Introduction and Course Outline

Lecture 1

Why Study Algorithms?

- Important for all other branches of computer science (networks, database, bioinformatics, public key cryptogrpaphy, computer graphics)
- Plays key role in modern technological innovation
- Provides novel “lens” on processes outside of computer science and technology
  - Quantum mechanics, economic markets, evolution
- Challenging (i.e., good for brain)
- fun

Why Study Algorithms?

- plays a key role in modern technological innovation

“Everyone knows Moore’s Law a prediction made in 1965 by Intel cofounder Gordon Moore that the density of transistors in integrated circuits would continue to double every 1 to 2 years.... in many areas, performance gains due to improvements in algorithms have vastly exceeded even the dramatic performance gains due to increased processor speed.”

- Excerpt from *Report to the President and Congress: Designing a Digital Future,* December 2010 (page 71).

Algorithms as technology

- Insertion sort runs in $O(n^2)$ time
- Merge sort runs in $O(n \lg n)$ time
- Suppose computer A executes 1 billion instructions per second
- Suppose computer B executes 10 million instructions per second
- Computer A is 100 times faster than Computer B
- Suppose input size = 1 million = 10^6
- To make difference more dramatic, suppose insertion sort is coded by a very good programmer so leading constants are small = $2n^2$
- Suppose merge sort is coded by average programmer so constants are large = $50 n \lg n$

Algorithms as technology

- Run insertion sort on faster computer A (1 billion instructions per sec)
- Run merge sort on slower computer B (10 million instructions per second)
- Insertion sort number of instructions = $2n^2$
- Merge sort number of instructions = $50\ n \lg n$
- Input size = $n = 10^6$
- What is running time in seconds for both programs?

Algorithms as technology

- Running time for insertion sort = 2 * $(10^6)^2$ / 10^9 instructions per second
- = 2000 seconds
- Running time for merge sort = 50 * $(10^6 \lg 10^6)$/ 10^7 instructions per second
- = 100 seconds

- Merge sort runs 20 times faster than insertion sort even if we run it on 100 times slower computer

Course Topics

- Vocabulary for design and analysis of algorithms
- Divide and conquer design paradigm
- Randomization in algorithms
- Dynamic programming design paradigm
- Greedy algorithms design paradigm
- Graph algorithms

Skills you will learn

- Become a better programmer
- Sharpen your mathematical and analytical skills
- Start thinking algorithmically
- Literacy with computer science’s “greatest hits”
- Ace your technical interviews

The Algorithm Designer’s Mantra

- “Perhaps the most important principle for the good algorithm designer is to refuse to be content”
  - Aho, Hopcroft and Ullman, The Design and Analysis of Computer Algorithms, 1974

Can we do better?

[than the obvious method]

Analyzing Algorithms

- Has come to mean predicting the resources that the algorithm requires
- Usually computational time is resource of primary importance
- Aims to identify best choice among several alternate algorithms
- Requires an agreed-upon “model” of computation
- Shall use a generic, one-processor, random-access machine (RAM) model of computation

Random-Access Machine

- Instructions are executed one after another (no concurrency)
- Admits commonly found instructions in “real” computers, data movement operations, control mechanism
- Uses common data types (integer and float)

Algorithm Analysis

- Time resource requirement depends on ***input size***
- ***Input size*** depends on problem being studied; frequently, this is the number of items in the input
- Running time: number of primitive operations or “steps” executed for an input
- Assume constant amount of time for each line of pseudocode

Algorithms Analysis Guiding Principles

Guiding Principle #1

- “worst – case analysis” : our running time bound holds for every input of length n.
  -- Particularly appropriate for “general--purpose” routines

- As Opposed to
  -- --”average--case” analysis
  -- --benchmarks

Requires domain Knowledge

- BONUS : worst case usually easier to analyze.

Guiding Principle #2

- Won’t pay much attention to constant factors, lower-order terms
  Justifications

1. Way easier
2. Constants depend on architecture / compiler / programmer
3. Lose very little predictive power
   (as we’ll see)

Guiding Principle #3

- Asymptotic Analysis : focus on running time for large input sizes n

  Eg : $\underbrace{6\, n \lg n}_{\text{Merge sort}}$ "better than" $\underbrace{2n^2}_{\text{Insertion sort}}$

Justification: Only big problems are interesting!

What Is a “Fast” Algorithm?

This Course : adopt these three biases as guiding principles

fast algorithm $\approx$ worst--case running time grows slowly with input size

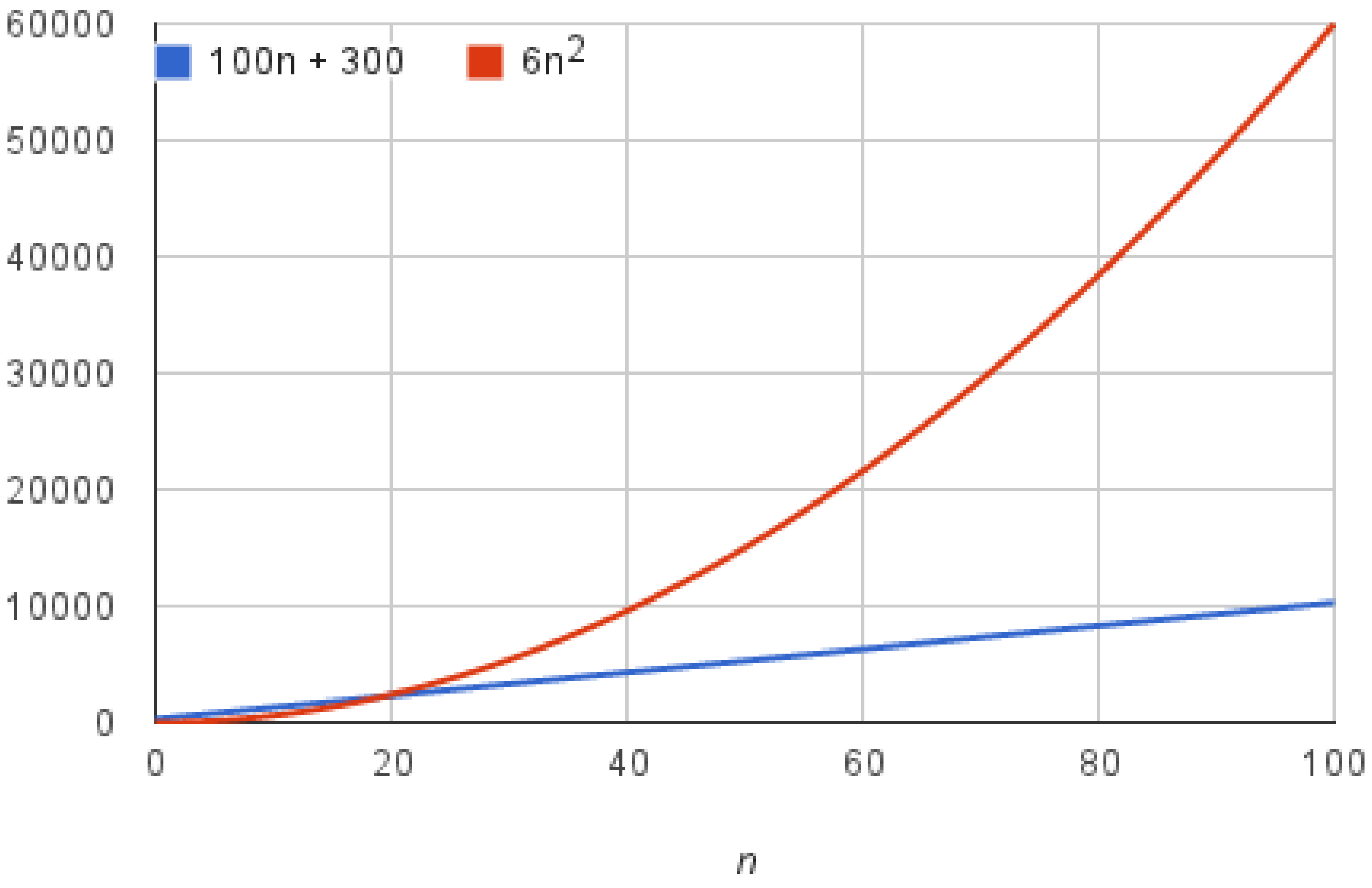
Usually : want as close to linear (O(n)) as possible

Rate of Growth

- An algorithm running on an input of size *n*, takes $6n^2+100n+300$ machine instructions.

- The $6n^2$ term becomes larger than the remaining terms, $100n+300$, once *n* becomes large enough

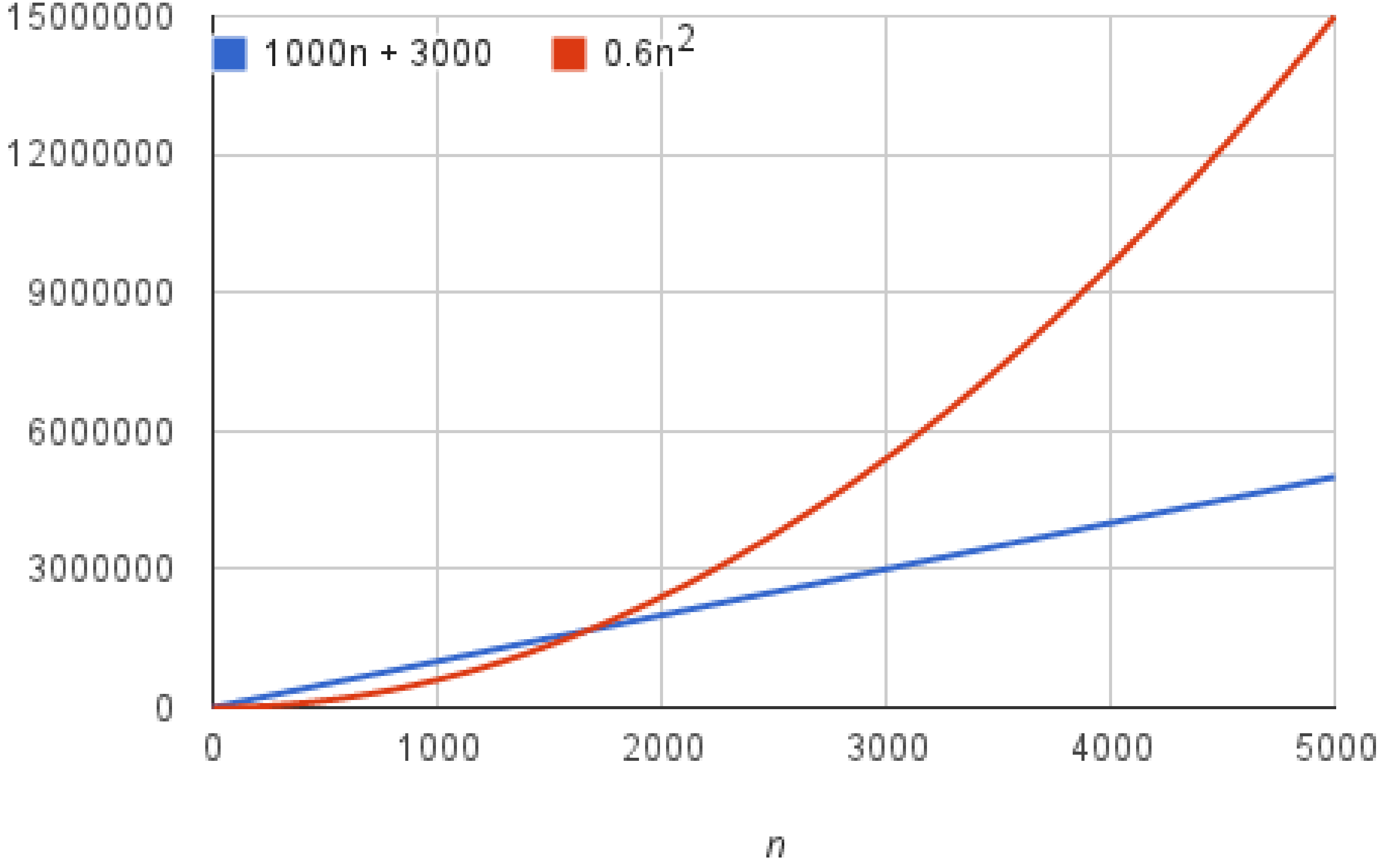
Comparison of $6n^2$ and $100n+300$



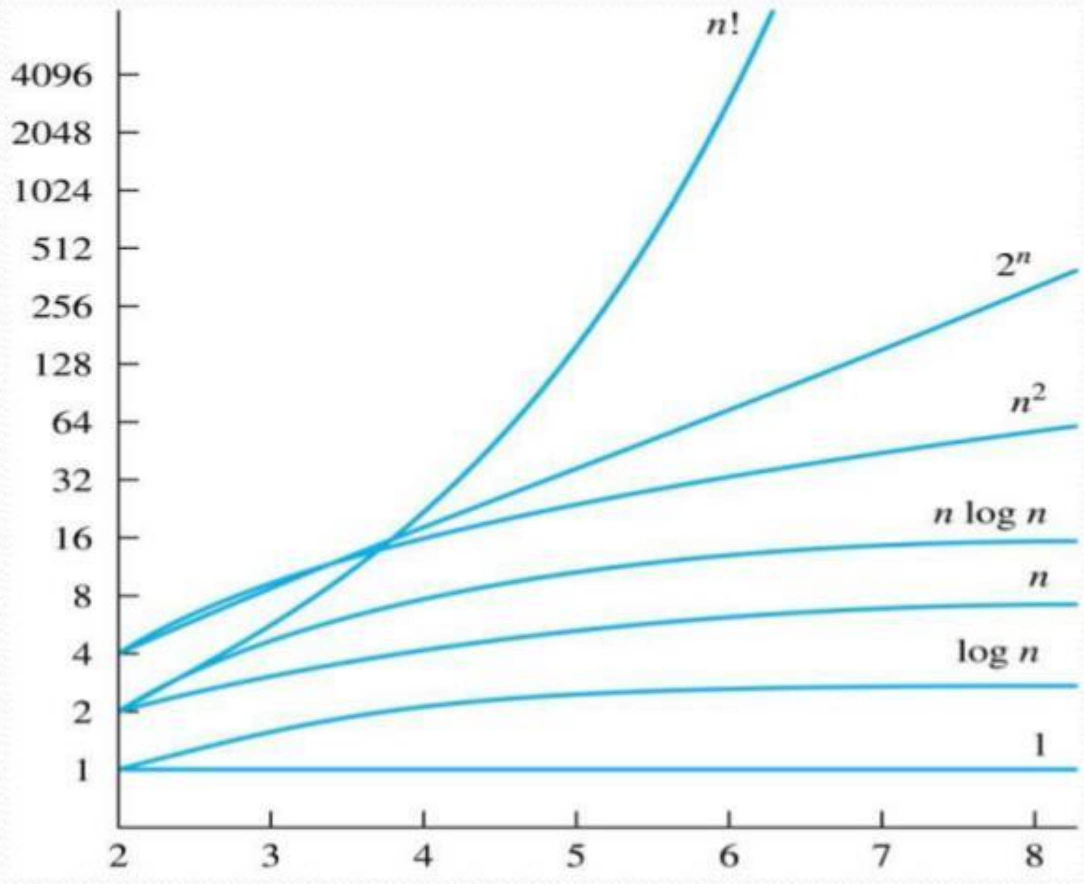
Rate of Growth

- It doesn't really matter what coefficients we use; as long as the running time is an^2+bn+c for some numbers $a > 0$, b, and c, there will always be a value of n for which an^2 is greater than $bn + c$, and this difference increases as n increases.

Comparison of $0.6n^2$ and $1000n+3000$



Display of Growth of Functions



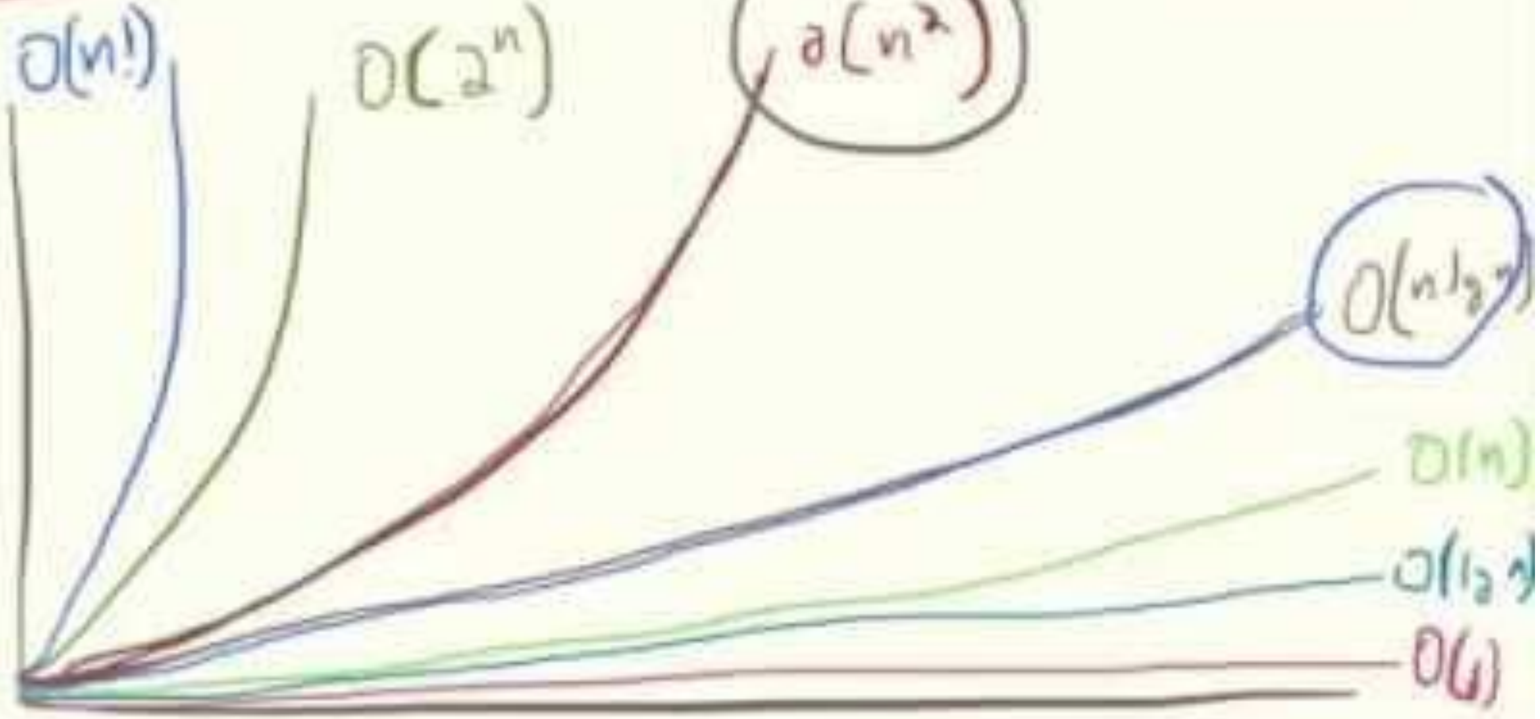
Note the difference in behavior of functions as *n* gets larger

Growth of Functions

1   3.32   10   33.22   100   1024   3MM

O(1), lg n, n, n lg n, n^2, 2^n, n!

n=10



CONSIDER TWO ALGORITHMS

- Algo X
- Input: A[1...n]
- Pseudocode:
- for i=1 to n
-     for j=1 to n
-         LightProcess(A[i])

- Cost of LightProcess:
-         5 msec

- Algo Y
- Input: A[1...n]
- Pseudocode:
- for i=1 to n
-         HeavyProcess(A[i])

- Cost of HeavyProcess:
-         5000 msec

CONSIDER TWO ALGORITHMS

- Which of the two algorithms is more efficient?

    - Algo X or Algo Y

- And WHY?

TIME TAKEN BY ALGO X

- ? Input: A[1...10]
- ? Pseudocode:
- ? for i=1 to n
- ? for j=1 to n
- ? LightProcess(A[i])
- ? **??? msec**

TIME TAKEN BY ALGO Y

- ? **Input: A[1...10]**
- ? **Pseudocode:**
- ? **for i=1 to n**
- ? **HeavyProcess(A[i])**
- ? **??? msec**

ALGO X VS. ALGO Y

TIME TAKEN BY ALGO X

- Input: A[1...10]
- Pseudocode:
- for i=1 to n
-     for j=1 to n
-         LightProcess(A[i])

When n = 10, 500 msec

TIME TAKEN BY ALGO Y

- **Input: A[1...10]**
- **Pseudocode:**
- **for i=1 to n**
-     **HeavyProcess(A[i])**

When n = 10, 50000 msec

ALGO X VS. ALGO Y

TIME TAKEN BY ALGO X

- Input: A[1...1000]
- Pseudocode:
- for i=1 to n
-     for j=1 to n
-         LightProcess(A[i])

When n = 1000, 5,000,000 msec

TIME TAKEN BY ALGO Y

- **Input: A[1...1000]**
- **Pseudocode:**
- **for i=1 to n**
- **HeavyProcess(A[i])**

When n = 1000, 5,000,000 msec

ALGO X VS. ALGO Y

TIME TAKEN BY ALGO X

- Input: A[1...10000]
- Pseudocode:
- for i=1 to n
-     for j=1 to n
-         LightProcess(A[i])

When n = 10000, 500,000,000 msec

TIME TAKEN BY ALGO Y

- **Input: A[1...10000]**
- **Pseudocode:**
- **for i=1 to n**
- **HeavyProcess(A[i])**

When n = 10000, 50,000,000 msec

ALGO X VS. ALGO Y

TIME TAKEN BY ALGO X

- Input: A[1...100000]
- Pseudocode:
- for i=1 to n
-     for j=1 to n
-         LightProcess(A[i])

When n = 100000, 50,000,000,000 msec

TIME TAKEN BY ALGO Y

- **Input: A[1...100000]**
- **Pseudocode:**
- **for i=1 to n**
- **HeavyProcess(A[i])**

When n = 100000, 500,000,000 msec

ALGO X VS. ALGO Y

How big is running time of exponential time algorithm ?