FINAL Study Guide

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Discrete Final

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1 The Foundations: Logic and Proofs

1.1 Propositional Logic

Examples of a proposition:
$p(x) = x$ is a cat.
$q(x) = x$ has fur.

1. Negation, $\neg p(x)$, changing the statement to x is not a cat
2. Conjunction, $p(x) \wedge q(x)$, changing the statement to say x is a cat and it has fur.
3. Disjunction, $p(x) \vee q(x)$, changing the statement to say x is a cat and it does not have fur.
4. Exclusive Or, $p(x) \oplus q(x)$, where the statement is true only when exactly one of $p(x)$ or $q(x)$ is true, otherwise the statement is false.
5. Conditional Statement, $p(x) \rightarrow q(x)$, where the statement is read "If p, then q" the statement is false when p is true and q is false, otherwise the statement is true.
6. Bi-Conditional Statement, $p(x) \leftrightarrow q(x)$, where the statement is true if the values of p and q match. The statement is read as any of the following;
   (a) "p if and only if q"
   (b) "p is necessary and sufficient for q"
   (c) "if p then q, and conversely"
   (d) "p iff q"

Definitions

1. Hypothesis - p is considered the hypothesis in the statement $p(x) \rightarrow q(x)$.
2. Conclusion- q is considered the conclusion in the statement $p(x) \rightarrow q(x)$.
3. Bi-Implications - Another way to express "Bi-Conditional statements"

1.2 Propositional Equivalences

1. Logically Equivalent, $p(x) \equiv q(x)$, when two statements share the same truth values then the statement is said to be logically equivalent.

2. Identity Laws
$$p(x) \wedge T \equiv p(x)$$
$$p(x) \vee F \equiv p(x)$$

3. Domination Laws
$$p(x) \vee T \equiv T$$
$$p(x) \wedge F \equiv F$$

4. Idempotent Laws
$$p(x) \vee p(x) \equiv p(x)$$
$$p(x) \wedge p(x) \equiv p(x)$$

5. Double Negation Law
$$\neg(\neg p(x)) \equiv p(x)$$

6. Commutative Laws
$$p(x) \vee q(x) \equiv q(x) \vee p(x)$$
$$p(x) \wedge q(x) \equiv q(x) \wedge p(x)$$

7. Associative Laws
$$(p(x) \vee q(x)) \vee r(x) \equiv p(x) \vee (q(x) \vee r(x))$$
$$(p(x) \wedge q(x)) \wedge r(x) \equiv p(x) \wedge (q(x) \wedge r(x))$$

8. Distributive Laws
$$p(x) \vee (q(x) \wedge r(x)) \equiv (p(x) \vee q(x)) \wedge (p(x) \vee r(x))$$
$$p(x) \wedge (q(x) \vee r(x)) \equiv (p(x) \wedge q(x)) \vee (p(x) \wedge r(x))$$

9. De Morgan's Laws
$$\neg\left(p(x) \wedge q(x)\right) \equiv \neg p(x) \vee \neg q(x)$$
$$\neg\left(p(x) \vee q(x)\right) \equiv \neg p(x) \wedge \neg q(x)$$

10. Absorption Laws
$$p(x) \vee (p(x) \wedge q(x)) \equiv p(x)$$
$$p(x) \wedge (p(x) \vee q(x)) \equiv p(x)$$

11. Negation Laws
$$p(x) \vee \neg p(x) \equiv T$$
$$p(x) \wedge \neg p(x) \equiv F$$

Definitions

1. Tautology - When all cases in the statement are determined to be true, it is said to be a tautology.

2. Contradiction- When all cases in the statement are determined to be false, it is said to be a contradiction.

3. Contingency - When the statement is neither a tautology, nor a contradiction then it is said to be a contingency.