Linear Algebra Final Study Guide

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1 The Definitions

Definition:

A vector $\mathbf{v}$ is a **linear combination** of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ if there are scalars, $c_1, c_2, \ldots, c_k$ such that $\mathbf{v} = c_1 \times \mathbf{v}_1 + c_2 \times \mathbf{v}_2 + \cdots + c_k + \mathbf{v}_k$.

Definition:

If $S = \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$, is a set of vectors in $\mathcal{R}^n$, then the set of all linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ is called a **span** of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ and denoted by $span(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k)$ or $span(S)$.

Definition:

A set of vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ is **linear dependent** if the scalars $c_1, c_2, \ldots, c_k$, at least one of which is not zero, such that

$$c_1 \times \mathbf{v}_1 + c_2 \times \mathbf{v}_2 + \cdots + c_k \times \mathbf{v}_k = 0$$

Definition:

A **basis** for a subspace S of $\mathcal{R}^n$ is a set of vectors in S that

1. spans S
2. Is linear independent.

2 Basic Vector Mathematics

Finding vectors from points

Given two points $P(a, b)$ and $Q(c, d)$ then a vector $\mathbf{v}$ where $\mathbf{v} = PQ$, then;

$$\mathbf{v} = \begin{bmatrix} c - a \\ d - b \end{bmatrix}$$

Unit Vectors

Given a vector $\mathbf{v}$ the unit vector for said vector is

$$\left(\frac{1}{||\mathbf{v}||} \right) \mathbf{v}$$

Distance

The distance between two vectors is denoted $d(\mathbf{u}, \mathbf{v})$ and defined as

$$d(\mathbf{u}, \mathbf{v})) = ||\mathbf{u} - \mathbf{v}||$$

3 Eigenvalues, Eigenvectors and Eigenspaces

4 Standard Matrix

5 Spans

Spans of Matrices Find $span(A_1, A_2)$ given

$$a_1 = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$$

Step 1: Establish an augmented matrix.

$$\begin{bmatrix} 1 & 0 & | & a \\ 2 & 1 & | & b \\ -1 & 3 & | & c \\ 4 & 4 & | & d \end{bmatrix}$$

Step 2: Row reduce to row echelon form

$$\begin{bmatrix} 1 & 0 & | & a \\ 2 & 1 & | & b \\ -1 & 3 & | & c \\ 4 & 4 & | & d \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & | & a \\ 0 & 1 & | & b-2a \\ 0 & 0 & | & 7a-3b+c \\ 0 & 0 & | & 4a-4b+d \end{bmatrix}$$

Step 3: Solve for 0 in rows which contain all 0's

$$7a - 3b + c = 0$$

$$c = -7a + 3b$$

$$4a - 4b + d = 0$$

$$d = -4a - 4b$$

Step 4: use the values obtained to create a matrix

$$\begin{bmatrix} a & b-2a \\ -7a+3b & -4a+4b \end{bmatrix}$$