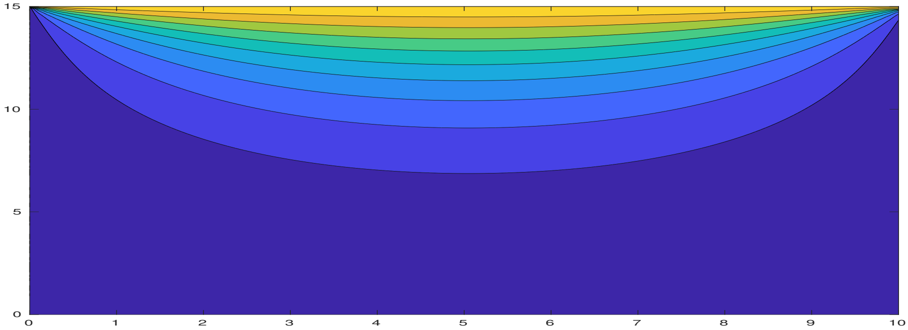
Szélessávú Hírközlés és Villamosságtan Tanszék
Antennák és EMC kutatócsoport
NES Laboratórium

# MűFi23

## PDEs - Partial Differential Equations



- PDE - Partial Differential Equation
- equation stating a relationship between a function of two or more independent variables and the partial derivatives of the function respect to these independent variables
- space variables $(x, y, z)$ or space and time variables $(x, y, z, t)$
- examples of these equations :

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0 \qquad f_{xx} + f_{yy} = 0$$

$$\frac{\partial f}{\partial t} = \alpha \frac{\partial^2 f}{\partial x^2} \qquad f_t = \alpha f_{xx}$$

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2} \qquad f_{tt} = c^2 f_{xx}$$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- in Cartesian coordinates :

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\text{so } f_t = \alpha(f_{xx} + f_{yy} + f_{zz}) \quad \Rightarrow \quad f_t = \nabla^2 f$$

- other than second-order PDEs

$$a f_t + b f_x = 0$$

$$f_{xxxx} + F_{yyyy} + f_{xxyy} =$$

- linear or non-linear

$$\text{linear : } a f_t + b \cdot x \cdot f_x = 0$$

$$\text{non-linear : } f \cdot f_x + b \; f_y = 0 \quad \text{or} \quad a f_x^2 + b f_y = 0$$


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- general quasi-linear nonhomogeneous PDE in two variables >

$$Af_{xx} + Bf_{xy} + Cf_{yy} + Df_x + Ef_y + Ff = G$$

- sign of discriminant : $B^2 - 4AC$ means types

| $B^2 - 4AC$ | Classification |
|---|---|
| Negative ($< 0$) | Elliptic |
| Zero ($= 0$) | Parabolic |
| Positive ($> 0$) | Hyperbolic |

- characteristics of PDE : $(n-1)$ dimensional hypersurface that have special features - ( in 2D cases these curves are paths) - information propagates throughout the solution domain along the characteristics paths
- if a PDE possesses real characteristics, then information propagates along these characteristics


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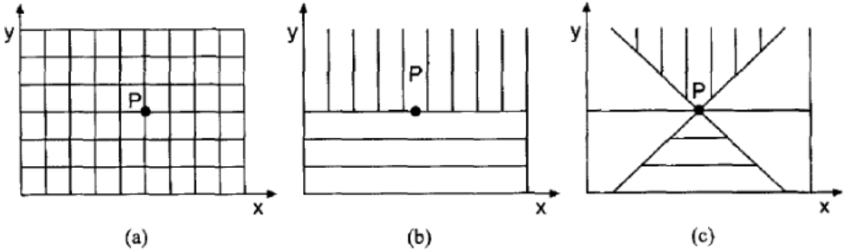
# DoD and RoI

- Domain of Dependence (DoD) - region of the solution domain upon which the solution at point $P, f(x_P, y_P)$ depends –
  $f(x_P, y_P)$ depends on everything that has happened in DoD
- Range of influence (RoI) - the region of the solution domain in which the solution $f(x, y)$ is influenced by the solution at point P –
  $f(x_P, y_P)$ influences the solution at all points in RoI





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- Equilibrium problems
  - steady-state problems in closed domains $D(x, y)$
  - jury problems - entire solution is passed on by a jury requiring satisfaction of all internal requirement(i.e. the PDE) and all the boundary conditions simultaneously
- example - steady heat diffusion (i.e. conduction) in a solid

$$\nabla^2 T = 0$$

where $T$ is the temperature of the solid, in 2D

$$T_{xx} + T_{yy} = 0$$

$$\text{with BC :} \qquad aT + bT_n = c$$

at each point other boundary


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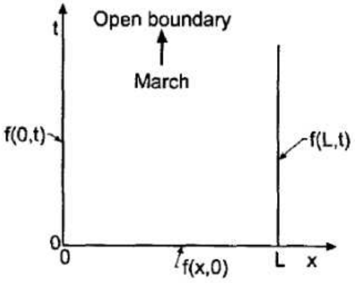
- initial value problems in open domains (open with respect to one of the independent variables)





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