# CSC8106:SYSTEM EVALUATION

B1 report

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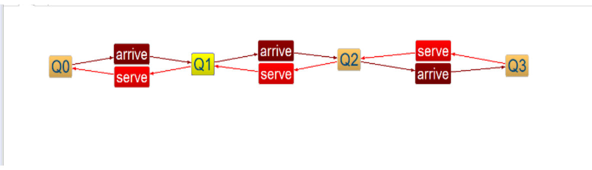
**Exercise B1 Solution.**

**B1.A)** I wrote a specification for M/M/1/3 and I applied it in PEPA tools as following using lecture Notes-3, where **(a)** is the arrival rate and **(s)** is the service rate

a =0.75;
s =1.35;
Q0 $\stackrel{def}{=}$(arrive, T).Q1;
Q1 $\stackrel{def}{=}$ (arrive, T).Q2+(serve, T).Q0;
Q2 $\stackrel{def}{=}$ (arrive, T).Q3+(serve, T).Q1;
Q3 $\stackrel{def}{=}$ (serve, T).Q2;
Arrival $\stackrel{def}{=}$ (arrive, a).Arrival;
Service $\stackrel{def}{=}$ (serve, s).Service;
Arrival <arrive> Q0 <serve> Service




**B1.B)** The result of form using PEPA to drive the states of the above mode;

**1.Using PEPA tool to derive the states.**

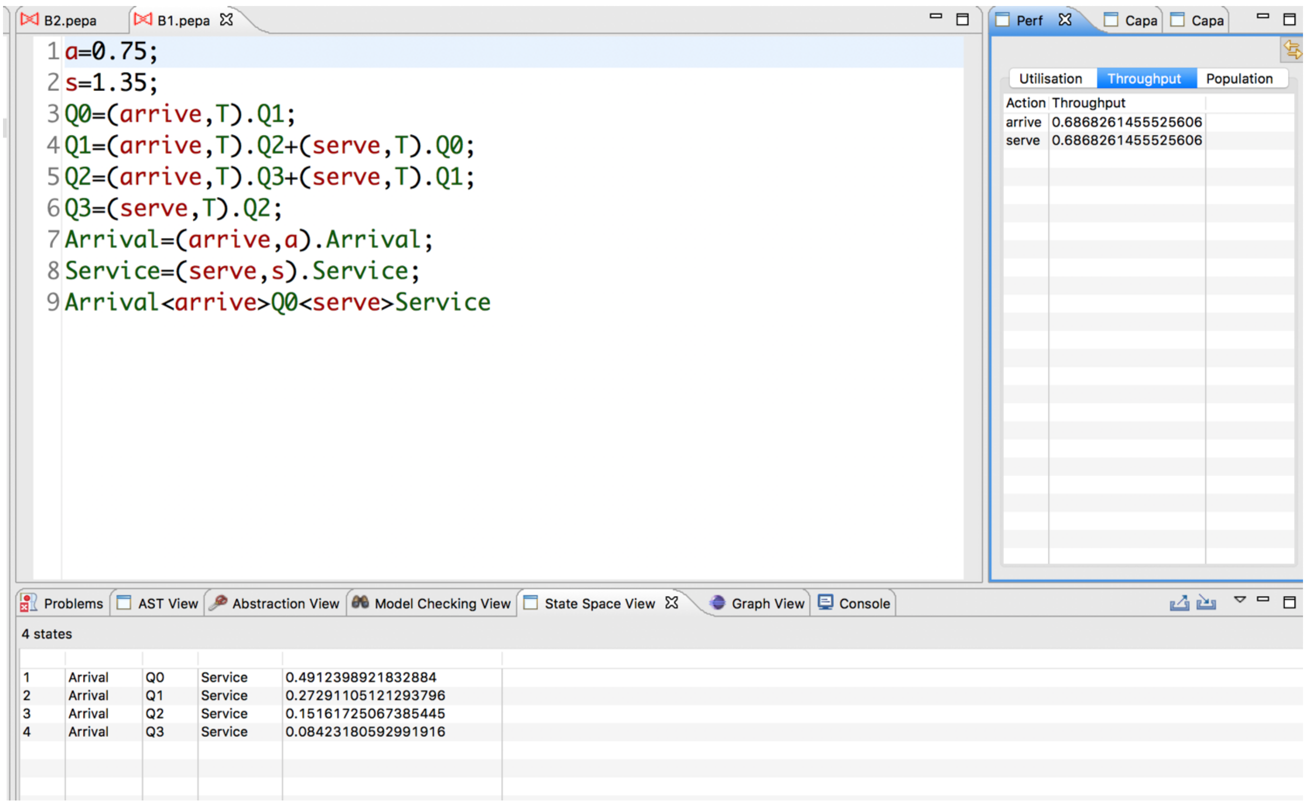



*Figure 1 Applying the Model in PEPA and Derive the states*

**2**.The derivation graph using the underlying CTMC in PEPA tool.

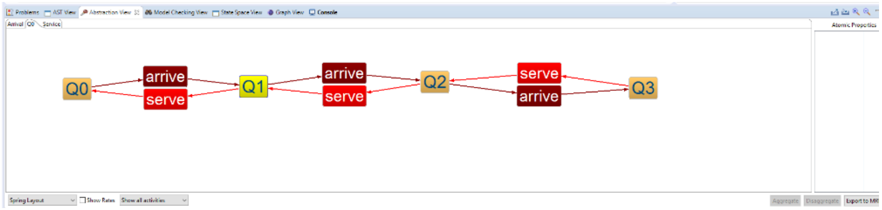



*Figure 2 The Derivation graph from PEPA*

**3.** The throughput for the model:

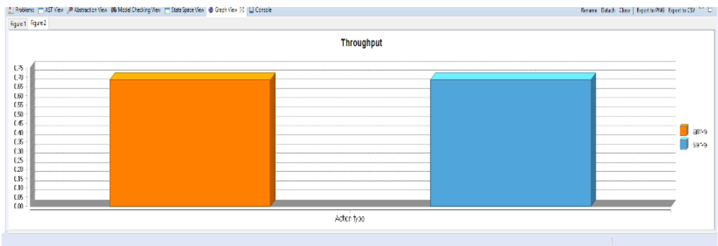
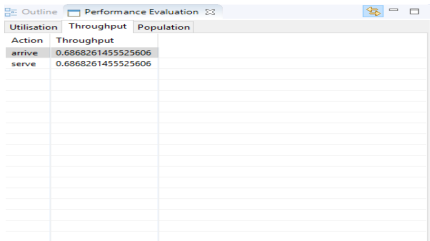


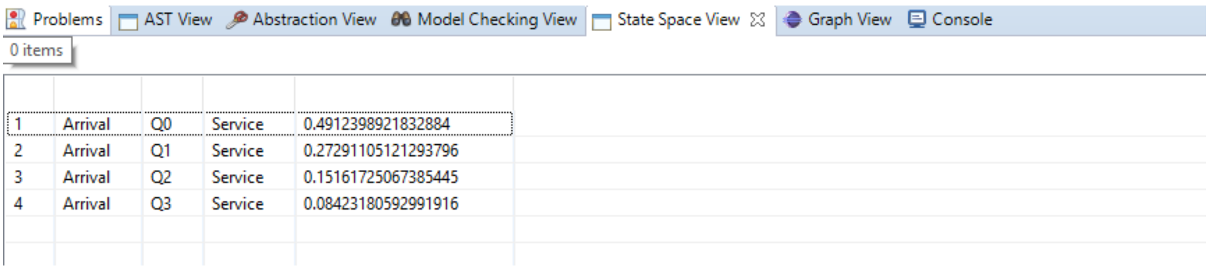

*Figure 3 Throughput graph*



| Action | Throughput |
|---|---|
| arrive | 0.6868261455525606 |
| serve | 0.6868261455525606 |

*Figure 4 throughput for arrive and serve*

3. The steady state probabilities of for each state:



| | | | | |
|---|---|---|---|---|
| 1 | Arrival | Q0 | Service | 0.4912398921832884 |
| 2 | Arrival | Q1 | Service | 0.27291105121293796 |
| 3 | Arrival | Q2 | Service | 0.15161725067385445 |
| 4 | Arrival | Q3 | Service | 0.08423180592991916 |

*Figure 5 The Steady state for each state*

**B1.C)** The equation that is suggested to be used here to find the steady state probabilities with finite size of the buffer and its size is 3.

First we need to calculate $\pi_0$ and from page 3 in lecture slides#2, $\pi_0 = \frac{1-r}{1-r^{N+1}}$ where $r = \frac{p}{c}$ and **p** is the arrival rate **a**=0.75 and **c** is the serve rate **s**=1.35, so we can compute $r$ as following:

$$r = \frac{p}{c} = \frac{a}{s} = \frac{0.75}{1.35} = 0.5555555555555556$$

Therefore we can compute $\pi_0$ as following:

$$\pi_0 = \frac{1-r}{1-r^{n+1}} = \frac{1-0.55}{1-0.55^4} = \frac{1-0.556}{1-0.0952} = \frac{0.44}{0.9047} = 0.4912398921832883771307241301647$$

The equation to find the steady state probabilities is

$$\pi_{i+1} = \frac{p}{c}\pi_i = \left(\frac{p}{c}\right)^{i+1}\pi_0$$

The steady state for $Q_0$ can be found as following:

$$\pi_0 = \pi_0 * \left(\frac{p}{c}\right)^0 = 0.4912398921832883771307241301647 * \left(\frac{0.75}{1.35}\right)^0 = 0.4912398921832883771307241301647 * (1) = 0.4912398921832883771307241301647$$

The steady state for $Q_1$ can be found as following:

$$\begin{aligned}\pi_1 = \pi_0 * \left(\frac{p}{C}\right)^1 &= 0.4912398921832883771307241301647 * \left(\frac{0.75}{1.35}\right)^1 \\ &= 0.4912398921832883771307241301647 * (0.5555555555555556) \\ &= 0.2729110512129380091277308360154\end{aligned}$$

The steady state for $Q_2$ can be found as following:

$$\begin{aligned}\pi_2 = \pi_0 * \left(\frac{p}{C}\right)^2 &= 0.4912398921832883771307241301647 * \left(\frac{0.75}{1.35}\right)^2 \\ &= 0.4912398921832883771307241301647 * (0.5555555555555556)^2 \\ &= 0.1516172506738544616447860739169\end{aligned}$$

The steady state for $Q_3$ can be found as following:

$$\begin{aligned}\pi_3 = \pi_0 * \left(\frac{p}{C}\right)^3 &= 0.4912398921832883771307241301647 * \left(\frac{0.75}{1.35}\right)^3 \\ &= 0.4912398921832883771307241301647 * (0.5555555555555556)^3 \\ &= 0.0842318059299191520967589599029\end{aligned}$$

**B1.D)**

**B1.D.1) Find the queue average size.**

-Finding the average queue size (**L**) can be computed via using the previous steady state for all states as following.

$$L = \sum_{i=0}^{n} (i * \pi_i) = \pi_0 * 0 + \pi_1 * 1 + \pi_2 * 2 + \pi_3 * 3$$
$$= 0 + (0.27291105121293800912773083601543 * 1)$$
$$+ ( 0.15161725067385446164478607391693 * 2)$$
$$+ ( 0.08423180592991915209675895990294 * 3)$$
$$= 0.82884097035040438870757986355811$$

**B1.D.1) Find the average response time.**

**-**To find the average response time **W** we need to use Little's Law:

**L= $\boldsymbol{\lambda}$ W => W=L / $\boldsymbol{\lambda}$,** the queue average size **L** is found previously but we need to find $\boldsymbol{\lambda}$ where $\boldsymbol{\lambda}$ is the average arrival rate, and it can be found as following:

$\boldsymbol{\lambda} = \boldsymbol{a} * (\mathbf{1} - \pi_3)$ = 0.75 * (1 - 0.08423180592991915209675895990294) = 0.68682614555256063592743078007 28

Now we can compute the average response time **W as following:**

**W = L /** $\boldsymbol{\lambda} = \frac{0.82884097035040438870757986355811}{0.6868261455525606359274307800728} = 1.2067696835908757738461763372952$

**References**:


- Thomas, Nigel, and Jane Hillston. *Using Markovian process algebra to specify interactions in queueing systems*. University of Edinburgh, 1997.
- Coursework notes