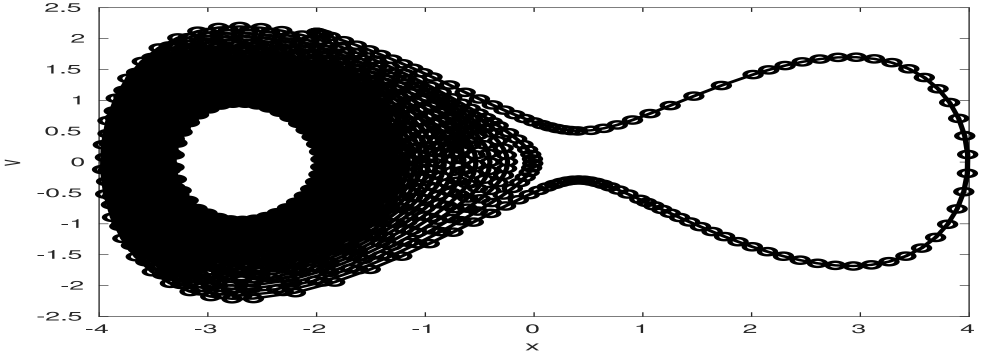
Szélessávú Hírközlés és Villamosságtan Tanszék
Antennák és EMC kutatócsoport
NES Laboratórium

MűFi23

ODEs - Applied Problems - HW assingments



reichardt.andras@vik.bme.hu

Reichardt András | (2023. október 12.)

ODE - Problem 1.

- Population growth of any species is frequently modeled by an ODE of the formed

$$\frac{dN}{dt} = aN - bN^2; \qquad N(0) = N_0$$

where N is the population, aN represents the birthrate, bN^2 represents the death rate due to all causes (such as disease, competition for fodd supplies, and so on).

- If $N_0 = 100000$, $a = 0.1$ and $b = 8 \cdot 10^{-7}$, calculate $N(1)$ for $t = 0$ to 20 years!

 reichardt.andras@vik.bme.hu

ODE - Problem 2.

- A lumped mass m initally at the temperature T_0 is cooled by convection to its surroundings ath the temperature T_a.
- From Newton's law of cooling, $\dot{q}_{conv} = hA(T - T_a)$, where h is the convective cooling coefficient and A is the surface area of the mass.
- The energy E stored in the mass is $E = m \cdot C \cdot T$, where C is the specific heat. From an energy balance, the rate of change of E must equal the rate of coolung due to convection $\dot{q}_{conv}$.

$$\frac{dT}{dt} = -\frac{hA}{mC} \cdot (T - T_a) \qquad T(0) = T_0$$

- Consider a sphere of radius $r = 1$ cm, made of an alloy for which $\varrho = 3000 kg/m^3$ and $C = 1000 J/(kg \cdot K)$.
- If $h = 500 \frac{J}{s \cdot m^2 \cdot K}$, $T(0) = 500C$ and $T_a = 50C$, calculate $T(t)$ for $t = 0$ to 10 s!

reichardt.andras@vik.bme.hu

Problem 3.

- A machine of mass m rests on a support that exerts both a damping force and a spring force on the machine. The support is subjected to the displacement

$$Y(t) = Y_0 \cdot \sin(\omega t)$$

- From Newton's second law of motion

$$m \cdot \frac{d^2y}{dt^2} = -C\left(\frac{\mathrm{d}y}{\mathrm{dt}} - \frac{\mathrm{d}Y}{\mathrm{dt}}\right) - K(y - Y)$$

  where $y - Y$ is the relative displacement between the machine and the support, C is the damping coefficient, and K is the spring constant.
- Determine the motion of the machine during the first cycle of oscillation of the support!
- $m = 1000$ kg, $C = 5000 N \cdot s/m$, $K = 50000 N/m$, $Y_0 = 1$ cm, $\omega = 100$ rad/s. Initial condtions are $y(0) = y'(0) = 0$.

reichardt.andras@vik.bme.hu

Problem 4.

- The population of two species competing for the same food supply can be modeled by the pair of ODEs:

$$\left.\begin{aligned} \frac{\mathrm{d}N_1}{\mathrm{d}t} &= N_1 \cdot (A_1 - B_1 N_1 - C_1 N_2) \qquad & N_1(0) = N_{1,0} \\ \frac{\mathrm{d}N_2}{\mathrm{d}t} &= N_2 \cdot (A_2 - B_2 N_2 - C_2 N_1) \qquad & N_2(0) = N_{2,0} \end{aligned}\right\}$$

  where AN is the birthrate, BN^2 models the death rate due to disease, and CN_1N_2 models the death rate due to competition for the food supply.
- Let's have $N_1(0) = N_2(0) = 100000$, $A_1 = 0.1$, $B_1 = 8 \cdot 10^{-7}$, $C_1 = 10^{-6}$, $A_2 = 0{,}1$, $B_2 = 8 \cdot 10^{-7}$ and $C_2 = 10^{-7}$.
- Calculate populations $N_1(t)$ and $N_2(t)$ for $t = 0$ to 10 years!

reichardt.andras@vik.bme.hu

Problem 5. - chemical reactions

- The inherent features of finite rate chemical reactions can be modeled by the prototype rate equation

$$\frac{\mathrm{d}C}{\mathrm{dt}} = \frac{C_e - C}{\tau} \qquad C(0) = C_0$$

where C is the instantaneous nonequilibrium mass fraction of the the species under consideration, C_e is its equilibrium mass fraction corresponding to the local conditions, and τ has the character of a chemical relaxaion time.

- Assume that C_e varies quadratically with time, that is

$$C_e = C_{e0} + \alpha \cdot t^2$$

- Let $C(0) = 0$, $C_e = 0.1$, $\alpha = 0.5$ and $\tau = 0.0001$.
- Solve for $C(t)$ from $t = 0$ to 0.01!

 reichardt.andras@vik.bme.hu

- The temperature distribution in the wall of a pipe through which a hot lkiquid is flowing is given by the ODE

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

with boundary conditions

$$T(1) = 100 \qquad \text{and} \qquad T(2) = 0$$

- Determine the temperature distribution in the wall!

- Let's see the pipe described in the previous problem! The pipe is cooled by convection on the outer surface. The heat conduction $\dot{q}_{cond}$ at the outer wall is equal to the heat convection $\dot{q}_{conv}$ to the surroundings :

$$\dot{q}_{cond} = -kA\frac{dT}{dr} = \dot{q}_{conv} = hA(T - T_a)$$

  where the thermal conductivity $k = 100\frac{J}{s \cdot m \cdot K}$, the convective cooling coefficient $h = 500J/(s \cdot m^2 \cdot K)$ and $T_a = 0C$ is the temperature of the surroundings.
- Determine the temperature distribution in the wall!

BVP - Problem 3.

- The temperature dependence in a cylindrical rod made of a radioactive isotope is governed by the ordinary differential equation

$$\frac{d^2T}{dr^2} + \frac{1}{r} \cdot \frac{dT}{dr} = A\left[1 + \left(\frac{r}{R}\right)^2\right]$$

with

$$T'(0) = 0 \qquad \text{and} \qquad T(R) = 0$$

- Solve this problem for $T(r)$ and $A = -100$!

- The velocity distribution in the laminar boundary layer formed when an incompressible fluid flows over a flat plate is related to the solution of the ordinary differential equation

$$\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

with

$$f(0) = 0; f'(0) = 0 \qquad \text{and} \qquad f'(\eta) \to 1 \text{ as } \eta \to \infty$$

where f is a dimensionless stream function, the velocity u is proportional to $f'(\eta)$, and η is proportional to distance normal to the plate.

- Solve this problem for $f(\eta)$!