FINAL Study Guide

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Discrete Final

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Contents

1 The Foundations: Logic and Proofs

1.1 Propositional Logic

Examples of a proposition:
$p(x) = x$ is a cat.
$q(x) = x$ has fur.

1. Negation, $\neg p(x)$, changing the statement to x is not a cat
2. Conjunction, $p(x) \wedge q(x)$, changing the statement to say x is a cat and it has fur.
3. Disjunction, $p(x) \vee q(x)$, changing the statement to say x is a cat and it does not have fur.
4. Exclusive Or, $p(x) \oplus q(x)$, where the statement is true only when exactly one of $p(x)$ or $q(x)$ is true, otherwise the statement is false.
5. Conditional Statement, $p(x) \rightarrow q(x)$, where the statement is read "If p, then q" the statement is false when p is true and q is false, otherwise the statement is true.
6. Bi-Conditional Statement, $p(x) \leftrightarrow q(x)$, where the statement is true if the values of p and q match. The statement is read as any of the following;
   (a) "p if and only if q"
   (b) "p is necessary and sufficient for q"
   (c) "if p then q, and conversely"
   (d) "p iff q"

Definitions

1. Hypothesis - p is considered the hypothesis in the statement $p(x) \rightarrow q(x)$.
2. Conclusion- q is considered the conclusion in the statement $p(x) \rightarrow q(x)$.
3. Bi-Implications - Another way to express "Bi-Conditional statements"

1.2 Propositional Equivalences

1. Logically Equivalent, $p(x) \equiv q(x)$, when two statements share the same truth values then the statement is said to be logically equivalent.

2. Identity Laws

$$p(x) \wedge T \equiv p(x)$$
$$p(x) \vee F \equiv p(x)$$

3. Domination Laws

$$p(x) \vee T \equiv T$$
$$p(x) \wedge F \equiv F$$

4. Idempotent Laws

$$p(x) \vee p(x) \equiv p(x)$$
$$p(x) \wedge p(x) \equiv p(x)$$

5. Double Negation Law

$$\neg(\neg p(x)) \equiv p(x)$$

6. Commutative Laws

$$p(x) \vee q(x) \equiv q(x) \vee p(x)$$
$$p(x) \wedge q(x) \equiv q(x) \wedge p(x)$$

7. Associative Laws

$$(p(x) \vee q(x)) \vee r(x) \equiv p(x) \vee (q(x) \vee r(x))$$
$$(p(x) \wedge q(x)) \wedge r(x) \equiv p(x) \wedge (q(x) \wedge r(x))$$

8. Distributive Laws

$$p(x) \vee (q(x) \wedge r(x)) \equiv (p(x) \vee q(x)) \wedge (p(x) \vee r(x))$$
$$p(x) \wedge (q(x) \vee r(x)) \equiv (p(x) \wedge q(x)) \vee (p(x) \wedge r(x))$$

9. De Morgan's Laws

$$\neg\left(p(x) \wedge q(x)\right) \equiv \neg p(x) \vee \neg q(x)$$
$$\neg\left(p(x) \vee q(x)\right) \equiv \neg p(x) \wedge \neg q(x)$$

10. Absorption Laws

$$p(x) \vee (p(x) \wedge q(x)) \equiv p(x)$$
$$p(x) \wedge (p(x) \vee q(x)) \equiv p(x)$$

11. Negation Laws

$$p(x) \vee \neg p(x) \equiv T$$
$$p(x) \wedge \neg p(x) \equiv F$$

Definitions

1. Tautology - When all cases in the statement are determined to be true, it is said to be a tautology.
2. Contradiction- When all cases in the statement are determined to be false, it is said to be a contradiction.
3. Contingency - When the statement is neither a tautology, nor a contradiction then it is said to be a contingency.

1.3 Predicates and Quantifiers

1. Universal Quantification, $\forall p(x)$ read as, "For all x, $p(x)$."
2. Existential Quantification, $\exists p(x)$ read as, "There exists an element x, that $p(x)$."
3. De Morgan's Laws for Quantifiers:

$$\neg \forall p(x) \equiv \exists \neg p(x)$$

$$\neg \exists p(x) \equiv \forall \neg p(x)$$

Definitions

1. Quantification - Used to express the extent that a predicate is true. In English, the words; all, some, many, none, and few are used in quantifications.
2. Counterexample - When there is a value, x in which $\forall p(x)$ is false, then that value of x is called a counterexample of $\forall p(x)$.

1.4 Rules of Inference (ROI)

Modus Ponens

$$\begin{array}{c} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$

Modus Tollens

$$\begin{array}{c} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array}$$

Hypothetical Syllogism

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Disjunctive Syllogism

$$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Addition

$$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array}$$

Simplification

$$\begin{array}{c} p \wedge q \\ \hline \therefore p \end{array}$$

Conjunction

$$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Universal Instantiation

$$\begin{array}{c} \forall p(x) \\ \hline \therefore p(c) \end{array}$$

Universal Generalization

$$\begin{array}{c} p(c) \text{ for an arbitrary c} \\ \hline \therefore \forall p(x) \end{array}$$

Existential Instantiation

$$\begin{array}{c} \exists p(x) \\ \hline \therefore p(c) \text{ for some element c} \end{array}$$

Existential Generalization

$$\begin{array}{c} p(c) \text{ for some element c} \\ \hline \therefore \exists p(x) \end{array}$$

Definitions

1. Argument - Sequence of propositions.
2. Premises - All propositions in the argument with the exclusion of the conclusion
3. Conclusion - The final proposition in the argument.
4. Argument Form - A sequence of compound propositions involving propositional variables.
5. Valid - A form that makes it impossible for the premises to be true and the conclusion nevertheless to be false.