FINAL Study Guide

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1 Trigonometry

1.1 Basic Trigonometry

$$sin(x) = \left[\frac{Opposite}{Hypotenuse}\right] \rightarrow csc(x) = \frac{1}{sin(x)}$$

$$cos(x) = \left[\frac{Adjacent}{Hypotenuse}\right] \rightarrow sec(x) = \frac{1}{cos(x)}$$

$$tan(x) = \left[\frac{Opposite}{Adjacent}\right] \rightarrow cot(x) = \frac{1}{tan(x)}$$

1.2 Trigonometry Derivatives

$$\frac{d}{dx}sin(x) = cos(x) \qquad \frac{d}{dx}csc(x) = -cot(x)csc(x)$$

$$\frac{d}{dx}cos(x) = -sin(x) \qquad \frac{d}{dx}sec(x) = sec(x)tan(x)$$

$$\frac{d}{dx}tan(x) = sec^2(x) \qquad \frac{d}{dx}cot(x) = -csc^2(x)$$

1.3 Inverse Trigonometry Derivatives

$$\frac{d}{dx}sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}tan^{-1}(x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}cot^{-1}(x) = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}csc^{-1}(x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

2 Integral

2.1 Table of Integrals

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$

$\int e^{kx} dx = \frac{1}{k} e^{kx} + C \quad (k \text{ constant})$

$\int \frac{du}{u} = \ln|u| + C \quad u \neq 0$

$\int \sin(kx) dx = -\frac{1}{k}\cos(kx) + C$

$\int \cos(kx) dx = \frac{1}{k}\sin(kx) + C$

$\int \sec^2 u \; du = \tan(u) + C$

$\int \tan u \; du = \ln|\sec u| + C$

$\int \sec u \; du = \ln|\sec u + \tan u| + C$

$\int a^u du = \frac{a^u}{\ln a} + C$

$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$

$\int \frac{du}{1+u^2} = \tan^{-1}(u) + C$

$\int -\frac{du}{\sqrt{1-u^2}} = \cos^{-1}(u) + C$

$\int e^u du = e^u + C$

$\int \csc u \; du = \ln|\csc u - \cot u| + C$

2.2 Mean Value Theorem (Average Value Theorem)

$$\frac{1}{a-b}\int_a^b f(x)dx = f(c)$$

2.3 Area Under A Curve

$$\int_a^b f(x)dx$$

2.4 Area Between Two Curves

$$\int_a^b [f(x) - g(x)]\, dx$$

2.5 Integration by Parts

$$\int udv = uv - \int vdu$$

2.6 Partial Fractions

$$\frac{p(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

$$\frac{p(x)}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$$

2.7 Simpson's Rule

$$\int_a^b f(x)dx \approx S_n$$

$$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

$$\Delta x = \frac{b-a}{n}$$

2.8 Improper Integrals

$$\int_a^\infty f(x)dx = \lim_{n\to\infty}\int_a^n f(x)dx$$

$$\int_\infty^b f(x)dx = \lim_{n\to\infty}\int_n^b f(x)dx$$

$$\int_\infty^\infty f(x)dx = \lim_{n\to\infty}\int_n^b f(x)dx + \lim_{n\to\infty}\int_n^b f(x)dx$$

2.9 Arc Length Formula

$$L = \int_a^b \left(\sqrt{1 + \left[\frac{dy}{dx}\right]^2}\right) dx$$

Volume

2.10 Disk Method

$$V = \pi \int_a^b \left[f(x)\right]^2 dx$$

2.11 Washer Method

$$V = \pi \int_a^b \left[(f(x))^2 - (g(x))^2\right] dx$$

2.12 Shells Method

$$V = 2\pi \int_a^b \left[xf(x)\right] dx$$

$$V = 2\pi \int_a^b x\left[f(x) - g(x)\right] dx$$

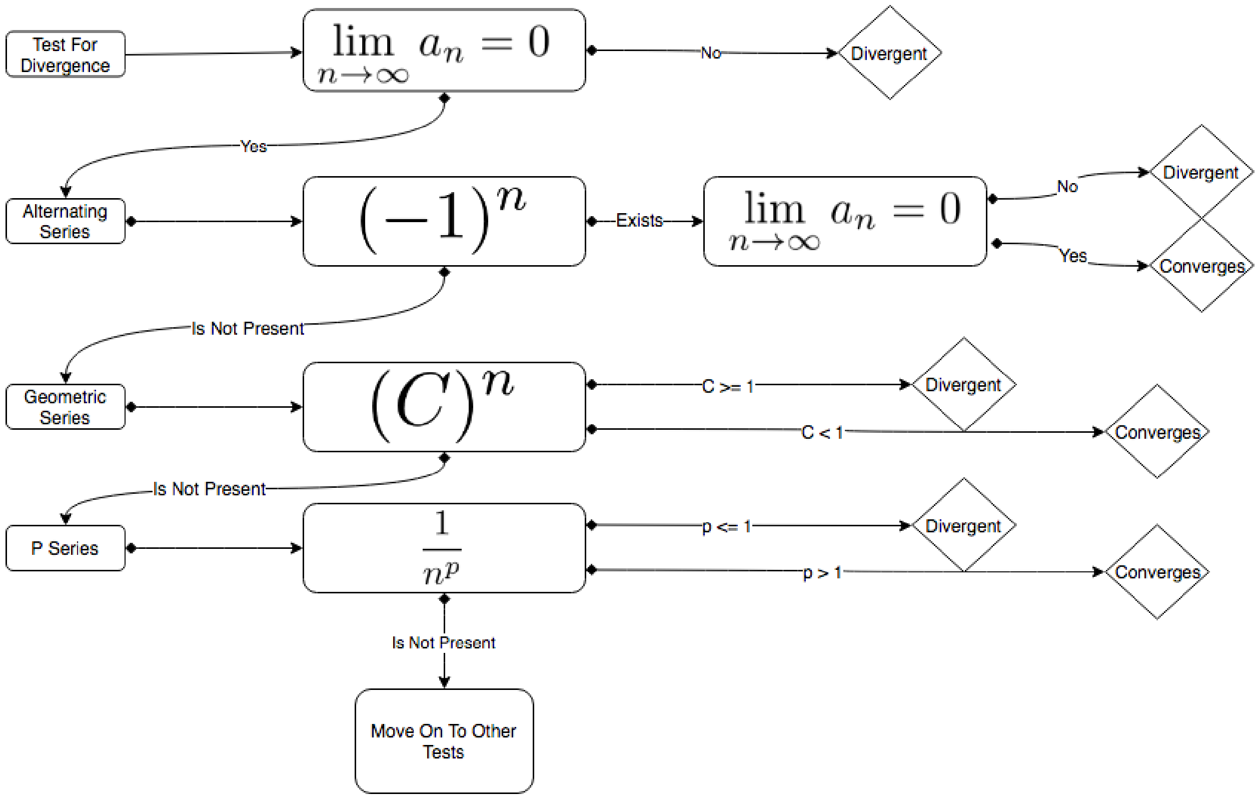
3 Polar Curves

3.1 Derivatives of Polar Curves

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad If \frac{dx}{dt} \neq 0$$

4 Series

4.1 Order Of Operations



4.2 Harmonic Series

1. $\frac{1}{x}$, is a harmonic Series and it Diverges

4.3 Test for Divergence

1. If [$\lim_{n\to\infty} a_n \neq 0$] is true than the series diverges

4.4 Alternating Series

1. Confirm $(-1)^n$ exists

2. If [$\lim_{n\to\infty} a_n = 0$] than the series Converges

3. If [$\lim_{n\to\infty} a_n \neq 0$] than the series Diverges

4.5 Geometric Series

1. For a series to be geometric it must have a value that is continuously added, such as a function $(c)^n$ where the series grows by a constant C each iteration.

2. If C $<$ 1 then the series converges
3. If C $>$ 1 or C $=$ 1 then the series diverges

4.6 P Series

1. Confirm $\left[\frac{1}{n^p}\right]$ exists
2. If p $>$ 1 than the series converges
3. If p $<$ 1 or p $=$ 1 than the series diverges

4.7 Integral Series

1. Set $a_n = f(x)$
2. Change $\sum\limits_{n=a}^{b} a_n$ To $\int_a^n f(x)$

4.8 Direct Comparison Test

1. $0 \leq a_n \leq b_n$
2. If $\sum\limits_{n=a}^{b} b_n$ converges then, $\sum\limits_{n=a}^{b} a_n$ converges
3. If $\sum\limits_{n=a}^{b} a_n$ diverges then, $\sum\limits_{n=a}^{b} a_n$ diverges

4.9 Limit Comparison Test

1. When you have the given series $\sum\limits_{n=a}^{b} a_n$ and what you believe looks like $\sum\limits_{n=a}^{b} b_n$ if $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists than either both series converge or both series diverge.

4.10 Ratio Test

1. $\rho = \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}|$
2. If $\rho < 1$, the series converges absolutely
3. If $\rho > 1$, the series diverges
4. If $\rho = 1$, the series is unable to be determined still

4.11 Root Test

1. If the series can be written as $(a_n)^k$ then root test is applicable.
2. $\rho = \lim_{n\to\infty} |a_n|^{\frac{1}{k}}$
3. If $\rho < 1$, the series converges absolutely
4. If $\rho > 1$, the series diverges
5. If $\rho = 1$, the series is unable to be determined still

4.12 Telescoping Series

1. Telescoping series are any series that can be written out as;
$$\lim_{n\to\infty} a_n = (b_1 - b_2) + (b_2 - b_3) + (b_3 - b_4) + ...$$
2. Almost every term should be canceled with its preceding term
3. Partial fractions is often used here
4. Partial sums is often recommended for these series

4.13 Power Series

1. a_n that is dominated by geometric growth
2. Most commonly seen as, $f(x) = \frac{a}{1-r}$

4.14 Radius of Convergence

1. Put in terms of $(a_n)^k$
2. solve r = $|a_n| < 1$
3. Radius of Convergence = $(-r, r)$

4.15 Interval of Convergence

1. If it is not obvious than using ratio test to find r is helpful.
2. $|r| < 1$
3. Put in terms of $(x - a) < b$ than the interval of convergence is $(b - a, b + a)$
4. If the point is convergent change (or) to [or]