Computational Methods & C++ Assignment

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Contents

Abstract

A one-space dimensional problem is considered, to examine the application of numerical schemes for the solution of partial differential equations.

Introduction

Methods

Several methods are used :

- Dufort-Frankel
- Richardson
- Laasonen simple implicit
- Crank-Nicholson

For all of the used schemes, the initialization of the result matrix is run the same way. First the study grid must be chosen. The initials conditions are:

- T_{int} of 100°F
- T_{sur} of 300°F
- $D = 0.1ft^2/hr$

First it was assumed that $\Delta x = 0.05$ and $\Delta t = 0.01$, so that the result matrix could be initialized. We set:

$$n_{space} = \frac{L}{\Delta x} + 1 \tag{1}$$

$$n_{time} = \frac{T}{\Delta t} \tag{2}$$

n_{space} is the number of iterations over the grid, according to the coordinate x. n_{time} is the number of iterations over the grid, according to the time t.

$$\begin{array}{cc} \begin{matrix} x=0 & x=\Delta x & \cdots & n_{space}-1 & n_{space} \end{matrix} & \\ \begin{pmatrix} T_{ext} & T_{int} & \cdots & T_{int} & T_{ext} \\ T_{ext} & 0 & \cdots & 0 & T_{ext} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ T_{ext} & 0 & \cdots & 0 & T_{ext} \end{pmatrix} & \begin{matrix} t=0 \\ t=\Delta t \\ \vdots \\ n_{time} \end{matrix} \end{array}$$

This is how the matrix is initialized, according to initial conditions. For most of the used schemes, we need two previous time steps to calculate the actual step. An order one scheme is used to find the second line of the matrix.

$$\begin{array}{cc} \begin{matrix} i=0 & i=1 & \cdots & n_{space}-1 & n_{space} \end{matrix} & \\ \begin{pmatrix} T_{ext} & T_{int} & \cdots & T_{int} & T_{ext} \\ T_{ext} & T_{i,j} & \cdots & T_{i,j} & T_{ext} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ T_{ext} & 0 & \cdots & 0 & T_{ext} \end{pmatrix} & \begin{matrix} j=0 \\ j=1 \\ \vdots \\ n_{time} \end{matrix} \end{array}$$

With this trick our matrix is ready to be used and filled up with every schemes.

Implicit schemes