

CS425: Computer Networks

Homework 1

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February 11, 2026

1 Problem 1: CRC C++ Implementation

The Cyclic Redundancy Check (CRC) mechanism was implemented using C++. The program utilizes string manipulation to simulate binary modulo-2 arithmetic.

Algorithm Overview

- **Generator:** The data block D is appended with $n - k$ zeros (where $n - k$ is the degree of the generator polynomial P). Modulo-2 division (XOR) is performed. The remainder R is appended to D to form the frame T .
- **Verifier:** The received frame is divided by P . If the remainder is all zeros, the frame is accepted; otherwise, it is discarded.

Execution Results

Using the specific parameters required by the assignment ($k = 10$ message 1010101101 and Polynomial 110101), the program successfully generated the frame and detected an injected error.

The screenshot below demonstrates the program output for the required inputs:

(Please see the attached `crc.cpp` file for the complete source code.)

```

==== TEST CASE 1 ====
1. Message: 1010101101
2. Poly: 110101
3. Tx Frame: 101010110101001
4. Rx Frame (Err at 3): 101110110101001
5. Status: DISCARDED (Error Detected)

==== TEST CASE 2 ====
1. Message: 10010011011
2. Poly: 10011
3. Tx Frame: 100100110111100
4. Rx Frame (Err at 7): 100100100111100
5. Status: DISCARDED (Error Detected)
PS C:\Users\Areen\Downloads\CS425\HW1> █

```

Figure 1: Program output showing CRC frame generation and error detection for two test cases.

2 Problem 2: Go-Back-N Window Size

Question: Why is the window size in Go-Back-N limited to $2^k - 1$ instead of 2^k ?

Analysis: In a sliding window protocol with k -bit sequence numbers, the sequence space size is $N = 2^k$. If the sender's window size W equals N , a critical ambiguity arises when acknowledgments are lost.

1. The sender transmits frames 0 through $N - 1$.
2. The receiver accepts them and expects frame 0 of the *next* cycle.
3. If all ACKs are lost, the sender times out and retransmits the *old* frame 0.
4. The receiver cannot distinguish the retransmitted *old* frame 0 from the *new* frame 0 it is currently expecting.

Therefore, we require $W \leq 2^k - 1$ to ensure the receiver's expected window never overlaps with the sender's retransmission window.

3 Problem 3: Selective-Reject Window Size

Question: What is the maximum window size for Selective-Reject ARQ?

Analysis: In Selective-Reject, the receiver accepts out-of-order frames within its window W_R . To prevent sequence number aliasing, the range of sequence numbers covered

by the sender's window (W_S) and the receiver's window (W_R) combined must not exceed the total sequence space.

$$W_S + W_R \leq 2^k$$

Assuming $W_S = W_R = W$, we derive:

$$2W \leq 2^k \implies W \leq 2^{k-1}$$

Thus, the maximum window size is half the sequence number space.

4 Problem 4: Stop-and-Wait Efficiency

Given:

- Bandwidth $R = 4 \text{ kbps} = 4000 \text{ bps}$
- Propagation Delay $T_p = 20 \text{ ms} = 0.02 \text{ s}$
- Target Efficiency $\eta \geq 50\%$

Calculation: Efficiency is defined as $\eta = \frac{T_{trans}}{T_{trans} + 2T_p}$, where $T_{trans} = \frac{L}{R}$.

$$\frac{L/R}{L/R + 2T_p} \geq 0.5$$

$$\frac{L}{L + 2RT_p} \geq \frac{1}{2}$$

$$2L \geq L + 2RT_p \implies L \geq 2RT_p$$

Substituting values:

$$L \geq 2 \times 4000 \times 0.02 = 160 \text{ bits}$$

Answer: The frame size must be at least **160 bits**.

5 Problem 5: Probability Analysis

Given: Frame size $n = 4$ bits, BER $p = 10^{-3}$.

(a) **Probability frame has no errors:**

$$P(\text{No Error}) = (1 - p)^n = (1 - 0.001)^4 = (0.999)^4 \approx 0.9960$$

(b) **Probability frame has at least one error:**

$$P(\text{Error}) = 1 - P(\text{No Error}) = 1 - 0.9960 = 0.0040$$

(c) **With Parity Bit ($n = 5$): Probability of undetected error:** A simple parity check fails to detect an error if an *even* number of bits flip (2 or 4 bits).

$$P(\text{Undetected}) = P(2 \text{ errors}) + P(4 \text{ errors})$$

$$P(k \text{ errors}) = \binom{n}{k} p^k (1-p)^{n-k}$$

For $k = 2$:

$$\binom{5}{2} (10^{-3})^2 (0.999)^3 = 10 \times 10^{-6} \times 0.997 \approx 1.0 \times 10^{-5}$$

For $k = 4$:

$$\binom{5}{4} (10^{-3})^4 (0.999)^1 \approx 5 \times 10^{-12} \text{ (negligible)}$$

Answer: The probability of an undetected error is approximately 10^{-5} .

6 Problem 6: CRC Calculation

Problem: Given Generator $P = 110011$ and Message $M = 11100011$. Find the CRC.

Step 1: Setup

- Degree of P is 5.
- Append 5 zeros to M : $M' = 1110001100000$.
- Perform Modulo-2 Division (XOR).

Step 2: Division Process

$$\begin{array}{r}
 & 10110110 \quad (\text{Quotient}) \\
 \hline
 110011) & 1110001100000 \\
 & 110011 \\
 \hline
 & 010111 \\
 & 000000 \\
 \hline
 & 101111 \\
 & 110011 \\
 \hline
 & 111000 \\
 & 110011 \\
 \hline
 & 010110 \\
 & 000000 \\
 \hline
 & 101100 \\
 & 110011 \\
 \hline
 & 111110 \\
 & 110011 \\
 \hline
 & 011010 \\
 & 000000 \\
 \hline
 & 11010 \quad (\text{Remainder})
 \end{array}$$

Result:

- **Remainder (CRC):** 11010
- **Transmitted Frame:** 1110001111010

7 Problem 7: Polynomial CRC

(a) Encoding

Given:

- Generator: $P(x) = X^4 + X + 1 \implies$ Binary: **10011**
- Message: $M = \textbf{10010011011}$
- Append 4 zeros ($r = 4$): $M' = \textbf{100100110110000}$

Division Result: Performing the modulo-2 division of M' by P yields a remainder of **1100**.

Transmitted Frame (T):

$$T = 100100110111100$$

(b) Error Pattern E_1

Given:

- Error Pattern: $E_1 = 100010000000000$ (Bits flipped at pos 1 and 5)
- Received Frame: $R_1 = T \oplus E_1$

To check for detection, we can simply divide the Error Pattern E_1 by the Generator P . If the remainder is non-zero, the error is detected.

Analysis: E_1 corresponds to polynomial $x^{14} + x^{10}$. Dividing E_1 (10001000000000) by P (10011):

Remainder Calculation:

$$10001000000000 / 10011 \Rightarrow \text{Remainder} = 1110$$

Conclusion: Since the remainder (**1110**) is not zero, the error is **DETECTED**.

(c) Error Pattern E_2

Given:

- Error Pattern: $E_2 = 100110000000000$

Analysis: Notice that the error pattern bits **10011** match the Generator P exactly, followed by zeros. Mathematically, this error polynomial is:

$$E_2(x) = x^{10} \cdot (x^4 + x + 1) = x^{10} \cdot P(x)$$

Since $E_2(x)$ is a multiple of the generator $P(x)$, the division will be perfect.

Remainder Calculation:

100110000000000 / 10011 => Remainder = 0000

Conclusion: Since the remainder is **0000**, the receiver assumes the frame is valid.
The error is **UNDETECTED**.