develop a plan that maximizes profit, subject to constraints on resources**.

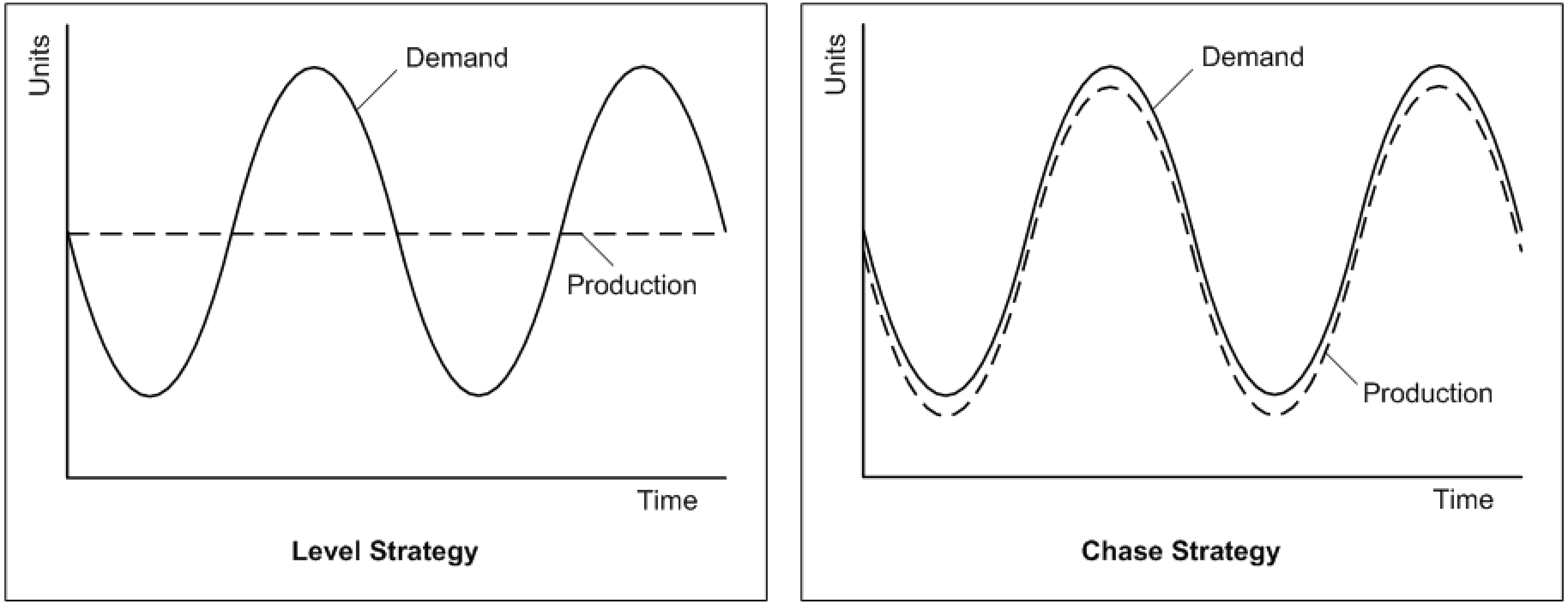
Aggregate Planning of Prod. & Capacity: Strategies

- **Level strategy**
  - This maintains constant production levels or workforce levels over the planning horizon.
  - It keeps the workforce adjustment low but may involve excessive inventory.

- **Chase strategy**
  - It adjusts the production rates or workforce levels to match the forecasted demand over the planning horizon.
  - It keeps inventory low but may involve excessive workforce adjustment.

- **Mixed strategy**
  - Various compromise strategies exist between the two extremes.

Aggregate Planning of Prod. & Capacity:
Strategies



Aggregate Planning of Prod. & Capacity:
Prototype Example

- The washing-machine plant is interested in determining work force and production levels for the next 8 months.

- **Forecasted demands for Jan-Aug**: 420, 280, 460, 190, 310, 145, 110, 125

- Starting inventory at the end of December is 200 and the firm would like to have 100 units on hand at the end of August.

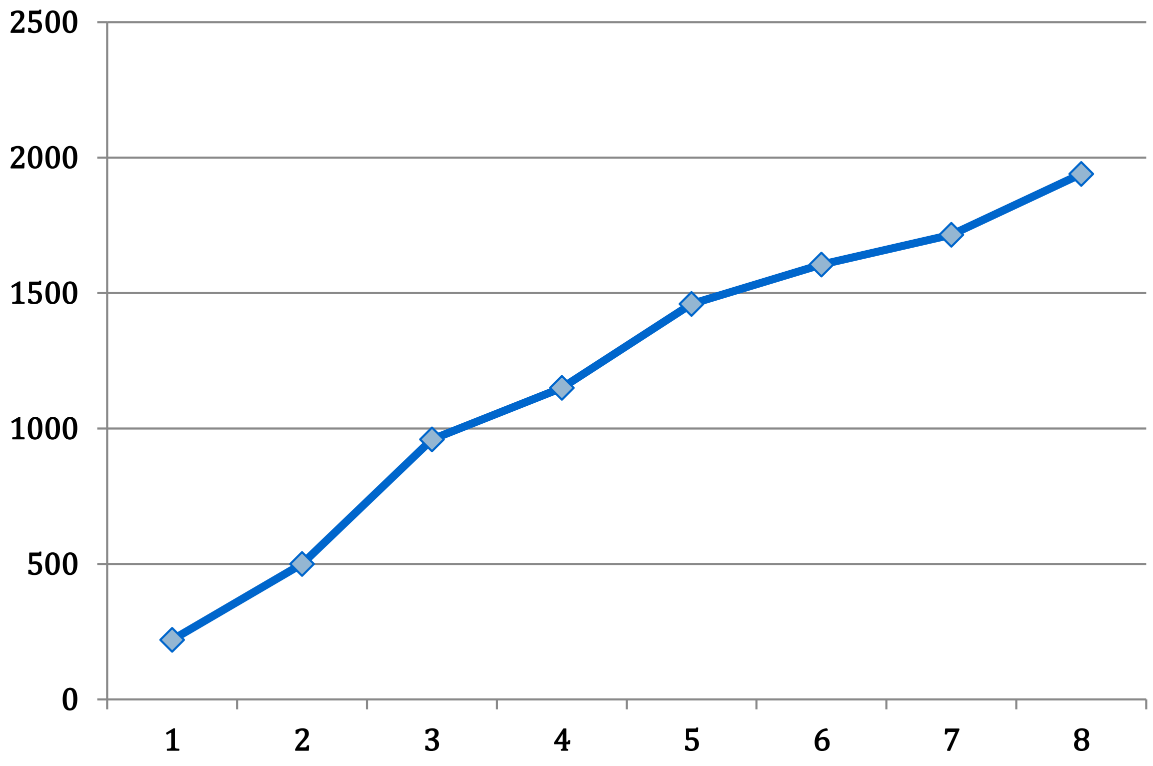
- Find monthly production levels.

Step 1: Determining Net Demand

Month	Forecasted Demand	Net Predicted Demand	Cumulative Net Demand
1 (Jan)	420	220	220
2 (Feb)	280	280	500
3 (Mar)	460	460	960
4 (Apr)	190	190	1150
5 (May)	310	310	1460
6 (Jun)	145	145	1605
7 (Jul)	110	110	1715
8 (Aug)	125	225	1940

Step 2: Graph Cumulative Net Demand to Find Plans Graphically

Month	Cumulated Net Demand
1 (Jan)	220
2 (Feb)	500
3 (Mar)	960
4 (Apr)	1150
5 (May)	1460
6 (Jun)	1605
7 (Jul)	1715
8 (Aug)	1940



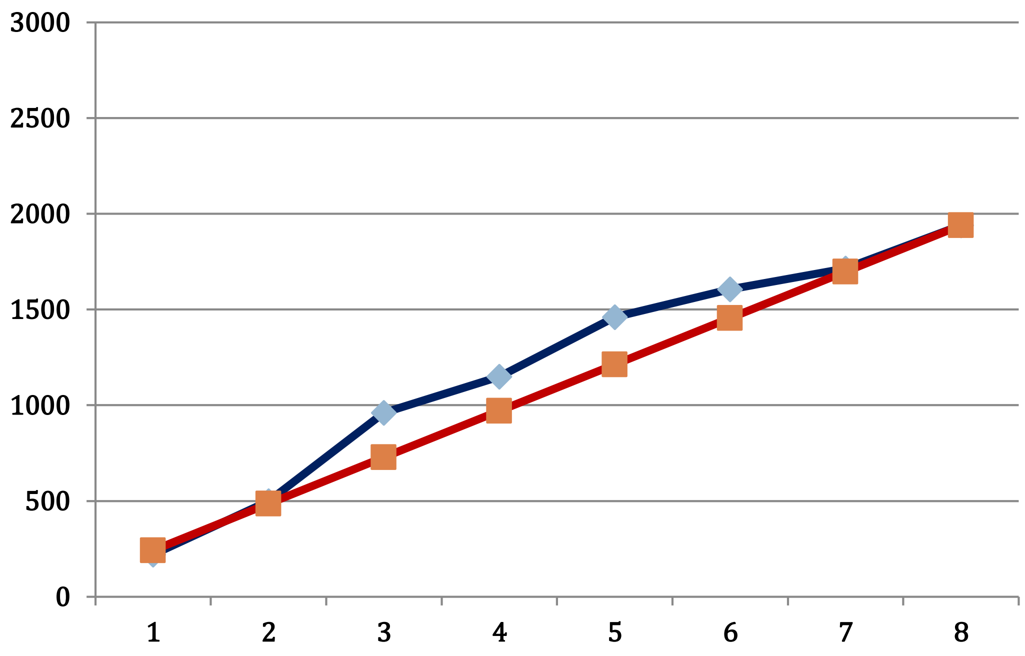
Constant Production Plan

- Suppose that we are interested in determining a production plan that doesn't change the size of the workforce over the planning horizon.

- Draw a straight line from origin to 1940 units in month 8.

- The slope of the line is the number of units to produce each month ($1940/8 = 242.2$ or 243 /month).

- **But, there are stockouts!**

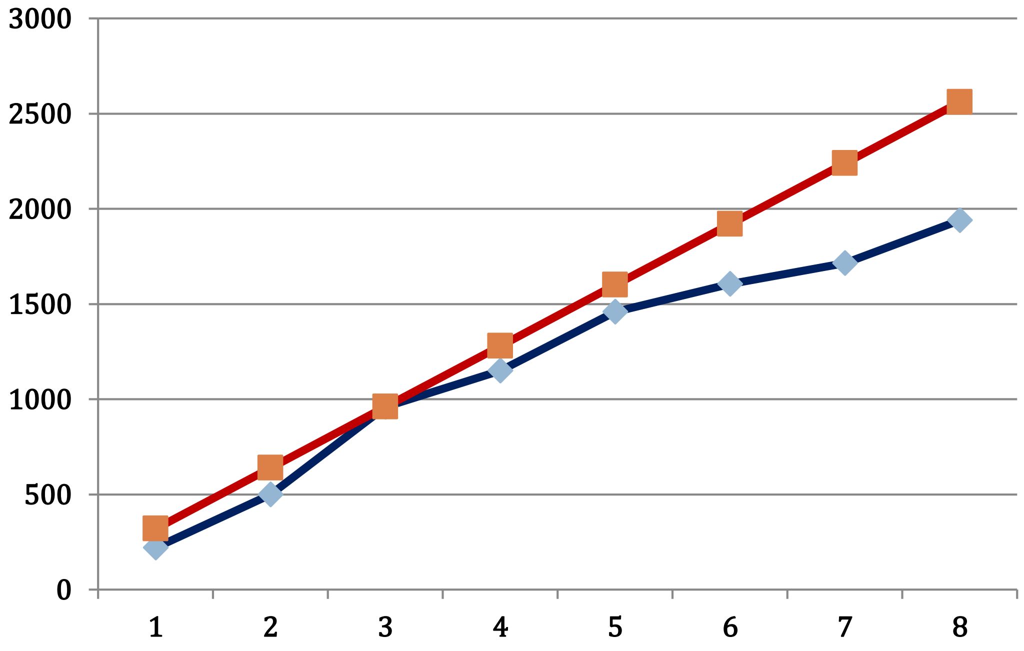


How to find a constant production plan with no stock-outs?

- Draw a straight line that goes through the origin and lies completely above the cumulative net demand curve:

The net demand curve is crossed at period 3.

So monthly production is 960/3 = 320.



Inventory based on a monthly production of 320

Month	Cumulative Net Demand	Cumulative Production	Inventory
1 (Jan)	220	320	100
2 (Feb)	500	640	140
3 (Mar)	960	960	0
4 (Apr)	1150	1280	130
5 (May)	1460	1600	140
6 (Jun)	1605	1920	315
7 (Jul)	1715	2240	525
8 (Aug)	1940	2560	620

But may not be realistic!

- **Why?**
  - It may not be possible to achieve the production level of 320 units/month with an integer number of workers
  - Since not all months have the same number of workdays, a constant production level may not translate to the same number of workers each month

- **Finding *K***
  - Suppose we are told that, over a period of 40 days, the plant had 38 workers who produced 520 units. It follows that:

$$K = 520/(38*40) = 0.3421$$ (quantity of aggregate unit produced by one man-day)

Constant Work Force Plan

Critical month is March.

Cum. net demand thru March = 960

Cum # of working days = 22 + 16 + 23 = 61

Find 960 / 61 = 15.7377 units/day

which implies 15.7377 /0.3421 = 46 workers required

Month	# wk days	Prod level	Cum Prod	Cum Net D	End Inv
Jan	22	346	346	220	126
Feb	16	252	598	500	98
Mar	23	362	960	960	0
Apr	20	315	1275	1150	125
May	21	330	1605	1460	145
Jun	22	346	1951	1605	346
Jul	21	330	2281	1715	566
Aug	22	346	2627	1940	687

Addition of Costs

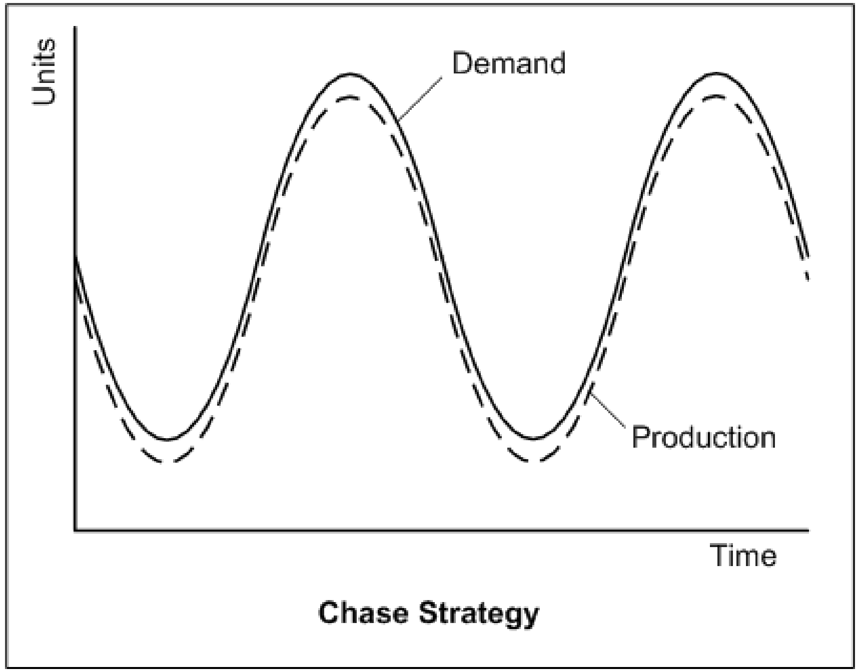
- Holding Cost (per unit per month): $8.50
- Hiring Cost per worker: $800
- Firing Cost per worker: $1,250
- Payroll Cost: $75/worker/day
- Shortage Cost: $50/unit short/month

Cost Evaluation of Constant Work-Force Plan

- Assume that the work force at end of Dec was 40.
- **Cost to hire** 6 workers: $6 \times 800 = \$4800$
- **Inventory Cost:**
  - Accumulate ending inventory: $(126 + 98 + 0 + \ldots + 687) = 2093.$
  - $\text{Add in 100 units netted out in Aug} = 2193.$
  - $\text{Hence Inventory Cost} = 2193 \times 8.5 = \$18{,}640.50$
- **Payroll cost**: $(\$75/\text{worker}/\text{day}) \times (46 \text{ workers}) \times (167\text{days}) = \$576{,}150$
- **Total cost of the plan**: $\$576{,}150 + \$18{,}640.50 + \$4800 = \mathbf{\$599{,}590.50}$

Zero Inventory Plan (Chase Strategy)

- Here the idea is to change the workforce each month in order to reduce ending inventory to nearly zero. Match the workforce with monthly demand as closely as possible.



Determine the Zero Inventory Plan (Chase Strategy)

Month A	# wk days B	Number of units produced per worker $C = B \times 0.3421$	Forecasted Net Demand D	Min # of workers required $E = D/C$ (round up)
Jan	22	7.5262	220	30
Feb	16	5.4736	280	52
Mar	23	7.8683	460	59
Apr	20	6.8420	190	28
May	21	7.1841	310	44
Jun	22	7.5262	145	20
Jul	21	7.1841	110	16
Aug	22	7.5262	225	30

Inventory based on chase plan

	# of worker	Hire	Fire	Prod / worker	Prod	Cum Prod	Cum D	End Inv
1	30		10	7.5262	226	226	220	6
2	52	22		5.4736	285	511	500	11
3	59	7		7.8683	464	975	960	15
4	28		31	6.8420	192	1167	1150	17
5	44	16		7.1841	316	1483	1460	23
6	20		24	7.5262	151	1634	1605	29
7	16		4	7.1841	115	1749	1715	34
8	30	14		7.5262	226	1975	1940	35

Modify the Zero-Inventory Plan

	# of worker	Hire	Fire	Prod / worker	Prod	Cum Prod	Cum D	End Inv
1	30		10	7.5262	226	226	220	6
2	**<u>50</u>**	20		5.4736	274	500	500	0
3	59	9		7.8683	464	964	960	4
4	28		31	6.8420	192	1156	1150	6
5	**<u>43</u>**	15		7.1841	309	1465	1460	5
6	**<u>19</u>**		24	7.5262	143	1608	1605	3
7	**<u>15</u>**		4	7.1841	108	1716	1715	1
8	30	15		7.5262	226	1942	1940	2

Cost Evaluation of Zero-Inventory Plan

- **Cost to hire** $(20+9+15+15)$ workers: $59 \times 800 = \$47{,}200$

- **Cost to fire** $(10+31+24+4)$ workers: $69 \times 1250 = \$86{,}250$

- **Inventory Cost**: accumulate ending inventory: $(6+0+4+6+5+3+1+2) = 27$
  - Add in 100 units netted out in Aug = 127.
  - Hence Inventory Cost $= 127 \times 8.5 = \$1{,}079.50$

- **Payroll cost:** $(\$75/\text{worker}/\text{day}) \times \sum[(\#\text{ workers each month })(\#\text{ wk days each month})] =$ $\$425{,}475$

- **Total cost of the plan:** $\$47{,}200 + \$86{,}250 + \$1079.50 + \$425{,}475 =$ **$560,004.50**

Aggregate planning problem:
A Linear Programming Model

- **Cost parameters and given values:**

The cost parameters are all time dependent.

These parameters can be defined for each time period, e.g. c_{Ht}: cost of hiring at period t

c_H = Cost of hiring one worker,
c_F = Cost of firing one worker,
c_I = Cost of holding one unit of stock for one period,
c_R = Cost of producing one unit on regular time,
c_O = Incremental cost of producing one unit on overtime,
c_U = Idle cost per unit of production,
c_S = Cost to subcontract one unit of production,
n_t = Number of production days in period t,
K = Number of aggregate units produced by one worker in one day,
I_0 = Initial inventory on hand at the start of the planning horizon,
W_0 = Initial workforce at the start of the planning horizon,
D_t = Forecast of demand in period t.

Solving aggregate planning problem by Linear Programming

- **Decision variables:**

W_t = Workforce level in period t,
P_t = Production level in period t,
I_t = Inventory level in period t,
H_t = Number of workers hired in period t,
F_t = Number of workers fired in period t,
O_t = Overtime production in units,
U_t = Worker idle time in units ("undertime"),
S_t = Number of units subcontracted from outside.

Solving aggregate planning problem by Linear Programming

- **Obtaining the overtime and idle time variables:**
  - Number of units produced by one worker in period *t*: Kn_t
  - Number of units produced by the entire workforce in period *t*: Kn_tW_t
  - If $P_t > Kn_tW_t$ then the number of units produced in overtime: $O_t = P_t - Kn_tW_t$
  - If $P_t < Kn_tW_t$: workforce is producing less than what should be on regular time. This means there is idle time. Idle time is measured in units of production rather than time and it is: $U_t = Kn_tW_t - P_t$

Solving aggregate planning problem by Linear Programming

- **Problem Constraints: (in addition to non-negative constraint)**

1. Conservation of workforce constraints.

$$\underset{\substack{\text{Number}\\ \text{of workers}\\ \text{in } t}}{W_t} = \underset{\substack{\text{Number}\\ \text{of workers}\\ \text{in } t-1}}{W_{t-1}} + \underset{\substack{\text{Number}\\ \text{hired}\\ \text{in t}}}{H_t} - \underset{\substack{\text{Number}\\ \text{fired}\\ \text{in } t}}{F_t} \qquad \text{for } 1 \le t \le T.$$

2. Conservation of units constraints.

$$\underset{\substack{\text{Inventory}\\ \text{in } t}}{I_t} = \underset{\substack{\text{Inventory}\\ \text{in } t-1}}{I_{t-1}} + \underset{\substack{\text{Number}\\ \text{of units}\\ \text{produced}\\ \text{in } t}}{P_t} + \underset{\substack{\text{Number}\\ \text{of units}\\ \text{subcontracted}\\ \text{in } t}}{S_t} - \underset{\substack{\text{Demand}\\ \text{in } t}}{D_t} \qquad \text{for } 1 \le t \le T.$$

3. Constraints relating production levels to workforce levels.

$$\underset{\substack{\text{Number}\\ \text{of units}\\ \text{produced}\\ \text{in } t}}{P_t} = \underset{\substack{\text{Number}\\ \text{of units}\\ \text{produced}\\ \text{by regular}\\ \text{workforce}\\ \text{in } t}}{Kn_tW_t} + \underset{\substack{\text{Number}\\ \text{of units}\\ \text{produced}\\ \text{on over-}\\ \text{time in } t}}{O_t} - \underset{\substack{\text{Number}\\ \text{of units}\\ \text{of idle}\\ \text{production}\\ \text{in } t}}{U_t} \qquad \text{for } 1 \le t \le T.$$

The LP model

$$\text{Minimize} \sum_{t=1}^{T} (c_H H_t + c_F F_t + c_I I_t + c_R P_t + c_O O_t + c_U U_t + c_S S_t)$$

subject to

$$W_t = W_{t-1} + H_t - F_t \qquad \text{for } 1 \le t \le T \quad \text{(conservation of workforce)}, \tag{A}$$

$$P_t = K n_t W_t + O_t - U_t \qquad \text{for } 1 \le t \le T \quad \text{(production and workforce)} \tag{B}$$

$$I_t = I_{t-1} + P_t + S_t - D_t \qquad \text{for } 1 \le t \le T \text{ (inventory balance)}, \tag{C}$$

$$H_t, F_t, I_t, O_t, U_t, S_t, W_t, P_t \ge 0 \quad \text{(nonnegativity)}, \tag{D}$$

Example

$$\text{Minimize}\left(8.50\sum_{i=1}^{8} I_i + 800\sum_{i=1}^{8} H_i + 1250\sum_{i=1}^{8} F_i + 75\sum_{i=1}^{8} n_i W_i\right)$$

s.t.

$$W_i - W_{i-1} - H_i + F_i = 0$$

$$P_i - I_i + I_{i-1} = D_i$$

$$P_i - 0.3421 n_i W_i = 0$$

$$W_i, P_i, H_i, F_i, I_i \geq 0$$

$$I_0 = 200$$

$$I_8 = 100$$

$$W_0 = 40$$

Solution from Linear Programming

Month	# of Workers	Hire	Fire	Production	Cumulative production	End month inventory
Jan	46	6		346	346	126
Feb	46			252	598	98
Mar	46			362	960	0
Apr	36		10	246	1206	56
May	36			259	1465	5
Jun	22		14	166	1631	26
Jul	22			158	1789	74
Aug	22			166	1955	15

Value of the objective function (total cost) is $467,450

Solution from Linear Programming

- **Rounding of Variables:**

- Rounding off the fractional values of variables may lead to an infeasible solution

- **Conservative rounding:**
  - Round the number of workers needed at period t (W_t) to the next larger integer
  - Then the other variables $(H_t, F_t, \text{and } P_t)$ can be found considering the cost of resulting plan

Extensions

- **Minimum buffer inventory:** to deal with uncertainty
  - This constraint is included in the model:

$$I_t \geq B_t \qquad \text{for } 1 \leq t \leq T$$

- **Capacity constraint:**

$$P_t \leq C_t \qquad for\, 1 \leq t \leq T$$

- **Excess or shortage in inventory:**

$$I_t = I_t^+ - I_t^- \quad , \; I_t^+ \geq 0 \;\; , \;\; I_t^- \geq 0$$

  - The holding cost will be charged against I_t^+ and the penalty (shortage) cost for I_t^-.

Reading

- Course notes
- Chapter Overview
- Textbook Sections 3.1-3.6
- Skim Sections 3.7-3.13