DATA VISUALIZATION AND MODELLING

Bayesian Networks

Assignment #3
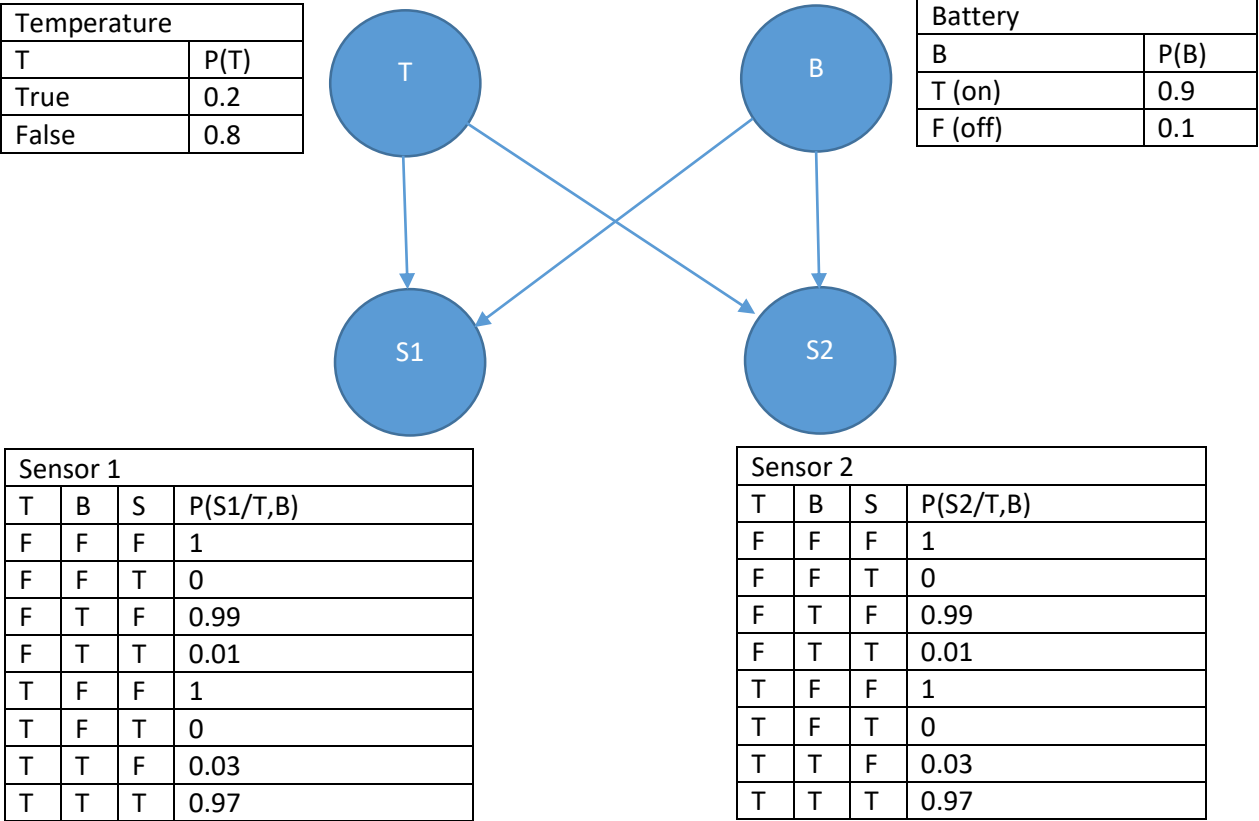
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We have two sensors that are meant to detect extreme temperature, which occurs 20% of the time. The sensors have identical specifications with a false positive rate of 1% and a false negative rate of 3%. If the power is off (dead battery), the sensors will read negative regardless of the temperature. Suppose now that we have two sensor kits: Kit A where both sensors receive power from the same battery, and Kit B where they receive power from independent batteries. Assuming that each battery has a 0.9 probability of power availability, compute the probability of extreme temperature given each of the following scenarios:

(a) The two sensors read negative.

(b) The two sensors read positive.

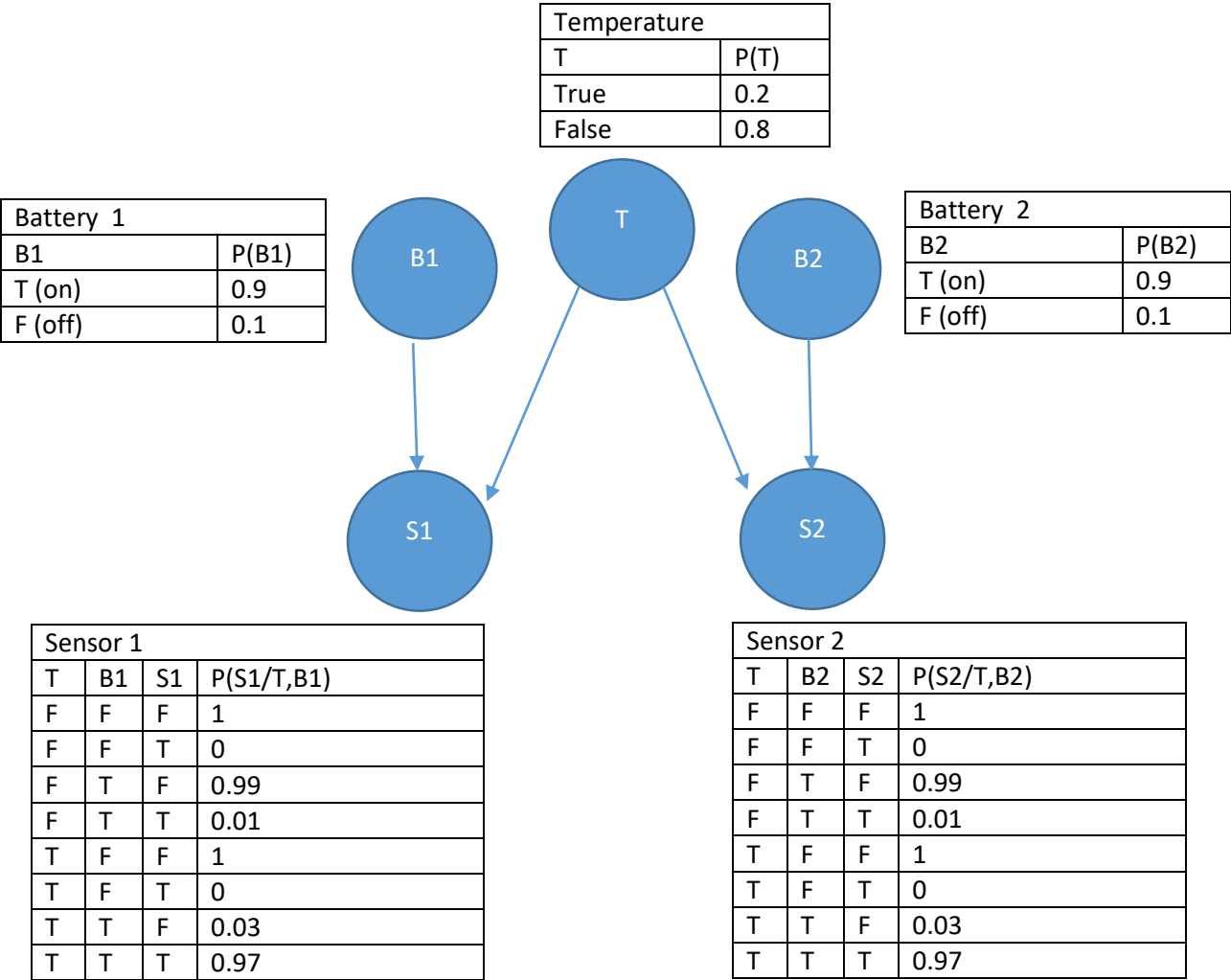(c) One sensor reads positive while the other reads negative.

Answer the previous questions with respect to each of the two kits. You must use direct computation "by hand" as well as gRain. In which of the three cases the answer is the same for the two kits? Why?

For solving the task, we should make Bayesian Network for each Kit. According to task we have three variables: temperature, batteries and sensors.

For Kit A Bayesian Network will be:

Temperature	
T	P(T)
True	0.2
False	0.8

Battery	
B	P(B)
T (on)	0.9
F (off)	0.1

Sensor 1			
T	B	S	P(S1/T,B)
F	F	F	1
F	F	T	0
F	T	F	0.99
F	T	T	0.01
T	F	F	1
T	F	T	0
T	T	F	0.03
T	T	T	0.97

Sensor 2			
T	B	S	P(S2/T,B)
F	F	F	1
F	F	T	0
F	T	F	0.99
F	T	T	0.01
T	F	F	1
T	F	T	0
T	T	F	0.03
T	T	T	0.97

For Kit B Bayesian Network will be:

Temperature	
T	P(T)
True	0.2
False	0.8

Battery 1	
B1	P(B1)
T (on)	0.9
F (off)	0.1

Battery 2	
B2	P(B2)
T (on)	0.9
F (off)	0.1

Sensor 1			
T	B1	S1	P(S1/T,B1)
F	F	F	1
F	F	T	0
F	T	F	0.99
F	T	T	0.01
T	F	F	1
T	F	T	0
T	T	F	0.03
T	T	T	0.97

Sensor 2			
T	B2	S2	P(S2/T,B2)
F	F	F	1
F	F	T	0
F	T	F	0.99
F	T	T	0.01
T	F	F	1
T	F	T	0
T	T	F	0.03
T	T	T	0.97

Kit A.

Task A.

$$P(T = T/S1 = F, S2 = F) = \frac{P(S1 = F, S2 = F|T = T) * P(T = T)}{P(S1 = F, S2 = F)}$$

$$P(S1 = F, S2 = F|T = T)$$

$$= P(S1 = F, S2 = F|T = T, B = T) * P(B = T) + P(S1 = F, S2 = F|T = T, B = F) * P(B = F)$$
$$= P(S1 = F|T = T, B = T) * P(S2 = F|T = T, B = T) * P(B = T)$$
$$+ P(S1 = F|T = T, B = F) * P(S2 = F|T = T, B = F) * P(B = F)$$

$$= 0.03 * 0.03 * 0.9 + 1 * 1 * 0.1 = 0.10081$$

$$P(S1 = F, S2 = F) =$$

$$P(S1 = F|T = T, B = T) * P(S2 = F|T = T, B = T) * P(T = T) * P(B = T)$$
$$+ P(S1 = F|T = T, B = F) * P(S2 = F|T = T, B = F) * P(T = T) * P(B = F)$$
$$+ P(S1 = F|T = F, B = F) * P(S2 = F|T = F, B = F) * P(T = F) * P(B = F)$$
$$+ P(S1 = F|T = F, B = T) * P(S2 = F|T = F, B = T) * P(T = F) * P(B = T)$$

$$= 0.03 * 0.03 * 0.2 * 0.9 + 1 * 1 * 0.2 * 0.1 + 1 * 1 * 0.8 * 0.1 + 0.99 * 0.99 * 0.8 * 0.9$$
$$= 0.000162 + 0.02 + 0.08 + 0.706 = 0.806$$

$$P(T = T/S1 = F, S2 = F) = \frac{0.10081 * 0.2}{0.806} = 0.025$$

Task B.

$$P(T = T/S1 = T, S2 = T) = \frac{P(S1 = T, S2 = T|T = T) * P(T = T)}{P(S1 = T, S2 = T)}$$

$$P(S1 = T, S2 = T|T = T)$$

$$= P(S1 = T, S2 = T|T = T, B = T) * P(B = T) + P(S1 = T, S2 = T|T = T, B = F) * P(B = F)$$
$$= P(S1 = T|T = T, B = T) * P(S2 = T|T = T, B = T) * P(B = T)$$
$$+ P(S1 = T|T = T, B = F) * P(S2 = T|T = T, B = F) * P(B = F)$$

$$= 0.97 * 0.97 * 0.9 + 0 * 0 * 0.1 = 0.85$$

$$P(S1 = T, S2 = T) =$$

$$P(S1 = T|T = T, B = T) * P(S2 = T|T = T, B = T) * P(T = T) * P(B = T)$$
$$+ P(S1 = T|T = T, B = F) * P(S2 = T|T = T, B = F) * P(T = T) * P(B = F)$$
$$+ P(S1 = T|T = F, B = F) * P(S2 = T|T = F, B = F) * P(T = F) * P(B = F)$$
$$+ P(S1 = T|T = F, B = T) * P(S2 = T|T = F, B = T) * P(T = F) * P(B = T)$$

$$= 0.97 * 0.97 * 0.2 * 0.9 + 0 * 0 * 0.2 * 0.1 + 0 * 0 * 0.8 * 0.1 + 0.01 * 0.01 * 0.8 * 0.9$$
$$= 0.169362 + 0.000072 = 0.169434$$

$$P(T = T/S1 = F, S2 = F) = \frac{0.85 * 0.2}{0.169434} \approx 0.99995$$

Task C.

$$P(T = T/S1 = F, S2 = T) = \frac{P(S1 = F, S2 = T|T = T) * P(T = T)}{P(S1 = F, S2 = T)}$$

$$P(S1 = F, S2 = T | T = T)$$

$$= P(S1 = F, S2 = T | T = T, B = T) * P(B = T) + P(S1 = F, S2 = T | T = T, B = F) * P(B = F)$$
$$= P(S1 = F | T = T, B = T) * P(S2 = T | T = T, B = T) * P(B = T)$$
$$+ P(S1 = F | T = T, B = F) * P(S2 = T | T = T, B = F) * P(B = F)$$

$$= 0.03 * 0.97 * 0.9 = 0.026$$

$$P(S1 = F, S2 = T) =$$

$$P(S1 = F | T = T, B = T) * P(S2 = T | T = T, B = T) * P(T = T) * P(B = T)$$
$$+ P(S1 = F | T = T, B = F) * P(S2 = T | T = T, B = F) * P(T = T) * P(B = F)$$
$$+ P(S1 = F | T = F, B = F) * P(S2 = T | T = F, B = F) * P(T = F) * P(B = F)$$
$$+ P(S1 = F | T = F, B = T) * P(S2 = T | T = F, B = T) * P(T = F) * P(B = T)$$

$$= 0.03 * 0.97 * 0.2 * 0.9 + 0.99 * 0.01 * 0.8 * 0.9 = 0.005 + 0.007 = 0.012$$

$$P(T = T / S1 = F, S2 = F) = \frac{0.026 * 0.2}{0.012} = 0.43$$

Kit B.

Task A.

$$P(T = T / S1 = F, S2 = F) = \frac{P(S1 = F, S2 = F | T = T) * P(T = T)}{P(S1 = F, S2 = F)}$$

$$P(S1 = F, S2 = F | T = T)$$

$$P(S1 = F, S2 = F | T = T, B1 = T, B2 = T) P(B1 = T, B2 = T)$$
$$+ P(S1 = F, S2 = F | T = T, B1 = T, B2 = F) P(B1 = T, B2 = F)$$
$$+ P(S1 = F, S2 = F | T = T, B1 = F, B2 = F) P(B1 = F, B2 = F)$$
$$+ P(S1 = F, S2 = F | T = T, B1 = F, B2 = T) P(B1 = F, B2 = T)$$

$$= P(S1 = F | T = T, B1 = T, B2 = T) * P(S2 = F | T = T, B1 = T, B2 = T) * P(B1 = T, B2 = T) +$$

$$P(S1 = F | T = T, B1 = T, B2 = F) * P(S2 = F | T = T, B1 = T, B2 = F) * P(B1 = T, B2 = F) +$$

$$P(S1 = F | T = T, B1 = F, B2 = F) * P(S2 = F | T = T, B1 = F, B2 = F) * P(B1 = F, B2 = F) +$$

$$P(S1 = F | T = T, B1 = T, B2 = T) * P(S2 = F | T = T, B1 = T, B2 = T) * P(B1 = T, B2 = T) =$$

$$P(S1 = F | T = T, B1 = T) * P(S2 = F | T = T, B2 = T) * P(B1 = T, B2 = T) +$$

$$P(S1 = F | T = T, B1 = T) * P(S2 = F | T = T, B2 = F) * P(B1 = T, B2 = F) +$$

$$P(S1 = F | T = T, B1 = F) * P(S2 = F | T = T, B2 = F) * P(B1 = F, B2 = F) +$$

$$P(S1 = F | T = T, B1 = F) * P(S2 = F | T = T, B2 = T) * P(B1 = F, B2 = T)$$

$$= 0.03 * 0.03 * 0.9 * 0.9 + 0.03 * 1 * 0.9 * 0.1 + 1 * 1 * 0.1 * 0.1 + 1 * 0.03 * 0.1 * 0.9$$
$$= 0.000729 + 0.0027 + 0.01 + 0.0027 = 0.016$$

$$P(S1 = F, S2 = F) =$$

$$P(S1=F, S2=F | T=T, B1=T, B2=T) P(T=T, B1=T, B2=T) +$$

$$P(S1=F, S2=F | T=T, B1=T, B2=F) P(T=T, B1=T, B2=F) +$$

$$P(S1=F, S2=F | T=T, B1=F, B2=F) P(T=T, B1=F, B2=F) +$$

$$P(S1=F, S2=F \mid T=T, B1=F, B2=T)P(T=T, B1=F, B2=T)+$$

$$P(S1=F, S2=F \mid T=F, B1=T, B2=T)P(T=F, B1=T, B2=T) +$$

$$P(S1=F, S2=F \mid T=F, B1=T, B2=F)P(T=F, B1=T, B2=F) +$$

$$P(S1=F, S2=F \mid T=F, B1=F, B2=F)P(T=F, B1=F, B2=F) +$$

$$P(S1=F, S2=F \mid T=F, B1=F, B2=T)P(T=F, B1=F, B2=T)=$$

$$P(S1=F \mid T=T, B1=T)*P(S2=F \mid T=T, B2=T)P(T=T, B1=T, B2=T) +$$

$$P(S1=F \mid T=T, B1=T)*P(S2=F \mid T=T, B2=F)P(T=T, B1=T, B2=F) +$$

$$P(S1=F \mid T=T, B1=F)*P(S2=F \mid T=T, B2=F)P(T=T, B1=F, B2=F) +$$

$$P(S1=F \mid T=T, B1=F)*P(S2=F \mid T=T, B2=T)P(T=T, B1=F, B2=T)+$$

$$P(S1=F \mid T=F, B1=T)*P(S2=F \mid T=F, B2=T)P(T=F, B1=T, B2=T) +$$

$$P(S1=F \mid T=F, B1=T)*P(S2=F \mid T=F, B2=F)P(T=F, B1=T, B2=F) +$$

$$P(S1=F \mid T=F, B1=F)*P(S2=F \mid T=F, B2=F)P(T=F, B1=F, B2=F) +$$

$$P(S1=F \mid T=F, B1=F)*P(S2=F \mid T=F, B2=T)P(T=F, B1=F, B2=T)=$$

0.03*0.03*0.2*0.9*0.9+0.03*1*0.2*0.9*0.1+1*1*0.2*0.1*0.1+1*0.03*0.2*0.1*0.9+0.99*0.99*0.8*0.9*0.9+0.99*1*0.8*0.9*0.1+1*1*0.8*0.1*0.1+1*0.99*0.8*0.9*0.1=0.00014+0.00054+0.002+0.00054+0.63+0.07+0.008+0.07=0.779

$$P(T=T/S1=F, S2=F) = \frac{0.016*0.2}{0.779} = 0.0041$$

Task B.

$$P(T=T/S1=T, S2=T) = \frac{P(S1=T, S2=T \mid T=T)*P(T=T)}{P(S1=T, S2=T)}$$

$$P(S1=T, S2=T \mid T=T)$$

$$P(S1=T, S2=T \mid T=T, B1=T, B2=T)P(B1=T, B2=T)$$
$$+ P(S1=T, S2=T \mid T=T, B1=T, B2=F)P(B1=T, B2=F)$$
$$+ P(S1=T, S2=T \mid T=T, B1=F, B2=F)P(B1=F, B2=F)$$
$$+ P(S1=T, S2=T \mid T=T, B1=F, B2=T)P(B1=F, B2=T)$$

$$= P(S1=T \mid T=T, B1=T, B2=T)*P(S2=T \mid T=T, B1=T, B2=T)*P(B1=T, B2=T) +$$

$$P(S1=T \mid T=T, B1=T, B2=F)*P(S2=T \mid T=T, B1=T, B2=F)*P(B1=T, B2=F) +$$

$$P(S1=T \mid T=T, B1=F, B2=F)*P(S2=T \mid T=T, B1=F, B2=F)*P(B1=F, B2=F) +$$

$$P(S1=T \mid T=T, B1=T, B2=T)*P(S2=T \mid T=T, B1=T, B2=T)*P(B1=T, B2=T) =$$

$$P(S1=T \mid T=T, B1=T)*P(S2=T \mid T=T, B2=T)*P(B1=T, B2=T) +$$

$$P(S1=T \mid T=T, B1=T)*P(S2=T \mid T=T, B2=F)*P(B1=T, B2=F) +$$

$$P(S1=T \mid T=T, B1=F)*P(S2=T \mid T=T, B2=F)*P(B1=F, B2=F) +$$

$$P(S1=T \mid T=T, B1=F)*P(S2=T \mid T=T, B2=T)*P(B1=F, B2=T)$$

$$= \ 0.97*0.97*0.9*0.9 = 0.75$$

$$P(S1 = T, S2 = T) =$$

P(S1=T, S2=T |T=T, B1=T, B2=T)P(T=T, B1=T, B2=T) +

P(S1=T, S2=T |T=T, B1=T, B2=F)P(T=T, B1=T, B2=F) +

P(S1=T, S2=T |T=T, B1=F, B2=F)P(T=T, B1=F, B2=F) +

P(S1=T, S2=T | T=T, B1=F, B2=T)P(T=T, B1=F, B2=T)+

P(S1=T, S2=T |T=F, B1=T, B2=T)P(T=F, B1=T, B2=T) +

P(S1=T, S2=T |T=F, B1=T, B2=F)P(T=F, B1=T, B2=F) +

P(S1=T, S2=T |T=F, B1=F, B2=F)P(T=F, B1=F, B2=F) +

P(S1=T, S2=T | T=F, B1=F, B2=T)P(T=F, B1=F, B2=T)=

P(S1=T |T=T, B1=T)*P(S2=T|T=T,B2=T)P(T=T, B1=T, B2=T) +

P(S1=T |T=T, B1=T)*P(S2=T|T=T,B2=F)P(T=T, B1=T, B2=F) +

P(S1=T |T=T, B1=F)*P(S2=T|T=T,B2=F)P(T=T, B1=F, B2=F) +

P(S1=T| T=T, B1=F)*P(S2=T|T=T,B2=T)P(T=T, B1=F, B2=T)+

P(S1=T |T=F, B1=T)*P(S2=T|T=F,B2=T)P(T=F, B1=T, B2=T) +

P(S1=T|T=F, B1=T)*P(S2=T|T=F,B2=F)P(T=F, B1=T, B2=F) +

P(S1=T|T=F, B1=F)*P(S2=T|T=F,B2=F)P(T=F, B1=F, B2=F) +

P(S1=T| T=F, B1=F) *P(S2=T|T=F,B2=T)P(T=F, B1=F, B2=T)=

0.97*0.97*0.2*0.9*0.9+0.01*0.01*0.8*0.9*0.9=0.15+0.0000648≈0.15

$$P(T = T/S1 = T, S2 = T) = \frac{0.75 * 0.2}{0.150000648} \approx 0.999$$

Task C.

$$P(T = T/S1 = F, S2 = T) = \frac{P(S1 = F, S2 = T|T = T) * P(T = T)}{P(S1 = F, S2 = T)}$$

$$P(S1 = F, S2 = T|T = T)$$

$$P(S1 = F, S2 = T |T = T, B1 = T, B2 = T)P(B1 = T, B2 = T)$$
$$+ \ P(S1 = F, S2 = T|T = T, B1 = T, B2 = F)P(B1 = T, B2 = F)$$
$$+ \ P(S1 = F, S2 = T |T = T, B1 = F, B2 = F)P(B1 = F, B2 = F)$$
$$+ \ P(S1 = F, S2 = T|T = T, B1 = F, B2 = T)P(B1 = F, B2 = T)$$

$$= \ P(S1 = F|T = T, B1 = T, B2 = T) * P(S2 = T|T = T, B1 = T, B2 = T) * P(B1 = T, B2 = T) +$$

$$P(S1 = F|T = T, B1 = T, B2 = F) * P(S2 = T|T = T, B1 = T, B2 = F) * P(B1 = T, B2 = F) +$$

$$P(S1 = F|T = T, B1 = F, B2 = F) * P(S2 = T|T = T, B1 = F, B2 = F) * P(B1 = F, B2 = F) +$$

$$P(S1 = F|T = T, B1 = T, B2 = T) * P(S2 = T|T = T, B1 = T, B2 = T) * P(B1 = T, B2 = T) =$$

$$P(S1 = F|T = T, B1 = T) * P(S2 = T|T = T, B2 = T) * P(B1 = T, B2 = T) +$$

$$P(S1 = F|T = T, B1 = T) * P(S2 = T|T = T, B2 = F) * P(B1 = T, B2 = F) +$$

$$P(S1 = F|T = T, B1 = F) * P(S2 = T|T = T, B2 = F) * P(B1 = F, B2 = F) +$$

$$P(S1 = F|T = T, B1 = F) * P(S2 = T|T = T, B2 = T) * P(B1 = F, B2 = T)$$

$$= 0.03 * 0.97 * 0.9 * 0.9 + 1 * 0.97 * 0.9 * 0.1 = 0.023 + 0.0873 = 0.1103$$

$$P(S1 = F, S2 = T) =$$

$$P(S1=F, S2=T \,|T=T, B1=T, B2=T\,)P(T=T, B1=T, B2=T\,) +$$

$$P(S1=F, S2=T \,|T=T, B1=T, B2=F\,)P(T=T, B1=T, B2=F\,) +$$

$$P(S1=F, S2=T \,|T=T, B1=F, B2=F\,)P(T=T, B1=F, B2=F\,) +$$

$$P(S1=F, S2=T \,| T=T, B1=F, B2=T\,)P(T=T, B1=F, B2=T\,)+$$

$$P(S1=F, S2=T \,|T=F, B1=T, B2=T\,)P(T=F, B1=T, B2=T\,) +$$

$$P(S1=F, S2=T \,|T=F, B1=T, B2=F\,)P(T=F, B1=T, B2=F\,) +$$

$$P(S1=F, S2=T \,|T=F, B1=F, B2=F\,)P(T=F, B1=F, B2=F\,) +$$

$$P(S1=F, S2=T \,| T=F, B1=F, B2=T\,)P(T=F, B1=F, B2=T\,)=$$

$$P(S1=F\,|T=T, B1=T\,)*P(S2=T|T=T,B2=T)P(T=T, B1=T, B2=T\,) +$$

$$P(S1=F\,|T=T, B1=T\,)*P(S2=T|T=T,B2=F)P(T=T, B1=T, B2=F\,) +$$

$$P(S1=F\,|T=T, B1=F\,)*P(S2=T|T=T,B2=F)P(T=T, B1=F, B2=F\,) +$$

$$P(S1=F|\, T=T, B1=F\,)*P(S2=T|T=T,B2=T)P(T=T, B1=F, B2=T\,)+$$

$$P(S1=F\,|T=F, B1=T\,)*P(S2=T|T=F,B2=T)P(T=F, B1=T, B2=T\,) +$$

$$P(S1=F|T=F, B1=T\,\,)*P(S2=T|T=F,B2=F)P(T=F, B1=T, B2=F\,) +$$

$$P(S1=F|T=F, B1=F\,)*P(S2=T|T=F,B2=F)P(T=F, B1=F, B2=F\,) +$$

$$P(S1=F|\, T=F, B1=F\,) *P(S2=T|T=F,B2=T)P(T=F, B1=F, B2=T\,)=$$

$$0.03*0.97*0.2*0.9*0.9+1*0.97*0.2*0.1*0.9+0.99*0.01*0.8*0.9*0.9+1*0.01*0.8*0.1*0.9=$$

$$0.0047+0.02+0.0064+0.00072=0.03182$$

$$P(T = T/S1 = F, S2 = T) = \frac{0.1103 * 0.2}{0.03182} \approx 0.7$$

In which of the three cases the answer is the same for the two kits? Why?

In the case when both sensors measure "true the probability of having high temperature, i.e, T = T is the same for both kits A and B, since the fact that the sensors are true determine that the batteries are on (otherwise, if they were off, then we know that the sensors would measure false with probability 1). Thus, in this case the role of the batteries is not influencing the probability of observing or not observing T, and that is why the results are the same for both kits and in this particular this Task B.