





Properties of FDs

- Theorem (Basic rules for the TANE algorithm). Let a∈ X and b∉ X.
 - **r1:** If $X \setminus \{a\}$ → $\{a\}$ is functional, then $\forall Y \supset X$, $Y \setminus \{a\}$ → $\{a\}$ is a functional, but not minimal, dependency.
 - **r2:** If $X \setminus \{a\}$ → $\{a\}$ and X → $\{b\}$ are functional, then $\forall Y \supset X$, $Y \setminus \{b\}$ → $\{b\}$ is a functional, but not minimal, dependency.

7



Ilustration of property r1



r1: If X \ {a} → {a} is functional then Y \ {a} → {a} is functional, but not minimal.

Example (illustrating a method of deriving rule r1).

Let $X = \{acde\}$ and $Y = \{abcdef\}$. Then, $a \in X$, $b \notin X$, $Y \supset X$.

• **r1**: $X \setminus \{a\} \rightarrow \{a\}$ is functional \Rightarrow $\{cde\} \rightarrow \{a\}$ is functional \Rightarrow

 $\{bcde\} \rightarrow \{a\}$ is functional, but not minimal \Rightarrow

 $\{bcdef\} \rightarrow \{a\}$ is functional, but not minimal \Rightarrow

 $Y \setminus \{a\} \rightarrow \{a\}$ is functional, but not minimal.

8





Proof of Properties of FDs...

- Theorem (Basic rules for TANE). Let a∈ X.
 - r1: If $X \setminus \{a\}$ → $\{a\}$ is functional, then $\forall Y \supset X$, $Y \setminus \{a\}$ → $\{a\}$ is a functional, but not minimal, dependency.

Proof:

Since $a \in X$, then $Y \supset X \Leftrightarrow Y \mid \{a\} \supset X \mid \{a\}$. Hence, as $X \mid \{a\} \to \{a\}$ is functional, then $\forall Y \mid \{a\} \supset X \mid \{a\}$, $Y \mid \{a\} \to \{a\}$ is functional. Thus, $\forall Y \supset X$, $Y \mid \{a\} \to \{a\}$ is functional, but not minimal.





Ilustration of property r2

Theorem. Let $Y \supset X$, $a \in X$ and $b \notin X$.

 r2: If X \ {a} → {a} and X → {b} are functional, then Y \ {b} → {b} is functional, but not minimal.

Example (illustrating a method of deriving rule r2).

Let $X = \{acde\}$ and $Y = \{abcde\}$. Then, $a \in X$, $b \notin X$, $Y \supset X$.

 $Y \setminus \{b\} \rightarrow \{b\}$ j is functional, but not minimal.

r1: X \ {a} → {a} and X → {b} are functional ⇒
 {cde} → {a} i {acde} → {b} are functional ⇒
 {cde} → {acde} i {acde} → {b} are functional ⇒
 {acde} → {b} is functional, but not minimal ⇒
 {acde} → {b} is functional, but not minimal ⇒
 {acde} → {b} is functional, but not minimal ⇒
 {acde}

10

WARSAW UNIVERSITY OF TECHNOLOGY DEVELOPMENT PROGRAMME



Proof of Properties of FDs

Theorem (Basic rules for TANE). Let Y⊃X, a∈X, b∉X.
 r2: If X \ {a} → {a} and X → {b} are functional, then
 ∀Y⊃X, Y \ {b} → {b} is a non-minimal fun. dependency.

Proof

Since $X \mid \{a\} \rightarrow \{a\}$ and $X \rightarrow \{b\}$ are functional dependencies, then $X \mid \{a\} \rightarrow X$ and $X \rightarrow \{b\}$ are functional dependencies, so $X \rightarrow \{b\}$ is a functional dependency, but not minimal. As $b \notin X$, then $X \mid \{b\} = X$.

Hence, $X \mid \{b\} \rightarrow \{b\}$ is a non-minimal functional dependency. Therefore, $\forall Y \supset X, \ Y \mid \{b\} \rightarrow \{b\}$ is a a non-minimal functional dependency.

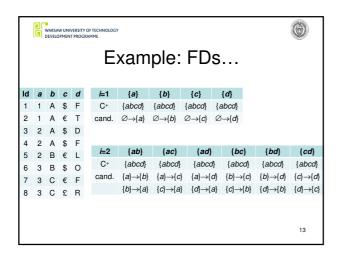
WARSAW UNIVERSITY OF TECHNOLOGY DEVELOPMENT PROGRAMME

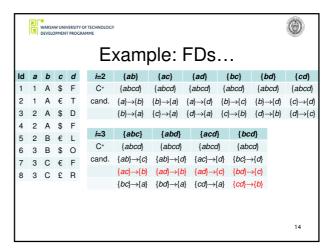


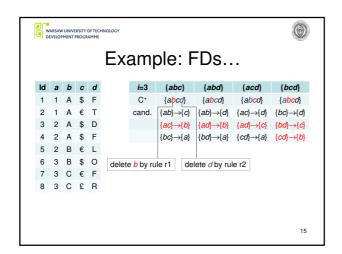
Discovering FDs with TANE

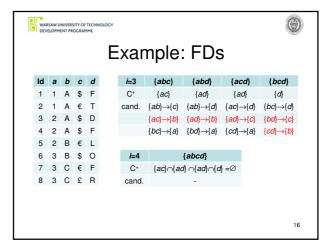
- TANE discovers minimal functional non-trivial dependencies with one dependent attribute.
- In iteration i, TANE builds candidate dependencies
 X \ {a} → {a} from attribute sets X of cardinality i.
- New attribute sets of cardinality i + 1 are built from attribute sets of cardinality i in an Apriori-like way.
- Basic rules r1 and r2 are applied to avoid creation of attribute sets of cardinality i + 1 from which no minimal functional dependency could be created.

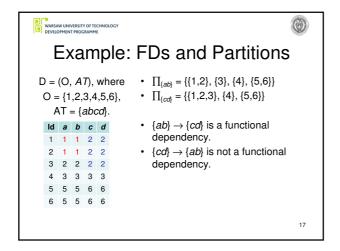
12

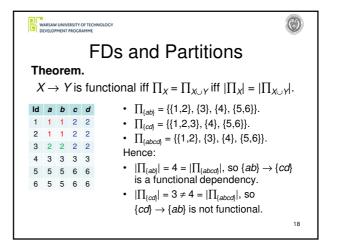


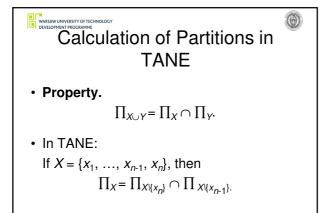


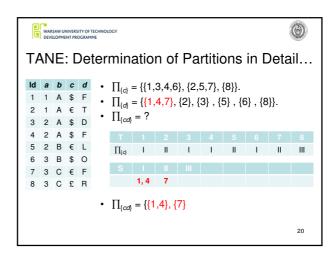


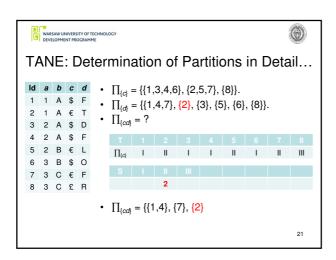


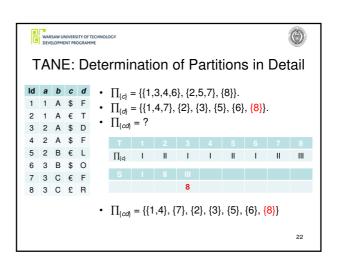


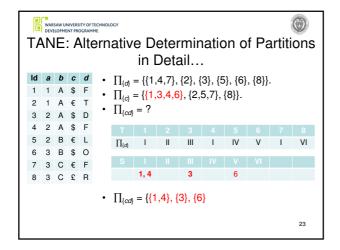


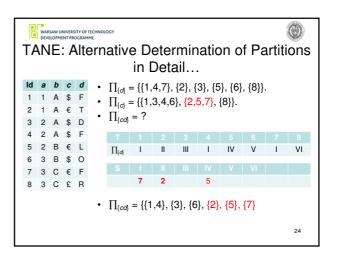


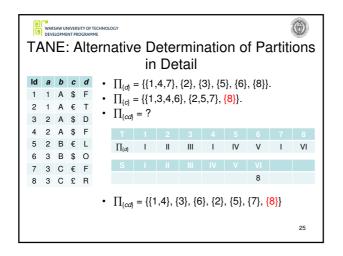


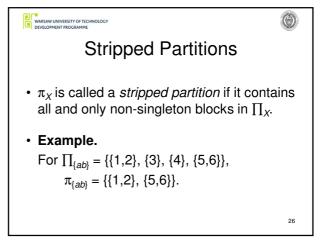


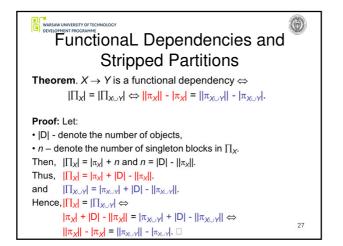


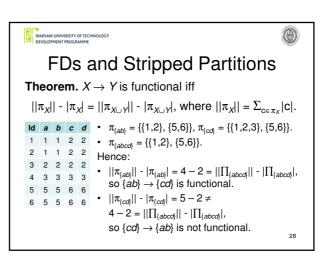


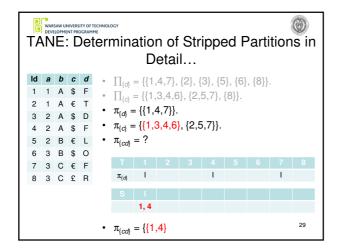


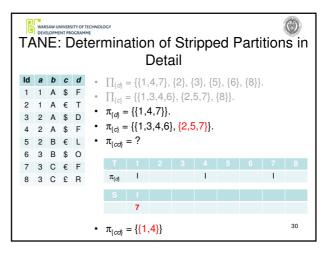


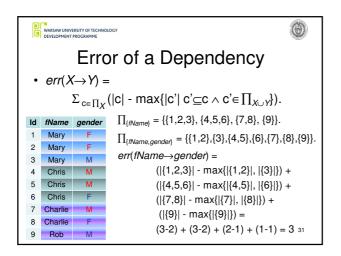


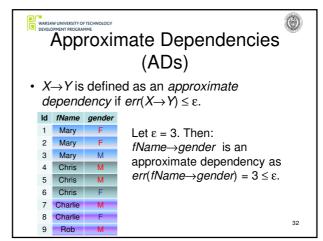


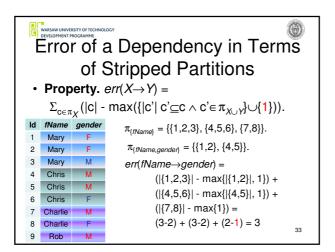


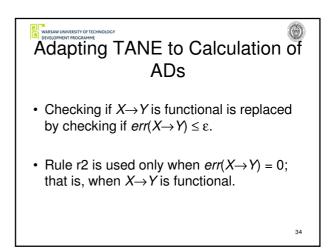


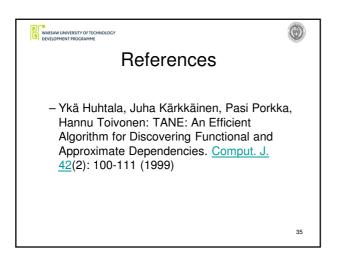












6