RD-1

1.) Radioactive decay

$A = -\frac{dN}{dt}$ radioactive activity

1 Bq = 1 Becquerel
(1 transformation
[or decay or disintegration]
in 1 second)

1 Ci (1 Curie)
exp. quantity

$$\boxed{-\frac{dN}{dt} = \lambda N}$$

· half-life
· third-life
·

One-decay process

Two-decay chain

$A \longrightarrow B \longrightarrow C$

$$\left.\begin{aligned} \frac{dN_A}{dt} &= \lambda_A \cdot N_A \\ \frac{dN_B}{dt} &= -\lambda_B N_B + \lambda_A N_A \end{aligned}\right\}$$

$\boxed{\text{RD-2}}$

Multiple products

$$
\begin{tikzcd}
A \arrow[r, "\lambda_1"] \arrow[d, "\lambda_3"'] & B \arrow[r, "\lambda_2"] & D \\
C \arrow[rru, "\lambda_4"'] & &
\end{tikzcd}
$$

$$\frac{dN_A}{dt} = -\lambda_1 N_A - \lambda_3 N_A$$

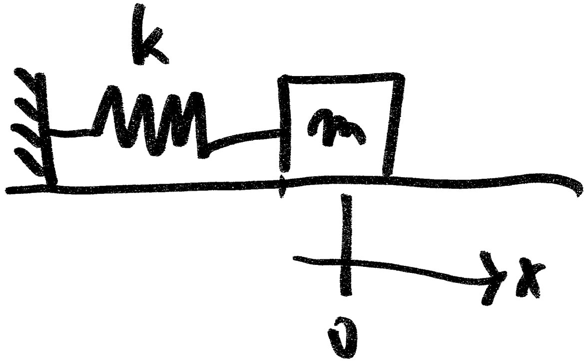
$$\frac{dN_B}{dt} = \lambda_1 N_A - \lambda_2 N_B$$

$$\frac{dN_C}{dt} = \lambda_3 N_A - \lambda_4 N_C$$

$$\frac{dN_D}{dt} = \lambda_2 N_B + \lambda_4 N_C$$

Mass-spring system

[SM 1]



$$m\ddot{x} = -kx$$

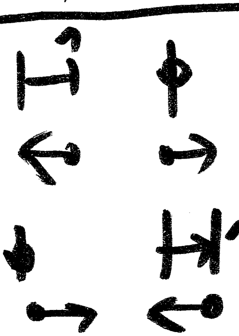
no loss effect

$$m\ddot{x} = -kx - \mu_0$$

constant loss

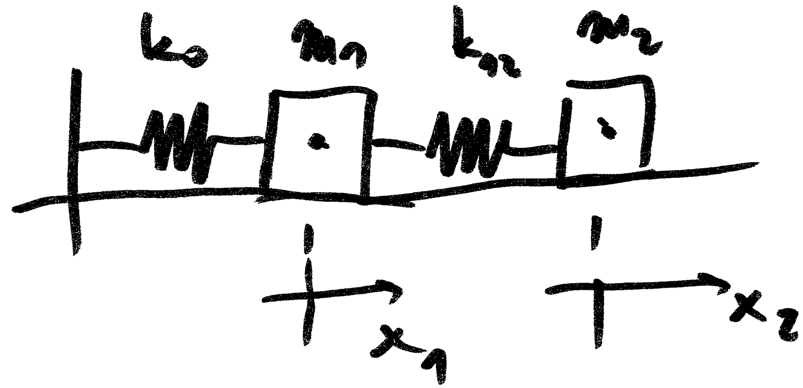
velocity dependent loss + external force

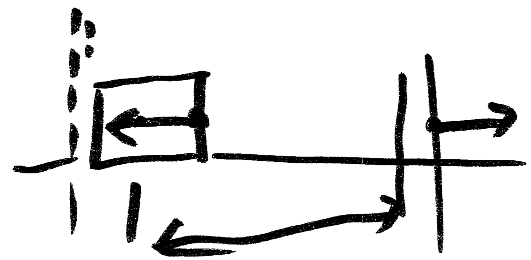
$$m\ddot{x} = -kx - d\dot{x} + f(t)$$



$(0-1)=-1$

$(1-0)=1$





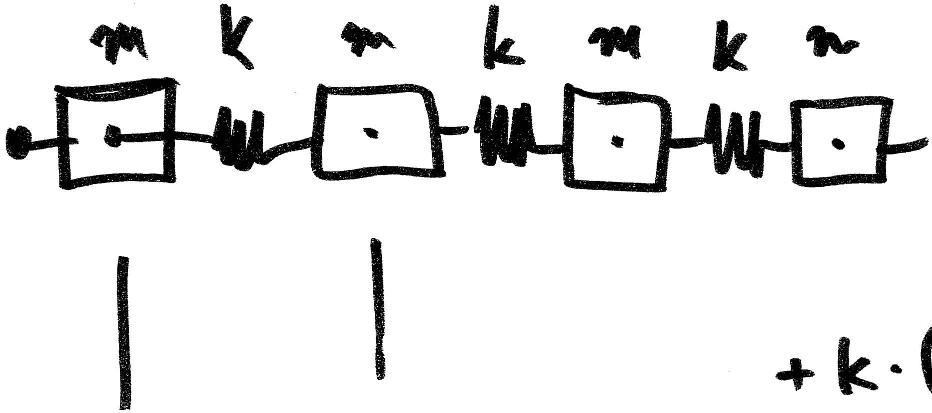
$k_{12}(x_2-x_1)$ →

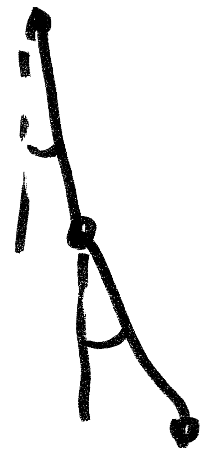
$-k_{12}(x_1-x_2)$

$$m_1\ddot{x}_1 = -k_0 x_1 - k_{12}(x_1-x_2)$$

$$m_2\ddot{x}_2 = -k_0 x_2 - k_{12}(x_2-x_1)$$

SM2

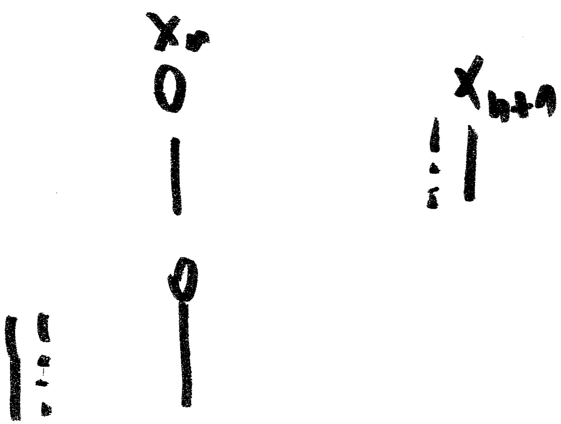




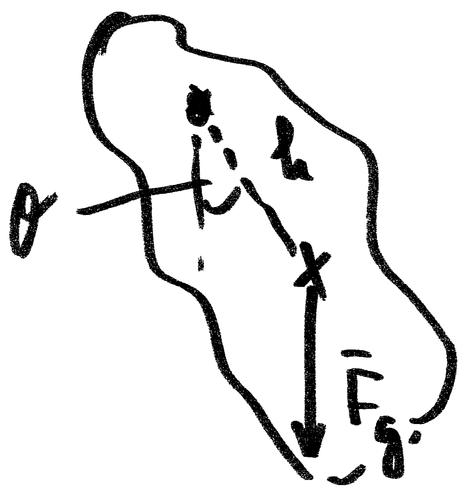
$+k\cdot(x_{n+1}-x_n)$

F_{R21} F_{R32} $k(x_n-x_{n-1})$

$$m_n\cdot\ddot{x}_n=k\cdot(x_{n+1}-x_n)-k(x_n-x_{n-1})$$



Phys Pen.



$M_g = T_g = I\cdot\alpha$ ← angular accel

moment of inertia

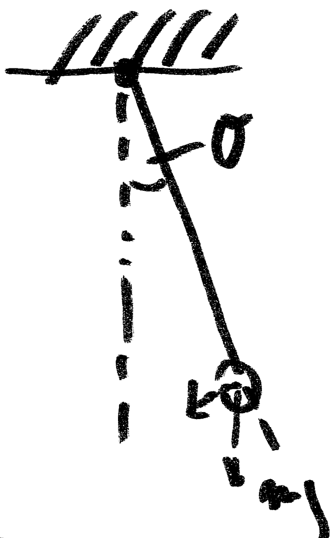
$$-m\cdot g\cdot h\cdot\sin\theta = I\cdot\alpha = I\cdot\frac{d^2\theta}{dt^2}$$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{m\cdot g\cdot h}{I}\cdot\sin\theta = -\omega^2\cdot\sin\theta}$$

↳ linearization $\quad \sin\theta = \theta +$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{l}\cdot\theta}$$

Mathematical pendulum



$$m\cdot l\cdot\frac{d^2\theta}{dt^2} = -m\cdot g\cdot\sin\theta$$

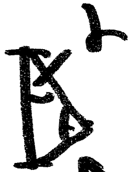
$$F = mg\cdot\sin\theta$$

$$l\cdot\frac{d^2\theta}{dt^2} = -g\cdot\sin\theta$$

$$s = l\cdot\theta$$

$$\frac{ds}{dt} = l\cdot\frac{d\theta}{dt} \;\Big|\; \frac{d}{dt}\left(\frac{ds}{dt}\right) = l\cdot\frac{d^2\theta}{dt^2}$$

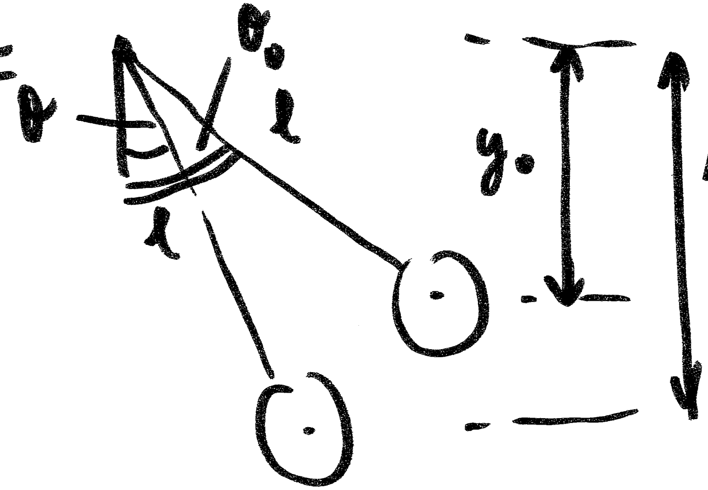
$$\boxed{\frac{d^2\theta}{dt^2} + \frac{g}{l}\cdot\sin\theta = 0}$$



$$\frac{F}{mg} = \sin\alpha$$

Energy based derivation

MP



$$y_0 = l \cdot \cos\theta_0$$

$$y_1 = l \cdot \cos\theta_1$$

$$h = l_1 - l_0 =$$

$$= l \cdot (\cos\theta - \cos\theta_0)$$

$$\frac{1}{2} m v^2 = mgh$$

$$v = \sqrt{2gh}$$

$$v = l \cdot \frac{d\theta}{dt} = \sqrt{2gh}$$

$$\frac{d\theta}{dt} = \frac{\sqrt{2gh}}{l}$$

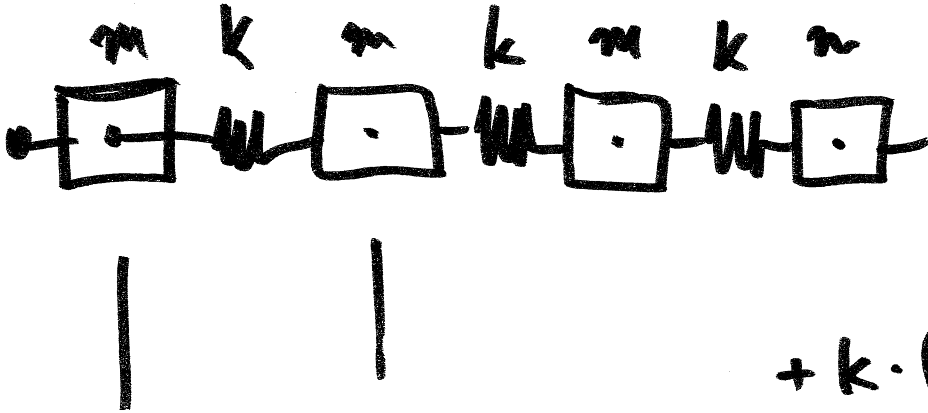
$$\frac{d\theta}{dt} = \sqrt{2g} \cdot \frac{1}{l} \sqrt{l \cdot (\cos\theta - \cos\theta_0)} = \sqrt{\frac{2g}{l}} \cdot \sqrt{\cos\theta - \cos\theta_0}$$

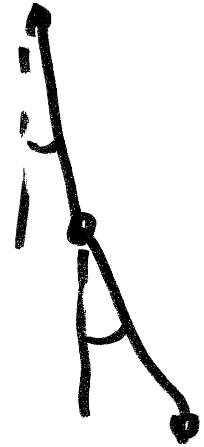
$$\frac{d^2\theta}{dt^2} = \sqrt{\frac{2g}{l}} \cdot \frac{1}{2} \cdot (\cos\theta - \cos\theta_0)^{-1/2} \cdot (-\sin\theta)\, \frac{d\theta}{dt}$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{2} \frac{\left(-\frac{2g}{l} \cdot \sin\theta\right)}{\cancel{\sqrt{\frac{2g}{l} \cdot (\quad)}}} \cdot \cancel{\sqrt{\frac{2g}{l} \cdot (\quad)}} = -\frac{g}{l} \cdot \sin\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{g}{l} \cdot \sin\theta = 0}$$

SM2





$+k\cdot(x_{n+1}-x_n)$

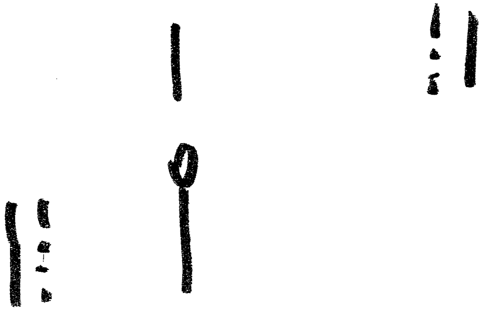
F_{R21} F_{R32}

$k(x_n-x_{n-1})$

$$m_n\cdot\ddot{x}_n=k\cdot(x_{n+1}-x_n)-k(x_n-x_{n-1})$$

x_n

x_{n+1}



$$0 = (m_1 + m_2)\, l_1^2 \cdot \ddot{\theta}_1 + m_2 \cdot l_1 l_2 \cdot \ddot{\theta}_2 \cdot \cos(\theta_2 - \theta_1) -$$

$$- m_2 l_1 l_2 \cdot \dot{\theta}_2^2 \cdot \sin(\theta_2 - \theta_1) + (m_1 + m_2)\, g\, l_1 \cdot \sin\theta_1$$

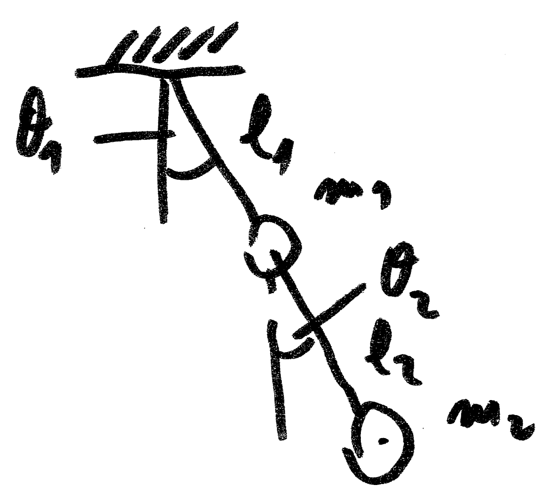
$$0 = m_2 \cdot l_2^2 \cdot \ddot{\theta}_2 + m_2 \cdot l_1 l_2 \cdot \ddot{\theta}_1 \cdot \cos(\theta_2 - \theta_1) +$$

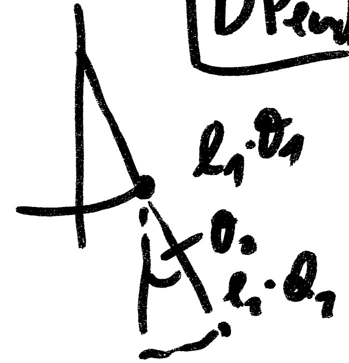
$$+ m_2 l_1 l_2 \dot{\theta}_1^2 \cdot \sin(\theta_2 - \theta_1) - m_2 g l_2 \cdot \sin\theta_2$$

$$\begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{pmatrix} 0 & B_1 \\ B_2 & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{pmatrix} + \begin{pmatrix} f_1(\theta_1) \\ f_2(\theta_2) \end{pmatrix}$$

DPend

1.) $L = T - V$





$T: \quad \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$v_1 = l_1 \cdot \dot{\theta}_1$

$$v_2 = \sqrt{l_1^2 \dot{\theta}_1^2 + l_2^2 \cdot \dot{\theta}_2^2 + 2 l_1 l_2 \cdot \cos(\theta_1 - \theta_2)}$$

$V: -m_1 \cdot g \cdot y_1 - m_2 \cdot g \cdot y_2$

$= -m_1 \cdot g \cdot l_1 \cdot \cos\theta_1 - m_2 g \left(l_1 \cdot \cos\theta_1 + l_2 \cdot \cos\theta_2 \right)$

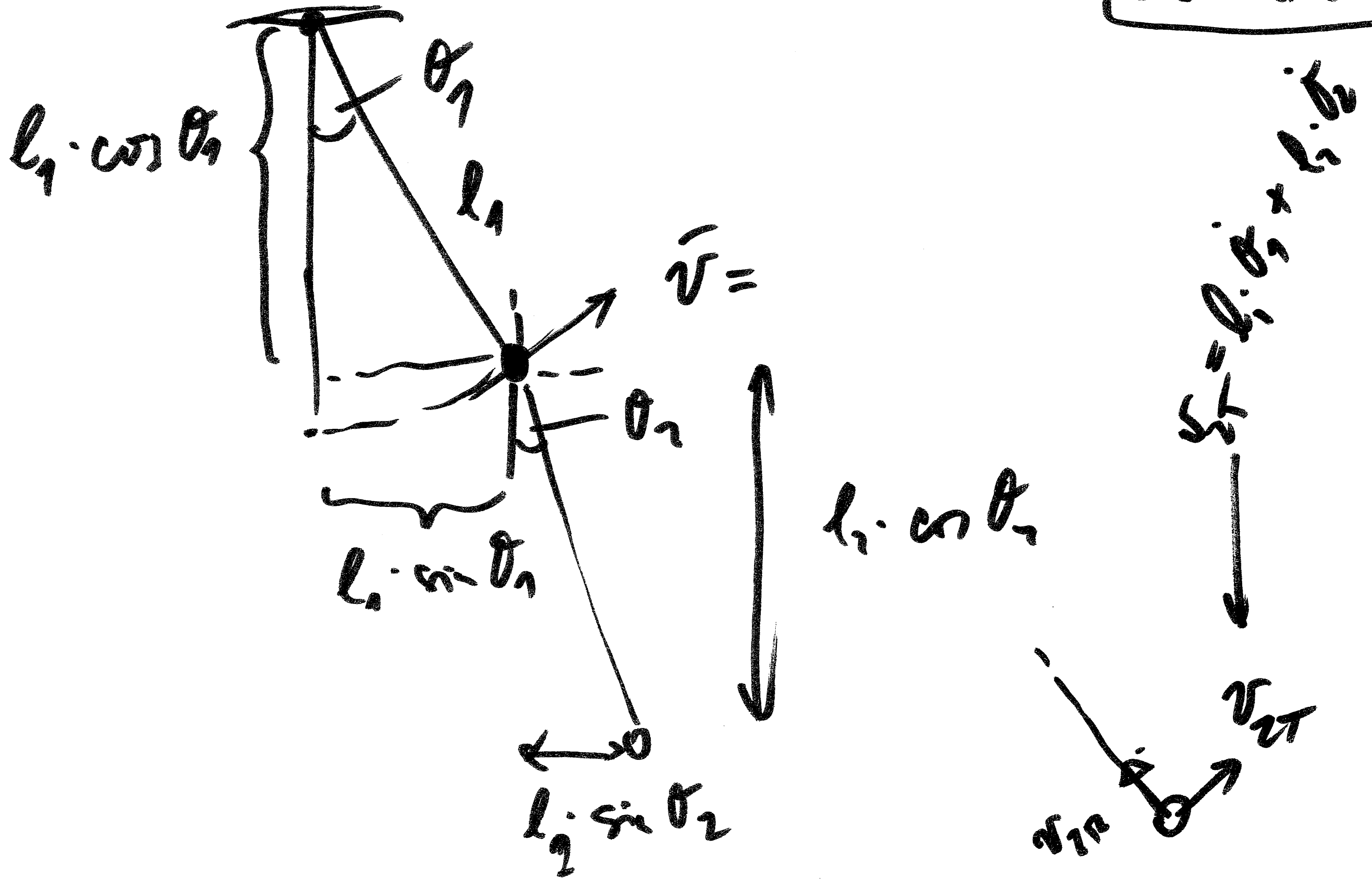
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_{i}}\right) - \frac{\partial L}{\partial q_{i}} = 0$$

$q_i \to \theta_i$

$\frac{\partial L}{\partial \dot{\theta}_1} = \ldots$ & $\frac{\partial L}{\partial \theta_1} = \ldots \Rightarrow 0 = \frac{d}{dt} \cdot \frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1}$

$\frac{\partial L}{\partial \dot{\theta}_2} = \ldots$ & $\frac{\partial L}{\partial \theta_2} = \ldots \Rightarrow 0 = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2}$

DoPend 2



$$v_{2R} = -l_1 \cdot \dot{\theta}_1 \cdot \sin(\theta_1 - \theta_2$$

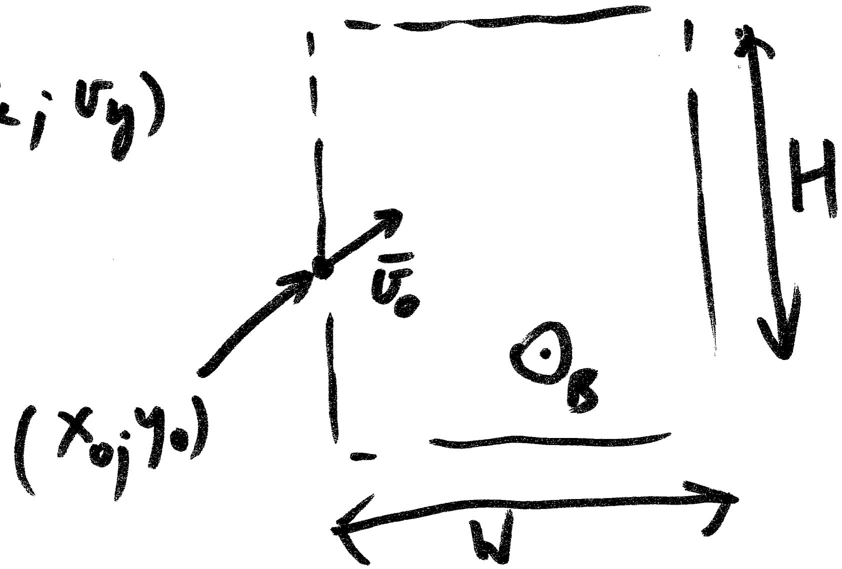
$$v_2 = \sqrt{v_{2R}^2 + v_{2T}^2} = \ldots$$

EOM (trajectory)

Equation **o**f **M**otion

m

$\bar{v} = (v_x ; v_y)$



in this case:

$$\bar{F} = q \cdot (\bar{v} \times \bar{B}) =$$

$$= q \cdot \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} =$$

$$= q \cdot \hat{e}_x (v_y \cdot B_z - v_z \cdot B_y) +$$

$$+ q \cdot \hat{e}_y (v_z \cdot B_x - v_x \cdot B_z) +$$

$$+ q \cdot \hat{e}_z (v_x \cdot B_y - B_x \cdot v_y)$$

$$\bar{F} = (F_x ; F_y) = \hat{e}_x \cdot F_x + \hat{e}_y \cdot F_y$$

$$\boxed{\begin{aligned} m \, a_x &= F_x \\ m \cdot a_y &= F_y \end{aligned}} \Leftrightarrow \boxed{\begin{aligned} v_x &= \dot{x} & v_y &= \dot{y} \\ \dot{v}_x &= \frac{1}{m} F_x & \dot{v}_y &= \frac{1}{m} F_y \end{aligned}}$$

unknowns: $\begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix}$

now $\bar{B} = e_z \cdot B_0$ i.e. $B_x = 0$

$B_y = 0$

$\rightarrow \boxed{\bar{F} = q\, v_y \cdot B_z \cdot \hat{e}_x - q\, v_x B_z\, \hat{e}_y}$ Eq 2