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[187. Simplicius, 170 n. Simultaneity, 116. Space, 73, 88, 103, 112 ff., 130. absolute and relative, 146, 159. antinomies of, 155 ff. perception of, 68. of perspectives, 88 ff. private, 89, 90. of touch and sight, 78, 113. Spencer, 4, 12, 236. Spinoza, 46, 166. Stadium, Zeno's argument of, 134 n., 175 ff. Subject-predicate, 45. Synthesis, 157, 185. Tannery, Paul, 169 n. Teleology, 223. Testimony, 67, 72, 82, 87, 96, 212. Thales, 3. Thing-in-itself, 75, 84. Things, 89 ff., 104 ff., 213. Time, 103, 116 ff., 130, 155 ff., 166, 215. absolute or relative, 146. local, 103. private, 121. Uniformities, 217. Unity, organic, 9. Universal and particular, 39 n. Volition, 223 ff. Whitehead, vi, 207. Wittgenstein, vii, 208 n. Worlds, actual and ideal, 111. possible, 186. private, 88. Zeller, 173. Zeno, 129, 134, 136, 165 ff. [1] Delivered as Lowell Lectures in Boston, in March and April 1914. [2] London and New York, 1912 (Home University Library). [3] The first volume was published at Cambridge in 1910, the second in 1912, and the third in 1913. [4] Appearance and Reality, pp. 3233. [5] Creative Evolution, English translation, p. 41. [6] Cf. Burnet, Early Greek Philosophy, pp. 85 ff. [7] Introduction to Metaphysics, p. 1. [8] Logic, book iii., chapter iii., 2. [9] Book iii., chapter xxi., 3. [10] Or rather a propositional function. [11] The subject of causality and induction will be discussed again in Lecture VIII. [12] See the translation by H. S. Macran, Hegel's Doctrine of Formal Logic, Oxford, 1912. Hegel's argument in this portion of his Logic depends throughout upon confusing the is of predication, as in Socrates is mortal, with the is of identity, as in Socrates is the philosopher who drank the hemlock. Owing to this confusion, he thinks that Socrates and mortal must be identical. Seeing that they are different, he does not infer, as others would, that there is a mistake somewhere, but that they exhibit identity in difference. Again, Socrates is particular, mortal is universal. Therefore, he says, since Socrates is mortal, it follows that the particular is the universaltaking the is to be throughout expressive of identity. But to say the particular is the universal is self-contradictory. Again Hegel does not suspect a mistake but proceeds to synthesise particular and universal in the individual, or concrete universal. This is an example of how, for want of care at the start, vast and imposing systems of philosophy are built upon stupid and trivial confusions, which, but for the almost incredible fact that they are unintentional, one would be tempted to characterise as puns. [13] Cf. Couturat, La Logique de Leibniz, pp. 361, 386. [14] It was often recognised that there was some difference between them, but it was not recognised that the difference is fundamental, and of very great importance. [15] Encyclopdia of the Philosophical Sciences, vol. i. p. 97. [16] This perhaps requires modification in order to include such facts as beliefs and wishes, since such facts apparently contain propositions as components. Such facts, though not strictly atomic, must be supposed included if the statement in the text is to be true. [17] The assumptions made concerning time-relations in the above are as follows: [18] The above paradox is essentially the same as Zeno's argument of the stadium which will be considered in our next lecture. [19] See next lecture. [20] Monist, July 1912, pp. 337341. [21] Le continu mathmatique, Revue de Mtaphysique et de Morale, vol. i. p. 29. [22] In what concerns the early Greek philosophers, my knowledge is largely derived from Burnet's valuable work, Early Greek Philosophy (2nd ed., London, 1908). I have also been greatly assisted by Mr D. S. Robertson of Trinity College, who has supplied the deficiencies of my knowledge of Greek, and brought important references to my notice. [23] Cf. Aristotle, Metaphysics, M. 6, 1080b, 18 sqq., and 1083b, 8 sqq. [24] There is some reason to think that the Pythagoreans distinguished between discrete and continuous quantity. G. J. Allman, in his Greek Geometry from Thales to Euclid, says (p. 23): The Pythagoreans made a fourfold division of mathematical science, attributing one of its parts to the how many, t? p?s??, and the other to the how much, t? p??????; and they assigned to each of these parts a twofold division. For they said that discrete quantity, or the how many, either subsists by itself or must be considered with relation to some other; but that continued quantity, or the how much, is either stable or in motion. Hence they affirmed that arithmetic contemplates that discrete quantity which subsists by itself, but music that which is related to another; and that geometry considers continued quantity so far as it is immovable; but astronomy (t?? sfa??????) contemplates continued quantity so far as it is of a self-motive nature. (Proclus, ed. Friedlein, p. 35. As to the distinction between t? p??????, continuous, and t? p?s??, discrete quantity, see Iambl., in Nicomachi Geraseni Arithmeticam introductionem, ed. Tennulius, p. 148.) Cf. p. 48. [25] Referred to by Burnet, op. cit., p. 120. [26] iv., 6. 213b, 22; H. Ritter and L. Preller, Historia Philosophi Grc, 8th ed., Gotha, 1898, p. 75 (this work will be referred to in future as R. P.). [27] The Pythagorean proof is roughly as follows. If possible, let the ratio of the diagonal to the side of a square be m/n, where m and n are whole numbers having no common factor. Then we must have m2 = 2n2. Now the square of an odd number is odd, but m2, being equal to 2n2, is even. Hence m must be even. But](59391.docx#chunk10804)

[the square of an even number divides by 4, therefore n2, which is half of m2, must be even. Therefore n must be even. But, since m is even, and m and n have no common factor, n must be odd. Thus n must be both odd and even, which is impossible; and therefore the diagonal and the side cannot have a rational ratio. [28] In regard to Zeno and the Pythagoreans, I have derived much valuable information and criticism from Mr P. E. B. Jourdain. [29] So Plato makes Zeno say in the Parmenides, apropos of his philosophy as a whole; and all internal and external evidence supports this view. [30] With Parmenides, Hegel says, philosophising proper began. Werke (edition of 1840), vol. xiii. p. 274. [31] Parmenides, 128 AD. [32] This interpretation is combated by Milhaud, Les philosophes-gomtres de la Grce, p. 140 n., but his reasons do not seem to me convincing. All the interpretations in what follows are open to question, but all have the support of reputable authorities. [33] Physics, vi. 9. 2396 (R.P. 136139). [34] Cf. Gaston Milhaud, Les philosophes-gomtres de la Grce, p. 140 n.; Paul Tannery, Pour l'histoire de la science hellne, p. 249; Burnet, op. cit., p. 362. [35] Cf. R. K. Gaye, On Aristotle, Physics, Z ix. Journal of Philology, vol. xxxi., esp. p. 111. Also Moritz Cantor, Vorlesungen ber Geschichte der Mathematik, 1st ed., vol. i., 1880, p. 168, who, however, subsequently adopted Paul Tannery's opinion, Vorlesungen, 3rd ed. (vol. i. p. 200). [36] Le mouvement et les partisans des indivisibles, Revue de Mtaphysique et de Morale, vol. i. pp. 382395. [37] Le mouvement et les arguments de Znon d'le, Revue de Mtaphysique et de Morale, vol. i. pp. 107125. [38] Cf. M. Brochard, Les prtendus sophismes de Znon d'le, Revue de Mtaphysique et de Morale, vol. i. pp. 209215. [39] Simplicius, Phys., 140, 28 D (R.P. 133); Burnet, op. cit., pp. 364365. [40] Op. cit., p. 367. [41] Aristotle's words are: The first is the one on the non-existence of motion on the ground that what is moved must always attain the middle point sooner than the end-point, on which we gave our opinion in the earlier part of our discourse. Phys., vi. 9. 939B (R.P. 136). Aristotle seems to refer to Phys., vi. 2. 223AB [R.P. 136A]: All space is continuous, for time and space are divided into the same and equal divisions. Wherefore also Zeno's argument is fallacious, that it is impossible to go through an infinite collection or to touch an infinite collection one by one in a finite time. For there are two senses in which the term infinite is applied both to length and to time, and in fact to all continuous things, either in regard to divisibility, or in regard to the ends. Now it is not possible to touch things infinite in regard to number in a finite time, but it is possible to touch things infinite in regard to divisibility: for time itself also is infinite in this sense. So that in fact we go through an infinite, [space] in an infinite [time] and not in a finite [time], and we touch infinite things with infinite things, not with finite things. Philoponus, a sixth-century commentator (R.P. 136A, Exc. Paris Philop. in Arist. Phys., 803, 2. Vit.), gives the following illustration: For if a thing were moved the space of a cubit in one hour, since in every space there are an infinite number of points, the thing moved must needs touch all the points of the space: it will then go through an infinite collection in a finite time, which is impossible. [42] Cf. Mr C. D. Broad, Note on Achilles and the Tortoise, Mind, N.S., vol. xxii. pp. 3189. [43] Op. cit. [44] Aristotle's words are: The second is the so-called Achilles. It consists in this, that the slower will never be overtaken in its course by the quickest, for the pursuer must always come first to the point from which the pursued has just departed, so that the slower must necessarily be always still more or less in advance. Phys., vi. 9. 239B (R.P. 137). [45] Phys., vi. 9. 239B (R.P. 138). [46] Phys., vi. 9. 239B (R.P. 139). [47] Loc. cit. [48] Loc. cit., p. 105. [49] Phil. Werke, Gerhardt's edition, vol. i. p. 338. [50] Mathematical Discourses concerning two new sciences relating to mechanics and local motion, in four dialogues. By Galileo Galilei, Chief Philosopher and Mathematician to the Grand Duke of Tuscany. Done into English from the Italian, by Tho. Weston, late Master, and now published by John Weston, present Master, of the Academy at Greenwich. See pp. 46 ff. [51] In his Grundlagen einer allgemeinen Mannichfaltigkeitslehre and in articles in Acta Mathematica, vol. ii. [52] The definition of number contained in this book, and elaborated in the Grundgesetze der Arithmetik (vol. i., 1893; vol. ii., 1903), was rediscovered by me in ignorance of Frege's work. I wish to state as emphatically as possiblewhat seems still often ignoredthat his discovery antedated mine by eighteen years. [53] Giles, The Civilisation of China (Home University Library), p. 147. [54] Cf. Principia Mathematica, 20, and Introduction, chapter iii. [55] In the above remarks I am making use of unpublished work by my friend Ludwig Wittgenstein. [56] Thus we are not using thing here in the sense of a class of correlated aspects, as we did in Lecture III. Each aspect will count separately in stating causal laws. [57] The above remarks, for purposes of illustration, adopt one of several possible opinions on each of several disputed points.](59391.docx#chunk10805)

[AN ESSAY ON THE FOUNDATIONS OF GEOMETRY By Bertrand A. W. Russell Fellow Of Trinity College, Cambridge 1897 CONTENTS INTRODUCTION. OUR PROBLEM DEFINED BY ITS RELATIONS TO LOGIC, PSYCHOLOGY AND MATHEMATICS. PAGE 1. The problem first received a modern form through Kant, who connected the priori with the subjective 1 2. A mental state is subjective, for Psychology, when its immediate cause does not lie in the outer world 2 3. A piece of knowledge is priori, for Epistemology, when without it knowledge would be impossible 2 4. The subjective and the priori belong respectively to Psychology and to Epistemology. The latter alone will be investigated in this essay 3 5. My test of the priori will be purely logical: what knowledge is necessary for experience? 3 6. But since the necessary is hypothetical, we must include, in the priori, the ground of necessity 4 7. This may be the essential postulate of our science, or the element, in the subject-matter, which is necessary to experience; 4 8. Which, however, are both at bottom the same ground 5 9. Forecast of the work 5  
CHAPTER I. A SHORT HISTORY OF METAGEOMETRY. 10. Metageometry began by rejecting the axiom of parallels 7 11. Its history may be divided into three periods: the synthetic, the metrical and the projective 7 12. The first period was inaugurated by Gauss, 10 [viii] 13. Whose suggestions were developed independently by Lobatchewsky 10 14. And Bolyai 11 15. The purpose of all three was to show that the axiom of parallels could not be deduced from the others, since its denial did not lead to contradictions 12 16. The second period had a more philosophical aim, and was inspired chiefly by Gauss and Herbart 13 17. The first work of this period, that of Riemann, invented two new conceptions: 14 18. The first, that of a manifold, is a class-conception, containing space as a species, 14 19. And defined as such that its determinations form a collection of magnitudes 15 20. The second, the measure of curvature of a manifold, grew out of curvature in curves and surfaces 16 21. By means of Gauss's analytical formula for the curvature of surfaces, 19 22. Which enables us to define a constant measure of curvature of a three-dimensional space without reference to a fourth dimension 20 23. The main result of Riemann's mathematical work was to show that, if magnitudes are independent of place, the measure of curvature of space must be constant 21 24. Helmholtz, who was more of a philosopher than a mathematician, 22 25. Gave a new but incorrect formulation of the essential axioms, 23 26. And deduced the quadratic formula for the infinitesimal arc, which Riemann had assumed 24 27. Beltrami gave Lobatchewsky's planimetry a Euclidean interpretation, 25 28. Which is analogous to Cayley's theory of distance; 26 29. And dealt with n-dimensional spaces of constant negative curvature 27 30. The third period abandons the metrical methods of the second, and extrudes the notion of spatial quantity 27 31. Cayley reduced metrical properties to projective properties, relative to a certain conic or quadric, the Absolute; 28 32. And Klein showed that the Euclidean or non-Euclidean systems result, according to the nature of the Absolute; 29 33. Hence Euclidean space appeared to give rise to all the kinds of Geometry, and the question, which is true, appeared reduced to one of convention 30 34. But this view is due to a confusion as to the nature of the coordinates employed 30 [ix] 35. Projective coordinates have been regarded as dependent on distance, and thus really metrical 31 36. But this is not the case, since anharmonic ratio can be projectively defined 32 37. Projective coordinates, being purely descriptive, can give no information as to metrical properties, and the reduction of metrical to projective properties is purely technical 33 38. The true connection of Cayley's measure of distance with non-Euclidean Geometry is that suggested by Beltrami's Saggio, and worked out by Sir R. Ball, 36 39. Which provides a Euclidean equivalent for every non-Euclidean proposition, and so removes the possibility of contradictions in Metageometry 38 40. Klein's elliptic Geometry has not been proved to have a corresponding variety of space 39 41. The geometrical use of imaginaries, of which Cayley demanded a philosophical discussion, 41 42. Has a merely technical validity, 42 43. And is capable of giving geometrical results only when it begins and ends with real points and figures 45 44. We have now seen that projective Geometry is logically prior to metrical Geometry, but cannot supersede it 46 45. Sophus Lie has applied projective methods to Helmholtz's formulation of the axioms, and has shown the axiom of Monodromy to be superfluous 46 46. Metageometry has gradually grown independent of philosophy, but has grown continually more interesting to philosophy 50 47. Metrical Geometry has three indispensable axioms, 50 48. Which we shall find to be not results, but conditions, of measurement, 51 49. And which are nearly equivalent to the three axioms of projective Geometry 52 50. Both sets of axioms are necessitated, not by facts, but by logic 52](59391.docx#chunk10806)

[CHAPTER II. CRITICAL ACCOUNT OF SOME PREVIOUS PHILOSOPHICAL THEORIES OF GEOMETRY. 51. A criticism of representative modern theories need not begin before Kant 54 52. Kant's doctrine must be taken, in an argument about Geometry, on its purely logical side 55 [x] 53. Kant contends that since Geometry is apodeictic, space must be priori and subjective, while since space is priori and subjective, Geometry must be apodeictic 55 54. Metageometry has upset the first line of argument, not the second 56 55. The second may be attacked by criticizing either the distinction of synthetic and analytic judgments, or the first two arguments of the metaphysical deduction of space 57 56. Modern Logic regards every judgment as both synthetic and analytic, 57 57. But leaves the priori, as that which is presupposed in the possibility of experience 59 58. Kant's first two arguments as to space suffice to prove some form of externality, but not necessarily Euclidean space, a necessary condition of experience 60 59. Among the successors of Kant, Herbart alone advanced the theory of Geometry, by influencing Riemann 62 60. Riemann regarded space as a particular kind of manifold, i.e. wholly quantitatively 63 61. He therefore unduly neglected the qualitative adjectives of space 64 62. His philosophy rests on a vicious disjunction 65 63. His definition of a manifold is obscure, 66 64. And his definition of measurement applies only to space 67 65. Though mathematically invaluable, his view of space as a manifold is philosophically misleading 69 66. Helmholtz attacked Kant both on the mathematical and on the psychological side; 70 67. But his criterion of apriority is changeable and often invalid; 71 68. His proof that non-Euclidean spaces are imaginable is inconclusive; 72 69. And his assertion of the dependence of measurement on rigid bodies, which may be taken in three senses, 74 70. Is wholly false if it means that the axiom of Congruence actually asserts the existence of rigid bodies, 75 71. Is untrue if it means that the necessary reference of geometrical propositions to matter renders pure Geometry empirical, 76 72. And is inadequate to his conclusion if it means, what is true, that actual measurement involves approximately rigid bodies 78 73. Geometry deals with an abstract matter, whose physical properties are disregarded; and Physics must presuppose Geometry 80 74. Erdmann accepted the conclusions of Riemann and Helmholtz, 81 [xi] 75. And regarded the axioms as necessarily successive steps in classifying space as a species of manifold 82 76. His deduction involves four fallacious assumptions, namely: 82 77. That conceptions must be abstracted from a series of instances; 83 78. That all definition is classification; 83 79. That conceptions of magnitude can be applied to space as a whole; 84 80. And that if conceptions of magnitude could be so applied, all the adjectives of space would result from their application 86 81. Erdmann regards Geometry alone as incapable of deciding on the truth of the axiom of Congruence, 86 82. Which he affirms to be empirically proved by Mechanics. 88 83. The variety and inadequacy of Erdmann's tests of apriority 89 84. Invalidate his final conclusions on the theory of Geometry 90 85. Lotze has discussed two questions in the theory of Geometry: 93 86. (1) He regards the possibility of non-Euclidean spaces as suggested by the subjectivity of space, 93 87. And rejects it owing to a mathematical misunderstanding, 96 88. Having missed the most important sense of their possibility, 96 89. Which is that they fulfil the logical conditions to which any form of externality must conform 97 90. (2) He attacks the mathematical procedure of Metageometry 98 91. The attack begins with a question-begging definition of parallels 99 92. Lotze maintains that all apparent departures from Euclid could be physically explained, a view which really makes Euclid empirical 99 93. His criticism of Helmholtz's analogies rests wholly on mathematical mistakes 101 94. His proof that space must have three dimensions rests on neglect of different orders of infinity 104 95. He attacks non-Euclidean spaces on the mistaken ground that they are not homogeneous 107 96. Lotze's objections fall under four heads 108 97. Two other semi-philosophical objections may be urged, 109 98. One of which, the absence of similarity, has been made the basis of attack by Delbouf, 110 99. But does not form a valid ground of objection 111 100. Recent French speculation on the foundations of Geometry has suggested few new views 112 101. All homogeneous spaces are priori possible, and the decision between them is empirical 114 [xii] CHAPTER III. Section A. the axioms of projective geometry. 102. Projective Geometry does not deal with magnitude, and applies to all spaces alike 117 103. It will be found wholly priori 117 104. Its axioms have not yet been formulated philosophically 118 105. Coordinates, in projective Geometry, are not spatial magnitudes, but convenient names for points 118 106. The possibility of distinguishing various points is an axiom 119 107. The qualitative relations between points, dealt with by projective Geometry, are presupposed by the quantitative treatment 119 108. The only qualitative relation between two points is the straight line, and all straight lines are qualitatively similar 120 109. Hence follows, by extension, the principle of projective transformation 121 110. By which figures qualitatively indistinguishable from a given](59391.docx#chunk10807)

[figure are obtained 122 111. Anharmonic ratio may and must be descriptively defined 122 112. The quadrilateral construction is essential to the projective definition of points, 123 113. And can be projectively defined, 124 114. By the general principle of projective transformation 126 115. The principle of duality is the mathematical form of a philosophical circle, 127 116. Which is an inevitable consequence of the relativity of space, and makes any definition of the point contradictory 128 117. We define the point as that which is spatial, but contains no space, whence other definitions follow 128 118. What is meant by qualitative equivalence in Geometry? 129 119. Two pairs of points on one straight line, or two pairs of straight lines through one point, are qualitatively equivalent 129 120. This explains why four collinear points are needed, to give an intrinsic relation by which the fourth can be descriptively defined when the first three are given 130 121. 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Which, however, is logically and philosophically untenable 153 148. Though Free Mobility is priori, actual measurement is empirical 154 [xiv] 149. Some objections remain to be answered, concerning 154 150. (1) The comparison of volumes and of Kant's symmetrical objects 154 151. (2) The measurement of time, where congruence is impossible 156 152. (3) The immediate perception of spatial magnitude; and 157 153. (4) The Geometry of non-congruent surfaces 158 154. Free Mobility includes Helmholtz's Monodromy 159 155. Free Mobility involves the relativity of space 159 156. From which, reciprocally, it can be deduced 160 157. Our axiom is therefore priori in a double sense 160 II. The Axiom of Dimensions. 158. Space must have a finite integral number of dimensions 161 159. But the restriction to three is empirical 162 160. The general axiom follows from the relativity of position 162 161. The limitation to three dimensions, unlike most empirical knowledge, is accurate and certain 163 III. The Axiom of Distance. 162. The axiom of distance corresponds, here, to that of the straight line in projective Geometry 164 163. The possibility of spatial measurement involves a magnitude uniquely determined by two points, 164 164. Since two points must have some relation, and the passivity of space proves this to be independent of external reference 165 165. There can be only one such relation 166 166. This must be measured by a curve joining the two points, 166 167. And the curve must be uniquely determined by the two points 167 168. Spherical Geometry contains an exception to this axiom, 168 169. Which, however, is not quite equivalent to Euclid's 168 170. The exception is due to the fact that two points, in spherical space, may have an external relation unaltered by motion, 169 171. Which, however, being a relation of linear magnitude, presupposes the possibility of linear magnitude 170 172. A relation between two points must be a line joining them 170 173. 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[CHAPTER IV. PHILOSOPHICAL CONSEQUENCES. 180. What is the relation to experience of a form of externality in general? 178 181. This form is the class-conception, containing every possible intuition of externality; and some such intuition is necessary to experience 178 182. What relation does this view bear to Kant's? 179 183. It is less psychological, since it does not discuss whether space is given in sensation, 180 184. And maintains that not only space, but any form of externality which renders experience possible, must be given in sense-perception 181 185. Externality should mean, not externality to the Self, but the mutual externality of presented things 181 186. Would this be unknowable without a given form of externality? 182 187. Bradley has proved that space and time preclude the existence of mere particulars, 182 188. And that knowledge requires the This to be neither simple nor self-subsistent 183 189. To prove that experience requires a form of externality, I assume that all knowledge requires the recognition of identity in difference 184 190. Such recognition involves time 184 191. And some other form giving simultaneous diversity 185 192. The above argument has not deduced sense-perception from the categories, but has shown the former, unless it contains a certain element, to be unintelligible to the latter 186 193. How to account for the realization of this element, is a question for metaphysics 187 [xvi] 194. What are we to do with the contradictions in space? 188 195. Three contradictions will be discussed in what follows 188 196. (1) The antinomy of the Point proves the relativity of space, 189 197. And shows that Geometry must have some reference to matter, 190 198. By which means it is made to refer to spatial order, not to empty space 191 199. The causal properties of matter are irrelevant to Geometry, which must regard it as composed of unextended atoms, by which points are replaced 191 200. (2) The circle in defining straight lines and planes is overcome by the same reference to matter 192 201. (3) The antinomy that space is relational and yet more than relational, 193 202. Seems to depend on the confusion of empty space with spatial order 193 203. Kant regarded empty space as the subject-matter of Geometry, 194 204. But the arguments of the Aesthetic are inconclusive on this point, 195 205. And are upset by the mathematical antinomies, which prove that spatial order should be the subject-matter of Geometry 196 206. The apparent thinghood of space is a psychological illusion, due to the fact that spatial relations are immediately given 196 207. The apparent divisibility of spatial relations is either an illusion, arising out of empty space, or the expression of the possibility of quantitatively different spatial relations 197 208. Externality is not a relation, but an aspect of relations. Spatial order, owing to its reference to matter, is a real relation 198 209. Conclusion 199  
  
  
  
  
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