Unsupervised Learning for Acoustic Applications

Discovering Hidden Patterns in Sound

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What is Unsupervised Learning?

Supervised Learning Algorithms: Learn a mapping from input samples to output targets, commonly used for solving **classification** and **regression** problems.

$$data_{training} = \{input_i, label_i\} \Rightarrow Relationship$$

Unsupervised Learning Algorithms: Discover hidden patterns, structures, or groupings within data without relying on labeled outputs, commonly used for **clustering**, **dimensionality reduction**, and **representation learning** problems.

$$data_{training} = \{input_i\} \Rightarrow label_i$$

Association of these structures to patterns
↔ Choice of dictionary ↔ Domain knowledge

Question

Supervised vs Unsupervised:

In practice which one should precede the other? Why?

Why Unsupervised Learning in Acoustics?

- Labeling acoustic data is expensive
- Useful for exploratory analysis
- Applications:
  - **Seismic interpretation:** Clustering groups seismic traces with similar waveforms to identify geological structures.
  - **Environmental sounds:** Clustering identifies distinct vocalization patterns in whale songs among marine mammals.
  - **Speech processing:** Clustering segments audio by speaker to distinguish different voices in conversations.
  - **Music software:** Clustering frequencies and durations for reverse engineering musical pieces

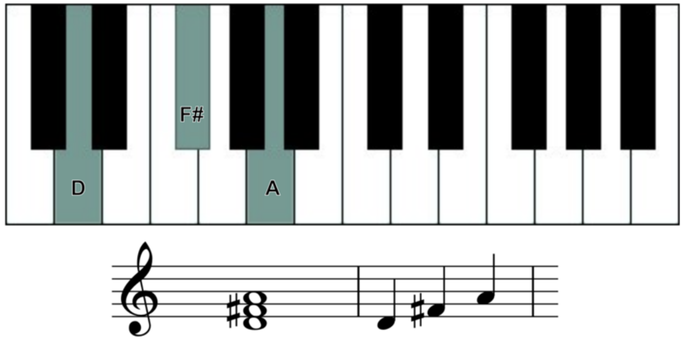
    Spotify by Daniel Ek and Martin Lorentzon.

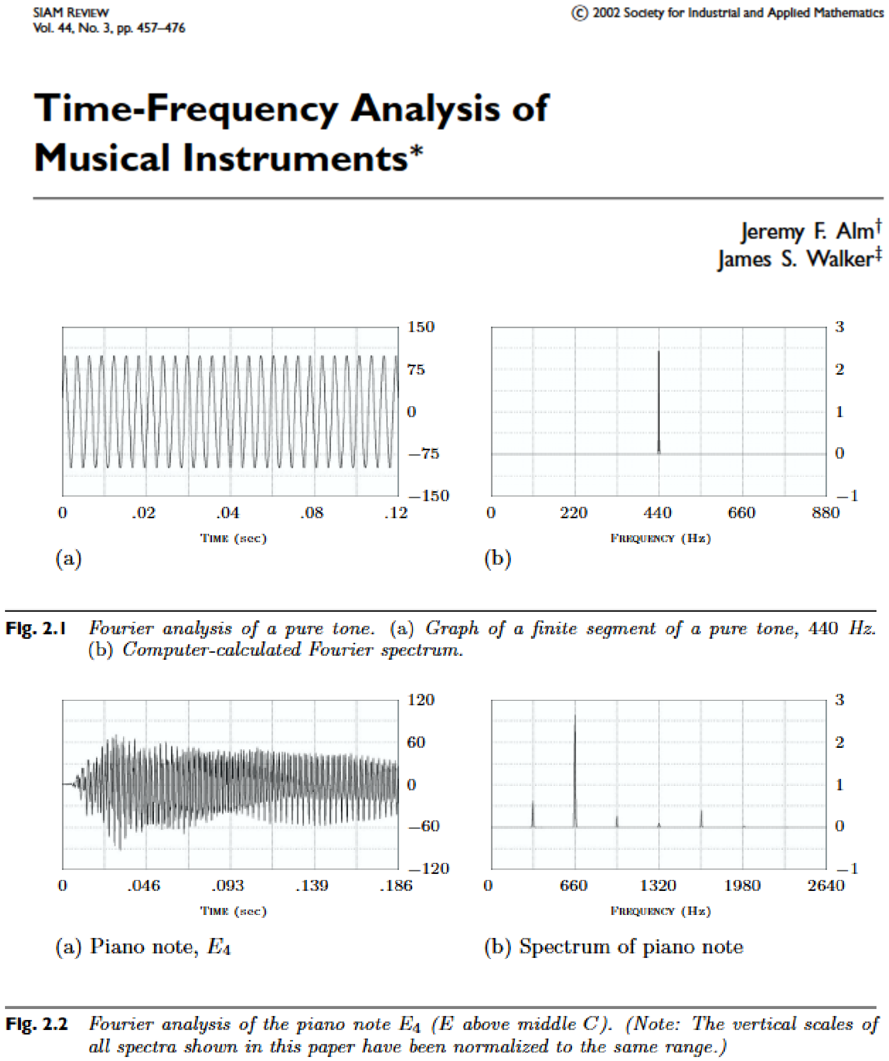
    Auto-Tune by Andy Hildebrand, from Exxon Geologist to Autotune Inventor.

Waveform Example

Examples of acoustic waveforms from music

$$f(t) = \mathrm{Re}\left\{\sum_m f_m e^{i\omega_m t}\right\}$$





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Time-Frequency Analysis of Musical Instruments*

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Fig. 2.1 *Fourier analysis of a pure tone.* (a) *Graph of a finite segment of a pure tone, 440 Hz.* (b) *Computer-calculated Fourier spectrum.*

(a) Piano note, E_4

(b) Spectrum of piano note

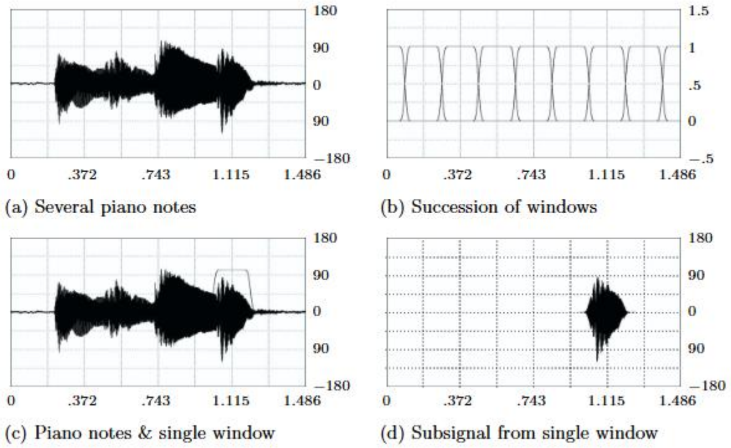
Fig. 2.2 *Fourier analysis of the piano note E_4 (E above middle C). (Note: The vertical scales of all spectra shown in this paper have been normalized to the same range.)*

Spectrogram Example

- Time-frequency representation (Importance of choice of dictionary)

$$f(t) = Re\left\{\sum_{m,n} f_{m,n} e^{i\omega_{mn}t} w(t - t_n)\right\}$$

- Useful for audio classification
- Example: song analysis



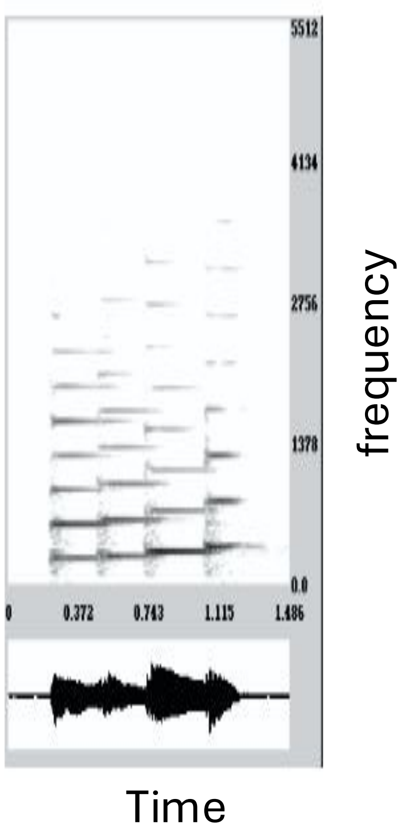
(a) Several piano notes

(b) Succession of windows

(c) Piano notes & single window

(d) Subsignal from single window

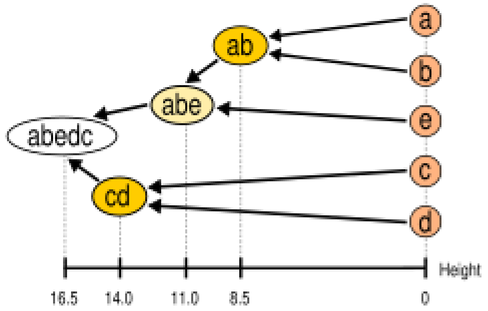
Fig. 3.2 *Components of a spectrogram.*



Types of Unsupervised Learning

- Clustering
- Dimensionality Reduction
- Association Rule Learning

Hierarchical Clustering



Description

- Group similar data points into clusters.
- There are two main types:
  - **Agglomerative (bottom-up)** – starts with each point as its own cluster and merges them.
  - **Divisive (top-down)** – starts with one big cluster and recursively splits it.
- Dendrograms for acoustic event grouping
- Example: Grouping similar acoustic emission events in material testing

Algorithmic steps

1. Start with each data point as its own cluster (i.e., n clusters).
2. Compute the distance (or similarity) between every pair of clusters using a linkage method.
3. Merge the two closest clusters based on the chosen linkage.
4. Update the distance matrix to reflect the merge.
5. Repeat steps 2–4 until all points are merged into a single cluster (or until a stopping criterion is met, like a desired number of clusters).
6. Cut the dendrogram at a chosen height.

Hierarchical Clustering – Linkage methods

Linkage Method	Distance Metric Used	Cluster Shape	Sensitivity to Outliers	Notes
Single Linkage	Minimum pairwise distance $D_{\min}(A,B)$	Elongated	High	Can cause chaining
Complete Linkage	Maximum pairwise distance $D_{\max}(A,B)$	Compact	High	Tends to break large clusters
Average Linkage	Average pairwise distance	Balanced	Moderate	Good general-purpose choice
Centroid	Distance between centroids	Elliptical	Moderate	May produce disconnected clusters
Ward's Method	Increase in within-cluster variance	Spherical	Low	Often preferred for numerical data

$$D_{\mathrm{min/max}}(A,B) = \mathrm{min/max}\{d(a,b), a \in A, b \in B\}$$

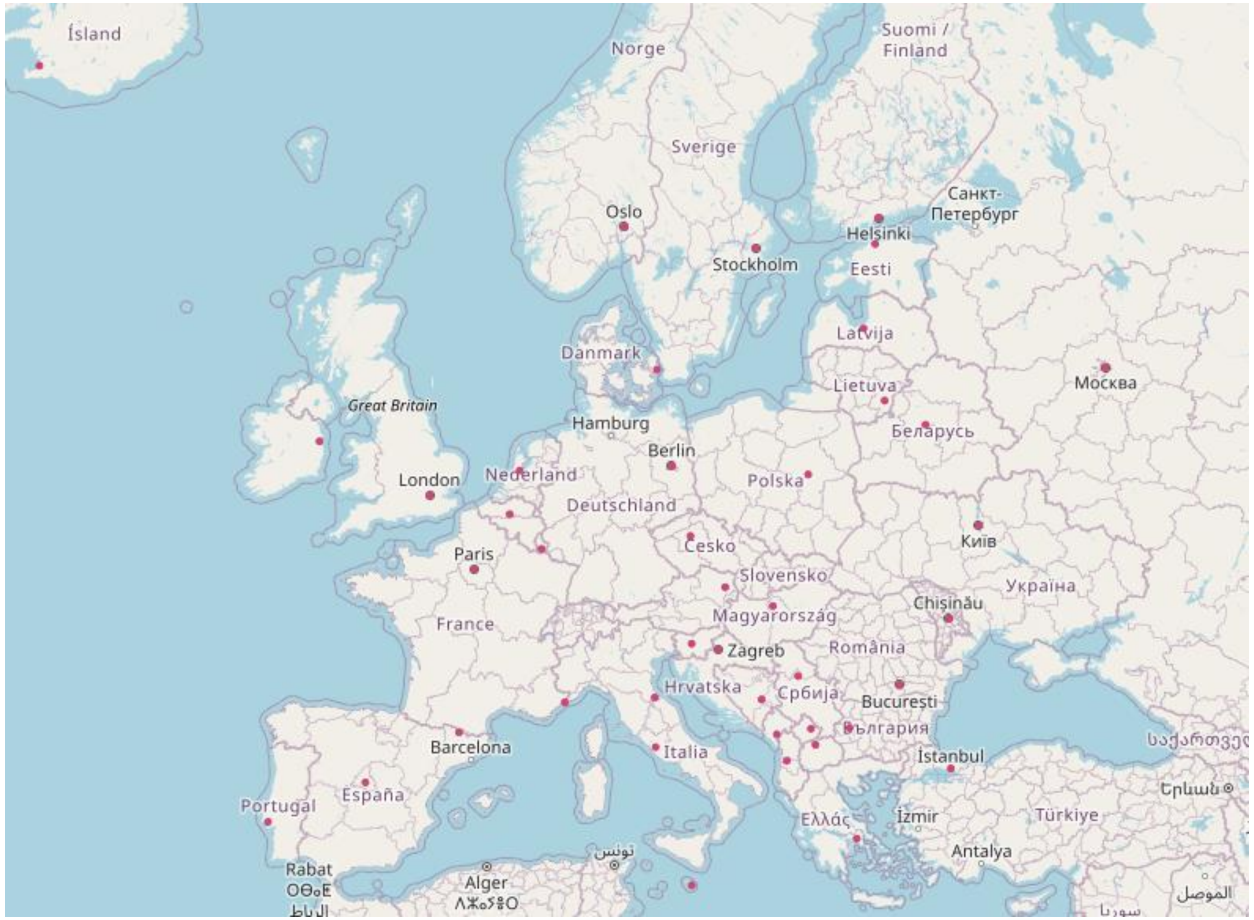
$$D_{\mathrm{cent}}(A,B) = d(\mu_A, \mu_B)$$

$$D_{\mathrm{avg}}(A,B) = |A|^{-1}|B|^{-1} \sum_{a \in A, b \in B} d(a,b)$$

$$D_{\mathrm{Ward}}(A,B) = |\boldsymbol{A}||\boldsymbol{B}|(|\boldsymbol{A} \cup \boldsymbol{B}|)^{\boldsymbol{-1}} d(\mu_A, \mu_B)$$

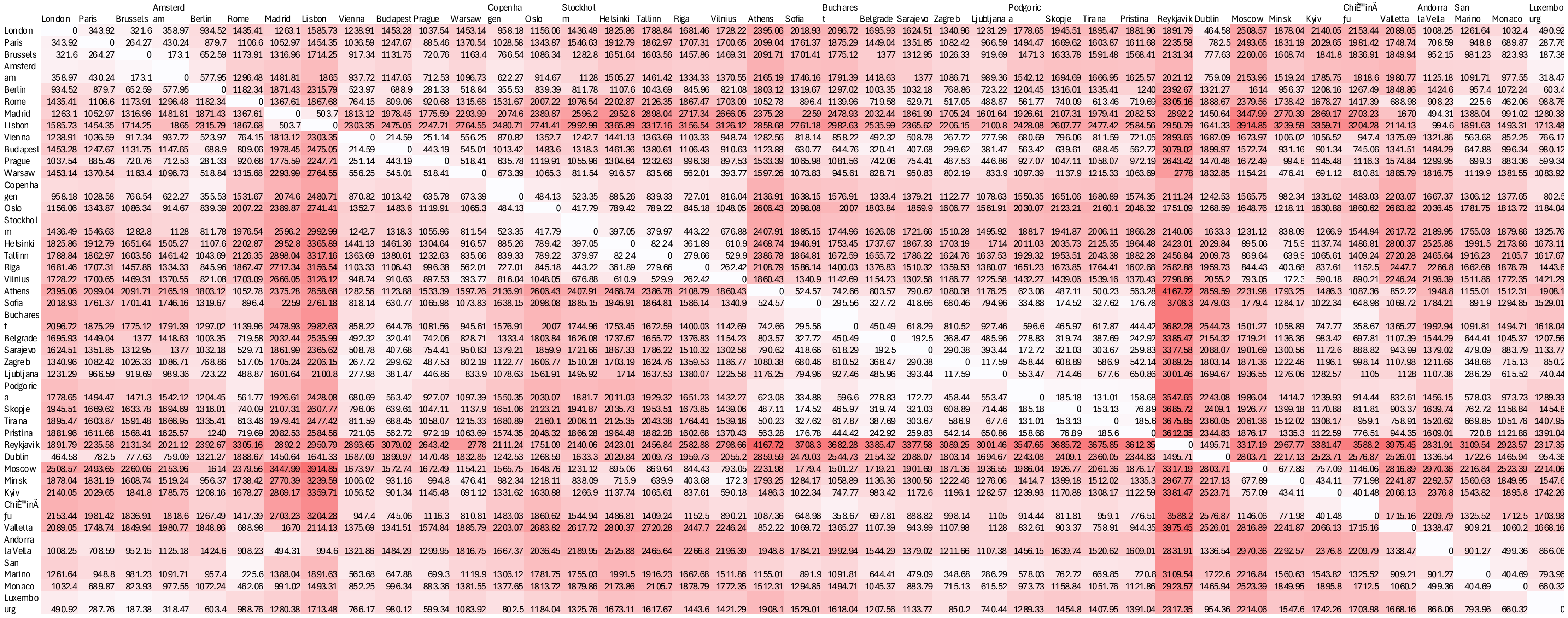
increase in within-cluster variance

Example – European cities



Example can be extended to earthquake locations using ObsPy.

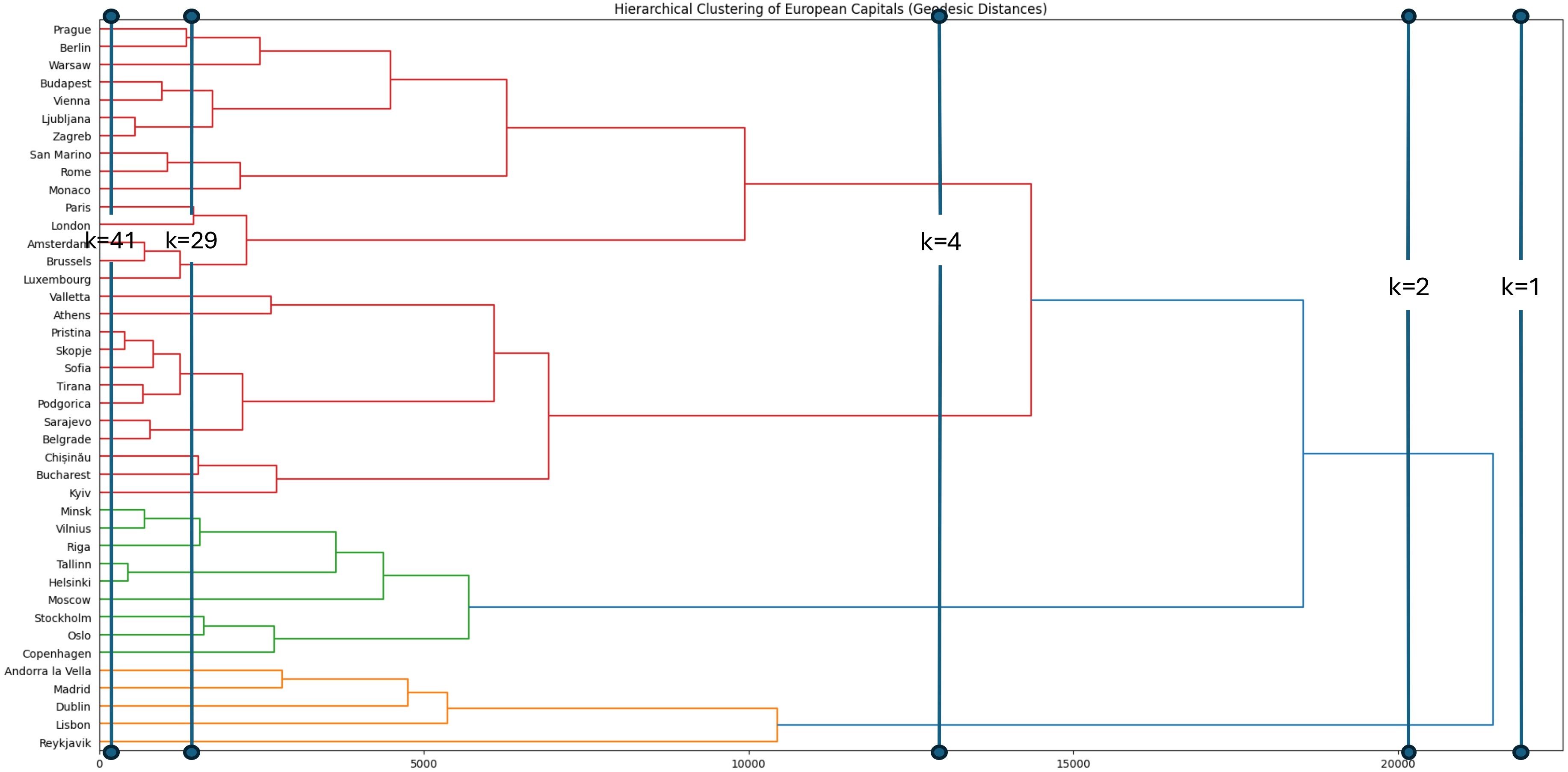
Example – European cities (distances)



	London	Paris	Brussels	Amsterdam	Berlin	Rome	Madrid	Lisbon	Vienna	Budapest	Prague	Warsaw	Copenhagen	Oslo	Stockholm	Helsinki	Tallinn	Riga	Vilnius	Athens	Sofia	Bucharest	Belgrade	Sarajevo	Zagreb	Ljubljana	Podgorica	Skopje	Tirana	Pristina	Reykjavik	Dublin	Moscow	Minsk	Kyiv	ChiÈ™inÄƒu	Valletta	Andorra la Vella	San Marino	Monaco	Luxembourg
London	0	343.92	321.6	358.97	934.52	1435.41	1263.1	1585.73	1238.91	1453.28	1037.54	1453.14	958.18	1156.06	1436.49	1825.86	1788.84	1681.46	1728.22	2395.06	2018.93	2096.72	1695.93	1624.51	1340.96	1231.29	1778.65	1945.51	1895.47	1881.96	1891.79	464.58	2508.57	1878.04	2140.05	2153.44	2089.05	1008.25	1261.64	1032.4	490.92
Paris	343.92	0	264.27	430.24	879.7	1106.6	1052.97	1454.35	1036.59	1247.67	885.46	1370.54	1028.58	1343.87	1546.63	1912.79	1862.97	1707.31	1700.65	2099.04	1761.37	1875.29	1449.04	1351.85	1082.42	966.59	1494.47	1669.62	1603.87	1611.68	2235.58	782.5	2493.65	1831.19	2029.65	1981.42	1748.74	708.59	948.8	689.87	287.76
Brussels	321.6	264.27	0	173.1	652.59	1173.91	1316.96	1714.25	917.34	1131.75	720.76	1163.4	766.54	1086.34	1282.8	1651.64	1603.56	1457.86	1469.31	2091.71	1701.41	1775.12	1377	1312.95	1026.33	919.69	1471.3	1633.78	1591.48	1568.41	2131.34	777.63	2260.06	1608.74	1841.8	1836.91	1849.94	952.15	981.23	823.93	187.38
Amsterdam	358.97	430.24	173.1	0	577.95	1296.48	1481.81	1865	937.72	1147.65	712.53	1096.73	622.27	914.67	1128	1505.27	1461.42	1334.33	1370.55	2165.19	1746.16	1791.39	1418.63	1377	1086.71	989.36	1542.12	1694.69	1666.95	1625.57	2021.12	759.09	2153.96	1519.24	1785.75	1818.6	1980.77	1125.18	1091.71	977.55	318.47
Berlin	934.52	879.7	652.59	577.95	0	1182.34	1871.43	2315.79	523.97	688.9	281.33	518.84	355.53	839.39	811.78	1107.6	1043.69	845.96	821.08	1803.12	1319.67	1297.02	1003.35	1032.18	768.86	723.22	1204.45	1316.01	1335.41	1240	2392.67	1321.27	1614	956.37	1208.16	1267.49	1848.86	1424.6	957.4	1072.24	603.4
Rome	1435.41	1106.6	1173.91	1296.48	1182.34	0	1367.61	1867.68	764.15	809.06	920.68	1315.68	1531.67	2007.22	1976.54	2202.87	2126.35	1867.47	1703.09	1052.78	896.4	1139.96	719.58	529.71	517.05	488.87	561.77	740.09	613.46	719.69	3305.16	1888.67	2379.56	1738.42	1678.27	1417.39	688.98	908.23	225.6	462.06	988.76
Madrid	1263.1	1052.97	1316.96	1481.81	1871.43	1367.61	0	503.7	1813.12	1978.45	1775.59	2293.99	2074.6	2389.87	2596.2	2952.8	2898.04	2717.34	2666.05	2375.28	2259	2478.93	2032.44	1861.99	1705.24	1601.64	1926.61	2107.31	1979.41	2082.53	2892.2	1450.64	3447.99	2770.39	2869.17	2703.23	1670	494.31	1388.04	991.02	1280.38
Lisbon	1585.73	1454.35	1714.25	1865	2315.79	1867.68	503.7	0	2303.35	2475.05	2247.71	2764.55	2480.71	2741.41	2992.99	3365.89	3317.16	3156.54	3126.12	2858.68	2761.18	2982.63	2535.99	2365.62	2206.15	2100.8	2428.08	2607.77	2477.42	2584.56	2950.79	1641.33	3914.85	3239.59	3359.71	3204.28	2114.13	994.6	1891.63	1493.31	1713.48
Vienna	1238.91	1036.59	917.34	937.72	523.97	764.15	1813.12	2303.35	0	214.59	251.14	556.25	870.82	1352.7	1242.7	1441.13	1363.69	1103.33	948.74	1282.56	818.14	858.22	492.32	508.78	267.72	277.98	680.69	796.06	811.59	721.05	2893.65	1687.09	1673.97	1006.02	1056.52	947.4	1375.69	1321.86	563.68	852.25	766.17
Budapest	1453.28	1247.67	1131.75	1147.65	688.9	809.06	1978.45	2475.05	214.59	0	443.19	545.01	1013.42	1483.6	1318.3	1461.36	1380.61	1106.43	910.63	1123.88	630.77	644.76	320.41	407.68	299.62	381.47	563.42	639.61	688.45	562.72	3079.02	1899.97	1572.74	931.16	901.34	745.06	1341.51	1484.29	647.88	996.34	980.12
Prague	1037.54	885.46	720.76	712.53	281.33	920.68	1775.59	2247.71	251.14	443.19	0	518.41	635.78	1119.91	1055.96	1304.64	1232.63	996.38	897.53	1533.39	1065.98	1081.56	742.06	754.41	487.53	446.86	927.07	1047.11	1058.07	972.19	2643.42	1470.48	1672.49	994.8	1145.48	1116.3	1574.84	1299.95	699.3	883.36	599.34
Warsaw	1453.14	1370.54	1163.4	1096.73	518.84	1315.68	2293.99	2764.55	556.25	545.01	518.41	0	673.39	1065.3	811.54	916.57	835.66	562.01	393.77	1597.26	1073.83	945.61	828.71	950.83	802.19	833.9	1097.39	1137.9	1215.33	1063.69	2778	1832.85	1154.21	476.41	691.12	810.81	1885.79	1816.75	1119.9	1381.55	1083.92
Copenhagen	958.18	1028.58	766.54	622.27	355.53	1531.67	2074.6	2480.71	870.82	1013.42	635.78	673.39	0	484.13	523.35	885.26	839.33	727.01	816.04	2136.91	1638.15	1576.91	1333.4	1379.21	1122.77	1078.63	1550.35	1651.06	1680.89	1574.35	2111.24	1242.53	1565.75	982.34	1331.62	1483.03	2203.07	1667.37	1306.12	1377.65	802.5
Oslo	1156.06	1343.87	1086.34	914.67	839.39	2007.22	2389.87	2741.41	1352.7	1483.6	1119.91	1065.3	484.13	0	417.79	789.42	789.22	845.18	1048.05	2606.43	2098.08	2007	1803.84	1859.9	1606.77	1561.91	2030.07	2123.21	2160.1	2046.32	1751.09	1268.59	1648.76	1218.11	1630.88	1860.62	2683.82	2036.45	1781.75	1813.72	1184.04
Stockholm	1436.49	1546.63	1282.8	1128	811.78	1976.54	2596.2	2992.99	1242.7	1318.3	1055.96	811.54	523.35	417.79	0	397.05	379.97	443.22	676.88	2407.91	1885.15	1744.96	1626.08	1721.66	1510.28	1495.92	1881.7	1941.87	2006.11	1866.28	2140.06	1633.3	1231.12	838.09	1266.9	1544.94	2617.72	2189.95	1755.03	1879.86	1325.76
Helsinki	1825.86	1912.79	1651.64	1505.27	1107.6	2202.87	2952.8	3365.89	1441.13	1461.36	1304.64	916.57	885.26	789.42	397.05	0	82.24	361.89	610.9	2468.74	1946.91	1753.45	1737.67	1867.33	1703.19	1714	2011.03	2035.73	2125.35	1964.48	2423.01	2029.84	895.06	715.9	1137.74	1486.81	2800.37	2525.88	1991.5	2173.86	1673.11
Tallinn	1788.84	1862.97	1603.56	1461.42	1043.69	2126.35	2898.04	3317.16	1363.69	1380.61	1232.63	835.66	839.33	789.22	379.97	82.24	0	279.66	529.9	2386.78	1864.81	1672.59	1655.72	1786.22	1624.76	1637.53	1929.32	1953.51	2043.38	1882.28	2456.84	2009.73	869.64	639.9	1065.61	1409.24	2720.28	2465.64	1916.23	2105.7	1617.67
Riga	1681.46	1707.31	1457.86	1334.33	845.96	1867.47	2717.34	3156.54	1103.33	1106.43	996.38	562.01	727.01	845.18	443.22	361.89	279.66	0	262.42	2108.79	1586.14	1400.03	1376.83	1510.32	1359.53	1380.07	1651.23	1673.85	1764.41	1602.68	2582.88	1959.73	844.43	403.68	837.61	1152.5	2447.7	2266.8	1662.68	1878.79	1443.6
Vilnius	1728.22	1700.65	1469.31	1370.55	821.08	1703.09	2666.05	3126.12	948.74	910.63	897.53	393.77	816.04	1048.05	676.88	610.9	529.9	262.42	0	1860.43	1340.9	1142.69	1154.23	1302.58	1186.77	1225.58	1432.27	1439.06	1539.16	1370.43	2798.66	2055.2	793.05	172.3	590.18	890.21	2246.24	2196.39	1511.86	1772.35	1421.29
Athens	2395.06	2099.04	2091.71	2165.19	1803.12	1052.78	2375.28	2858.68	1282.56	1123.88	1533.39	1597.26	2136.91	2606.43	2407.91	2468.74	2386.78	2108.79	1860.43	0	524.57	742.66	803.57	790.62	1080.38	1176.25	623.08	487.11	500.23	563.28	4167.72	2859.59	2231.98	1793.25	1486.3	1087.36	852.22	1948.8	1155.01	1512.31	1908.1
Sofia	2018.93	1761.37	1701.41	1746.16	1319.67	896.4	2259	2761.18	818.14	630.77	1065.98	1073.83	1638.15	2098.08	1885.15	1946.91	1864.81	1586.14	1340.9	524.57	0	295.56	327.72	418.66	680.46	794.96	334.88	174.52	327.62	176.78	3708.3	2479.03	1779.4	1284.17	1022.34	648.98	1069.72	1784.21	891.9	1294.85	1529.01
Bucharest	2096.72	1875.29	1775.12	1791.39	1297.02	1139.96	2478.93	2982.63	858.22	644.76	1081.56	945.61	1576.91	2007	1744.96	1753.45	1672.59	1400.03	1142.69	742.66	295.56	0	450.49	618.29	810.52	927.46	596.6	465.97	617.87	444.42	3682.28	2544.73	1501.27	1058.89	747.77	358.67	1365.27	1992.94	1091.81	1494.71	1618.04
Belgrade	1695.93	1449.04	1377	1418.63	1003.35	719.58	2032.44	2535.99	492.32	320.41	742.06	828.71	1333.4	1803.84	1626.08	1737.67	1655.72	1376.83	1154.23	803.57	327.72	450.49	0	192.5	368.47	485.96	278.83	319.74	387.69	242.92	3385.47	2154.32	1719.21	1136.36	983.42	697.81	1107.39	1544.29	644.41	1045.37	1207.56
Sarajevo	1624.51	1351.85	1312.95	1377	1032.18	529.71	1861.99	2365.62	508.78	407.68	754.41	950.83	1379.21	1859.9	1721.66	1867.33	1786.22	1510.32	1302.58	790.62	418.66	618.29	192.5	0	290.38	393.44	172.72	321.03	303.67	259.83	3377.58	2088.07	1901.69	1300.56	1172.6	888.82	943.99	1379.02	479.09	883.79	1133.77
Zagreb	1340.96	1082.42	1026.33	1086.71	768.86	517.05	1705.24	2206.15	267.72	299.62	487.53	802.19	1122.77	1606.77	1510.28	1703.19	1624.76	1359.53	1186.77	1080.38	680.46	810.52	368.47	290.38	0	117.59	458.44	608.89	586.9	542.14	3089.25	1803.14	1871.36	1222.46	1196.1	998.14	1107.98	1211.66	348.68	715.13	850.2
Ljubljana	1231.29	966.59	919.69	989.36	723.22	488.87	1601.64	2100.8	277.98	381.47	446.86	833.9	1078.63	1561.91	1495.92	1714	1637.53	1380.07	1225.58	1176.25	794.96	927.46	485.96	393.44	117.59	0	553.47	714.46	677.6	650.86	3001.46	1694.67	1936.55	1276.06	1282.57	1105	1128	1107.38	286.29	615.52	740.44
Podgorica	1778.65	1494.47	1471.3	1542.12	1204.45	561.77	1926.61	2428.08	680.69	563.42	927.07	1097.39	1550.35	2030.07	1881.7	2011.03	1929.32	1651.23	1432.27	623.08	334.88	596.6	278.83	172.72	458.44	553.47	0	185.18	131.01	158.68	3547.65	2243.08	1986.04	1414.7	1239.93	914.44	832.61	1456.15	578.03	973.73	1289.33
Skopje	1945.51	1669.62	1633.78	1694.69	1316.01	740.09	2107.31	2607.77	796.06	639.61	1047.11	1137.9	1651.06	2123.21	1941.87	2035.73	1953.51	1673.85	1439.06	487.11	174.52	465.97	319.74	321.03	608.89	714.46	185.18	0	153.13	76.89	3685.72	2409.1	1926.77	1399.18	1170.88	811.81	903.37	1639.74	762.72	1158.84	1454.8
Tirana	1895.47	1603.87	1591.48	1666.95	1335.41	613.46	1979.41	2477.42	811.59	688.45	1058.07	1215.33	1680.89	2160.1	2006.11	2125.35	2043.38	1764.41	1539.16	500.23	327.62	617.87	387.69	303.67	586.9	677.6	131.01	153.13	0	185.6	3675.85	2360.05	2061.36	1512.02	1308.17	959.1	758.91	1520.62	669.85	1051.76	1407.95
Pristina	1881.96	1611.68	1568.41	1625.57	1240	719.69	2082.53	2584.56	721.05	562.72	972.19	1063.69	1574.35	2046.32	1866.28	1964.48	1882.28	1602.68	1370.43	563.28	176.78	444.42	242.92	259.83	542.14	650.86	158.68	76.89	185.6	0	3612.35	2344.83	1876.17	1335.3	1122.59	776.51	944.35	1609.01	720.8	1121.86	1391.04
Reykjavik	1891.79	2235.58	2131.34	2021.12	2392.67	3305.16	2892.2	2950.79	2893.65	3079.02	2643.42	2778	2111.24	1751.09	2140.06	2423.01	2456.84	2582.88	2798.66	4167.72	3708.3	3682.28	3385.47	3377.58	3089.25	3001.46	3547.65	3685.72	3675.85	3612.35	0	1495.71	3317.19	2967.77	3381.47	3588.2	3975.45	2831.91	3109.54	2923.57	2317.35
Dublin	464.58	782.5	777.63	759.09	1321.27	1888.67	1450.64	1641.33	1687.09	1899.97	1470.48	1832.85	1242.53	1268.59	1633.3	2029.84	2009.73	1959.73	2055.2	2859.59	2479.03	2544.73	2154.32	2088.07	1803.14	1694.67	2243.08	2409.1	2360.05	2344.83	1495.71	0	2803.71	2217.13	2523.71	2576.87	2526.01	1336.54	1722.6	1465.94	954.36
Moscow	2508.57	2493.65	2260.06	2153.96	1614	2379.56	3447.99	3914.85	1673.97	1572.74	1672.49	1154.21	1565.75	1648.76	1231.12	895.06	869.64	844.43	793.05	2231.98	1779.4	1501.27	1719.21	1901.69	1871.36	1936.55	1986.04	1926.77	2061.36	1876.17	3317.19	2803.71	0	677.89	757.09	1146.06	2816.89	2970.36	2216.84	2523.39	2214.06
Minsk	1878.04	1831.19	1608.74	1519.24	956.37	1738.42	2770.39	3239.59	1006.02	931.16	994.8	476.41	982.34	1218.11	838.09	715.9	639.9	403.68	172.3	1793.25	1284.17	1058.89	1136.36	1300.56	1222.46	1276.06	1414.7	1399.18	1512.02	1335.3	2967.77	2217.13	677.89	0	434.11	771.98	2241.87	2292.57	1560.63	1849.95	1547.6
Kyiv	2140.05	2029.65	1841.8	1785.75	1208.16	1678.27	2869.17	3359.71	1056.52	901.34	1145.48	691.12	1331.62	1630.88	1266.9	1137.74	1065.61	837.61	590.18	1486.3	1022.34	747.77	983.42	1172.6	1196.1	1282.57	1239.93	1170.88	1308.17	1122.59	3381.47	2523.71	757.09	434.11	0	401.48	2066.13	2376.8	1543.82	1895.8	1742.26
ChiÈ™inÄƒu	2153.44	1981.42	1836.91	1818.6	1267.49	1417.39	2703.23	3204.28	947.4	745.06	1116.3	810.81	1483.03	1860.62	1544.94	1486.81	1409.24	1152.5	890.21	1087.36	648.98	358.67	697.81	888.82	998.14	1105	914.44	811.81	959.1	776.51	3588.2	2576.87	1146.06	771.98	401.48	0	1715.16	2209.79	1325.52	1712.5	1703.98
Valletta	2089.05	1748.74	1849.94	1980.77	1848.86	688.98	1670	2114.13	1375.69	1341.51	1574.84	1885.79	2203.07	2683.82	2617.72	2800.37	2720.28	2447.7	2246.24	852.22	1069.72	1365.27	1107.39	943.99	1107.98	1128	832.61	903.37	758.91	944.35	3975.45	2526.01	2816.89	2241.87	2066.13	1715.16	0	1338.47	909.21	1060.2	1668.16
Andorra la Vella	1008.25	708.59	952.15	1125.18	1424.6	908.23	494.31	994.6	1321.86	1484.29	1299.95	1816.75	1667.37	2036.45	2189.95	2525.88	2465.64	2266.8	2196.39	1948.8	1784.21	1992.94	1544.29	1379.02	1211.66	1107.38	1456.15	1639.74	1520.62	1609.01	2831.91	1336.54	2970.36	2292.57	2376.8	2209.79	1338.47	0	901.27	499.36	866.06
San Marino	1261.64	948.8	981.23	1091.71	957.4	225.6	1388.04	1891.63	563.68	647.88	699.3	1119.9	1306.12	1781.75	1755.03	1991.5	1916.23	1662.68	1511.86	1155.01	891.9	1091.81	644.41	479.09	348.68	286.29	578.03	762.72	669.85	720.8	3109.54	1722.6	2216.84	1560.63	1543.82	1325.52	909.21	901.27	0	404.69	793.96
Monaco	1032.4	689.87	823.93	977.55	1072.24	462.06	991.02	1493.31	852.25	996.34	883.36	1381.55	1377.65	1813.72	1879.86	2173.86	2105.7	1878.79	1772.35	1512.31	1294.85	1494.71	1045.37	883.79	715.13	615.52	973.73	1158.84	1051.76	1121.86	2923.57	1465.94	2523.39	1849.95	1895.8	1712.5	1060.2	499.36	404.69	0	660.32
Luxembourg	490.92	287.76	187.38	318.47	603.4	988.76	1280.38	1713.48	766.17	980.12	599.34	1083.92	802.5	1184.04	1325.76	1673.11	1617.67	1443.6	1421.29	1908.1	1529.01	1618.04	1207.56	1133.77	850.2	740.44	1289.33	1454.8	1407.95	1391.04	2317.35	954.36	2214.06	1547.6	1742.26	1703.98	1668.16	866.06	793.96	660.32	0

Example – European cities (dendrogram)

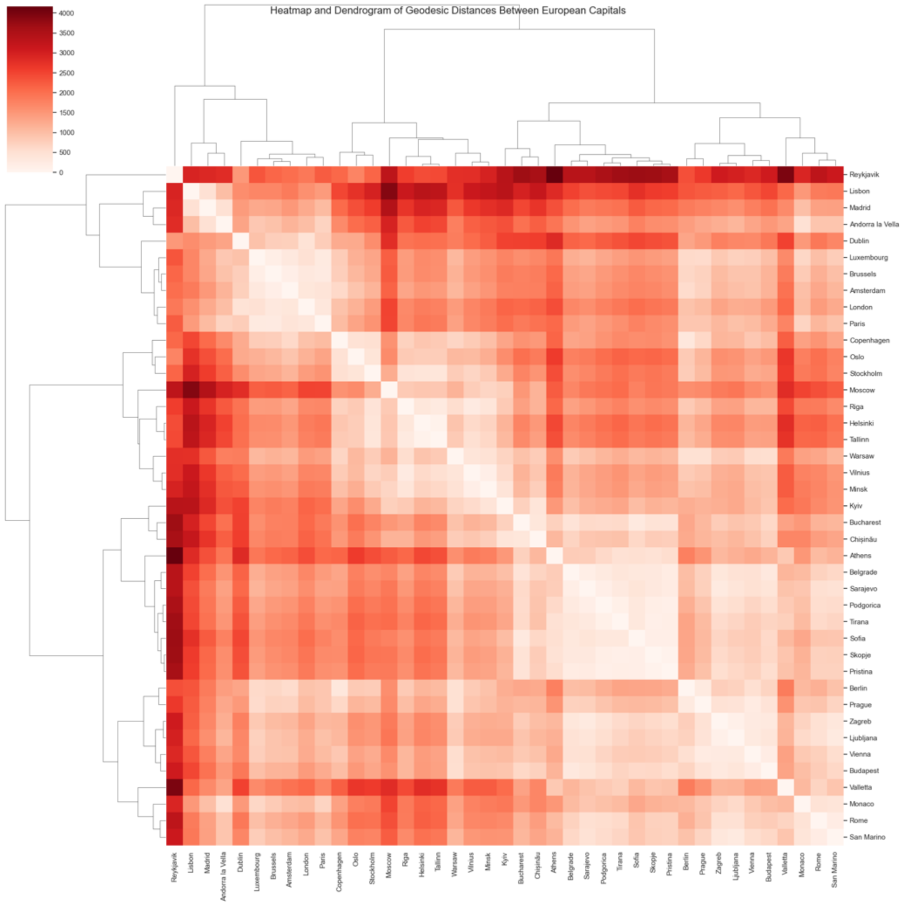
Agglomerative (bottom-up)



Divisive (top-down)

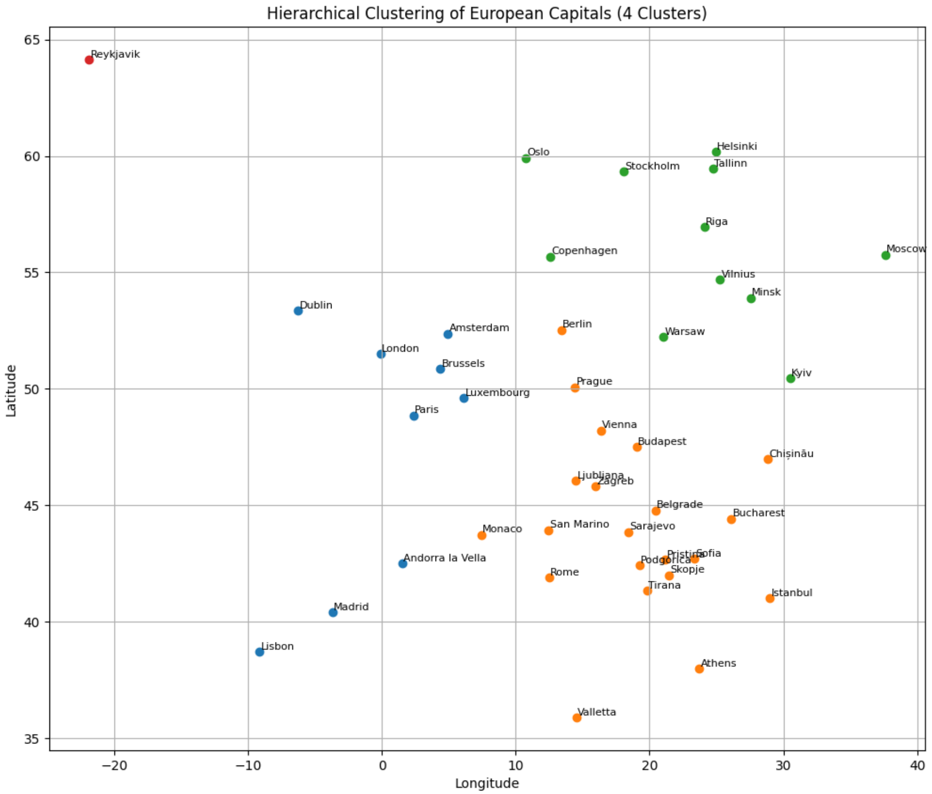
Example – European cities

(heat map + dendrogram)

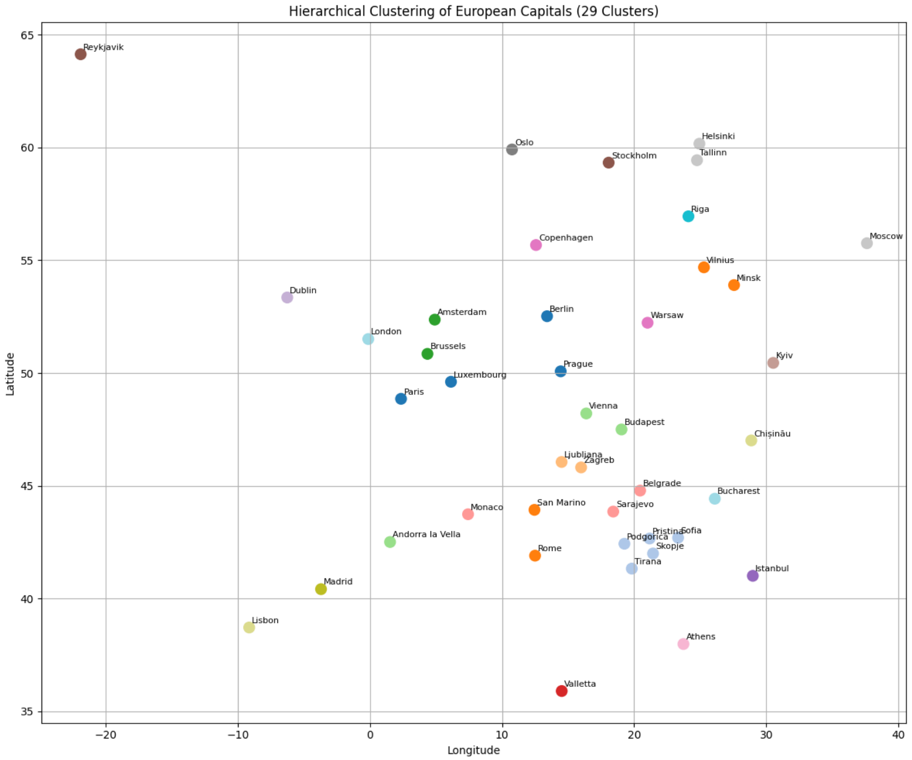


Example – European cities

k=4



k=29



Hierarchical Clustering - Properties

- Deterministic
- Computational cost $\mathcal{O}(n^3)$
- Dependent on distance definition
- Not necessarily optimal!
  Q: Why?
  A: Greedy at each iteration

K-Means Clustering

Description

- Partitions data into k-clusters

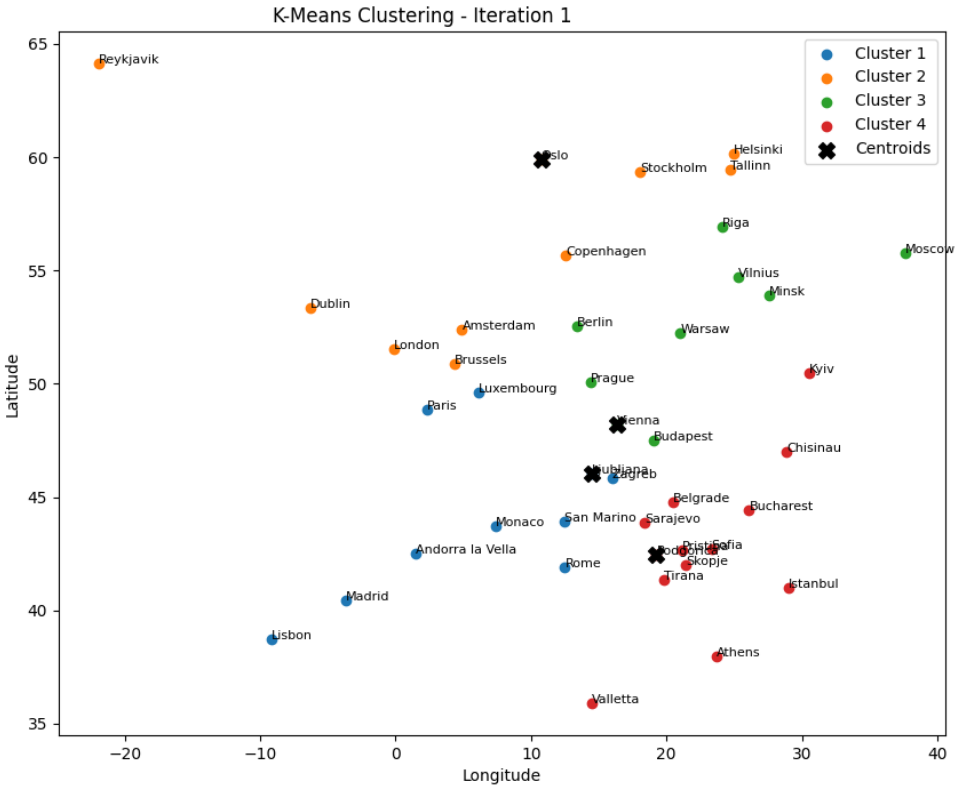
$$\arg\min_{\mu_k} \sum_{k=1}^{K} \sum_{x_i}^{\Omega_k} D(x_i, \mu_k)$$

- $D(x_i, \mu_k) = [x_i - \mu_k]^T [x_i - \mu_k]$
- $D(x_i, \mu_k) \neq$ **variance ! Q: Implications?**
- Assumes spherical clusters
- Fast and simple

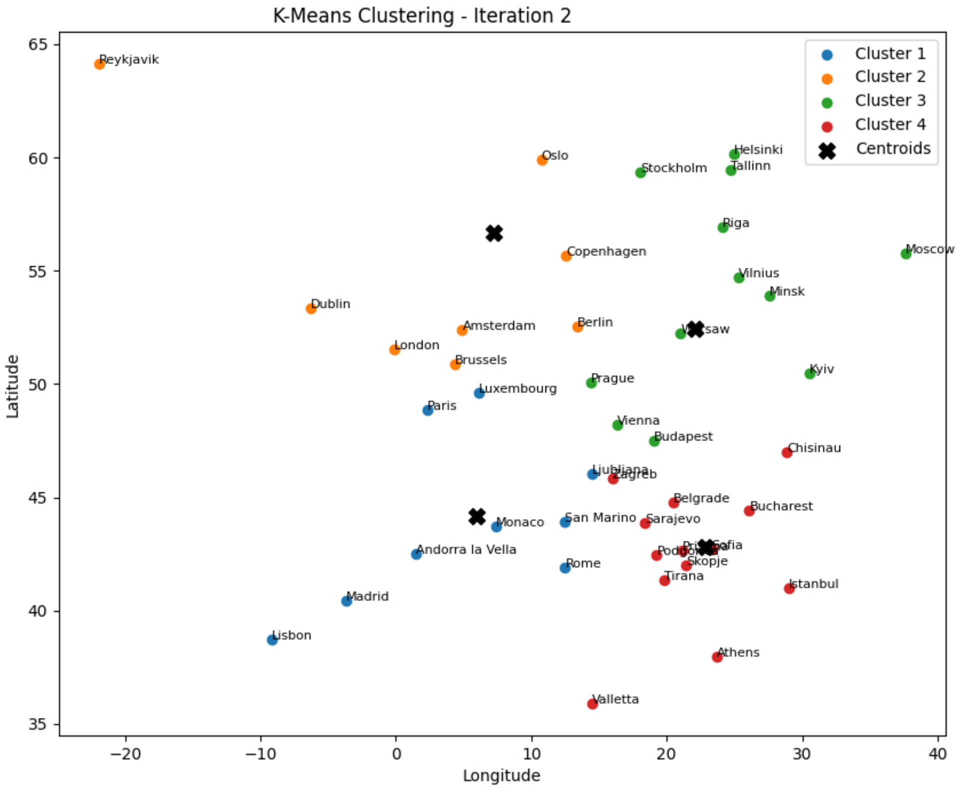
Algorithmic Steps

1. Choose the number of clusters (K) you want to find.
2. Initialize K centroids μ_k **randomly** (these are the centers of your clusters).
3. Assign each data point to the nearest centroid (based on distance, usually Euclidean): $x_i \in \Omega_k$
4. Update centroids by calculating the mean of all points assigned to each cluster: $\mu_k = |\Omega_k|^{-1} \sum_{x_i \in \Omega_k} x_i$
5. Repeat steps 3 and 4 until the centroids no longer change significantly (i.e., convergence).

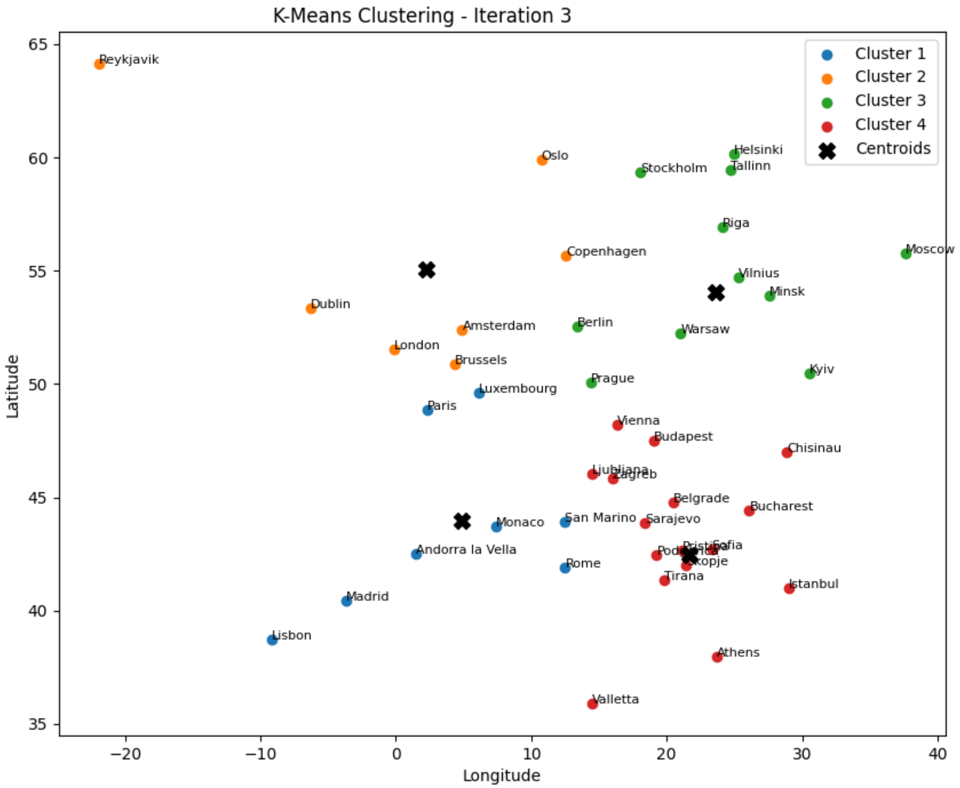
Example – European cities



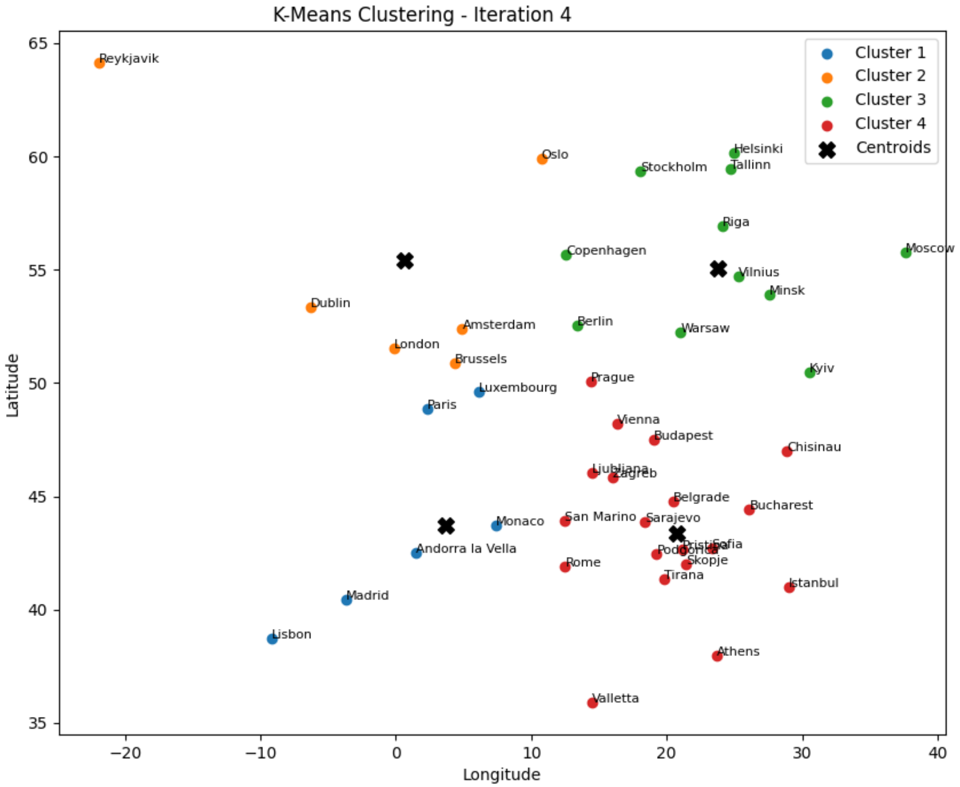
Example – European cities



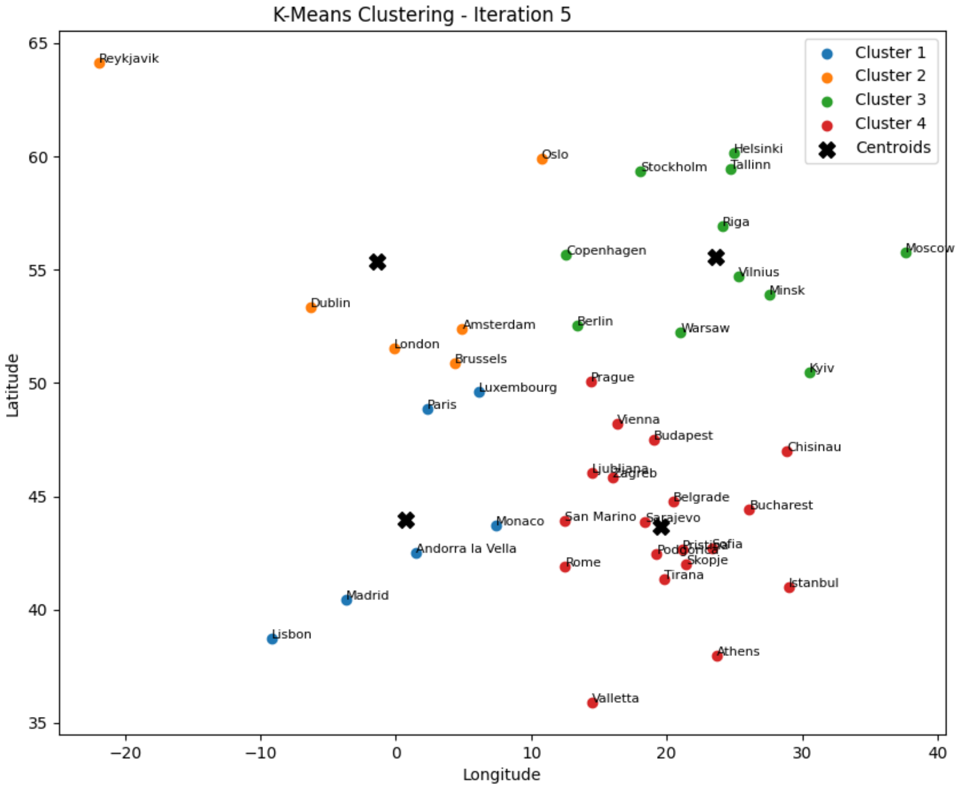
Example – European cities



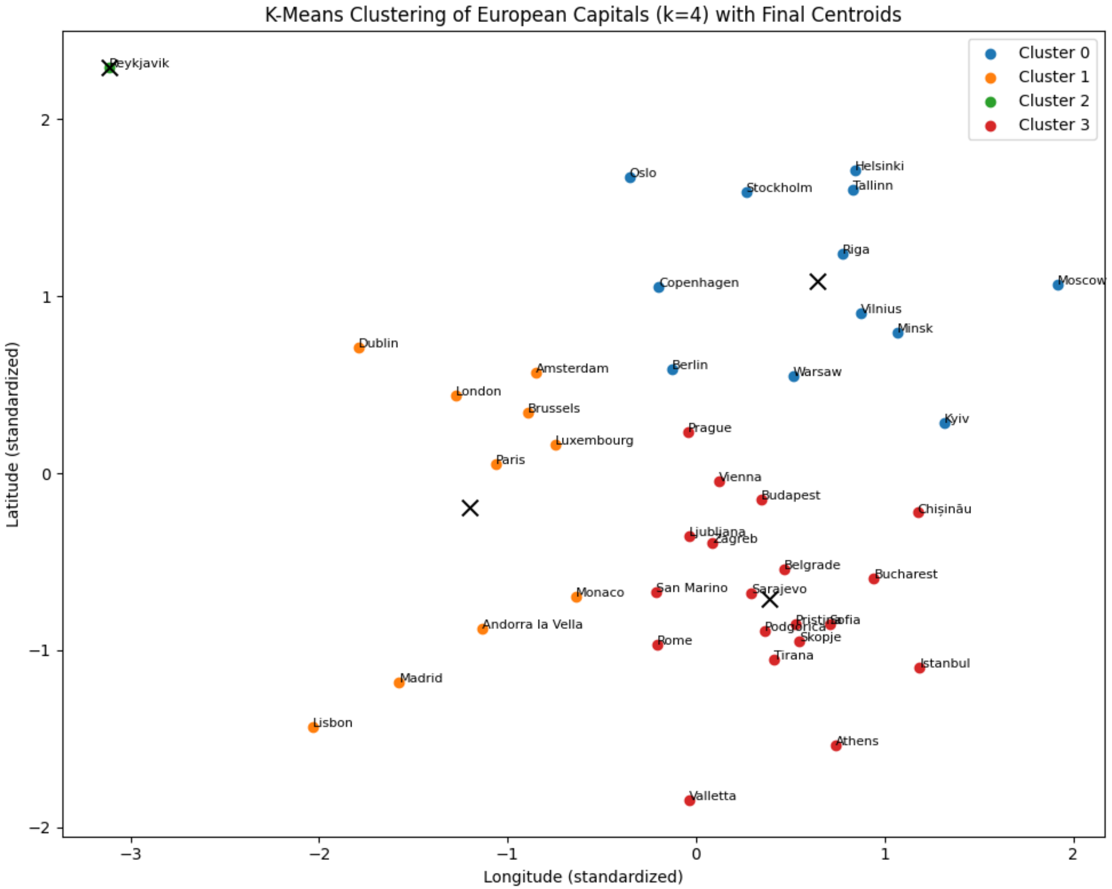
Example – European cities



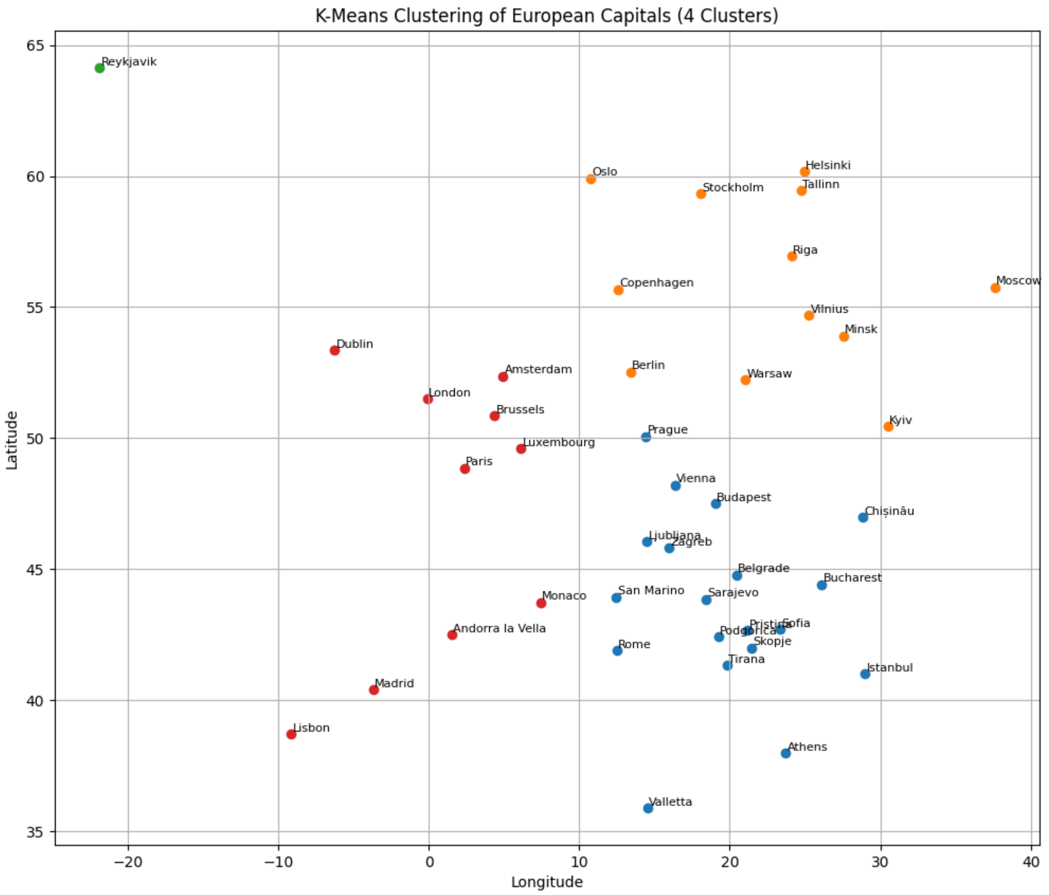
Example – European cities

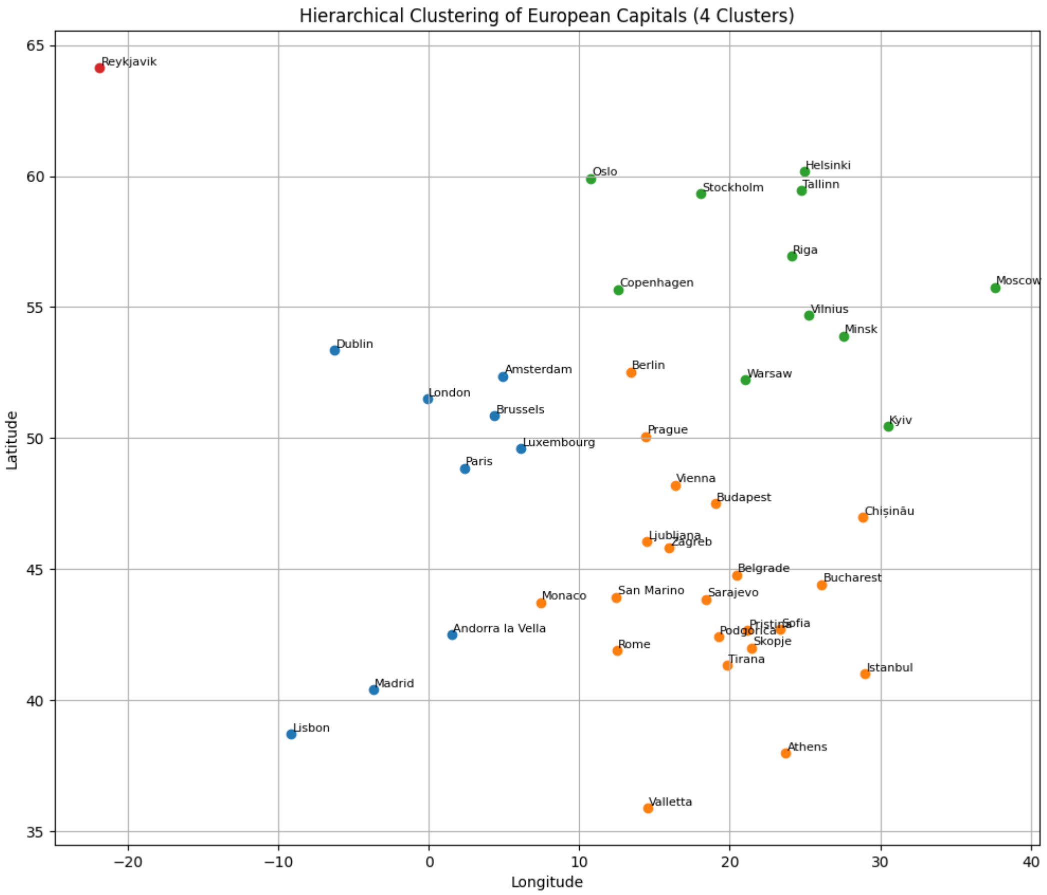


Example – European cities

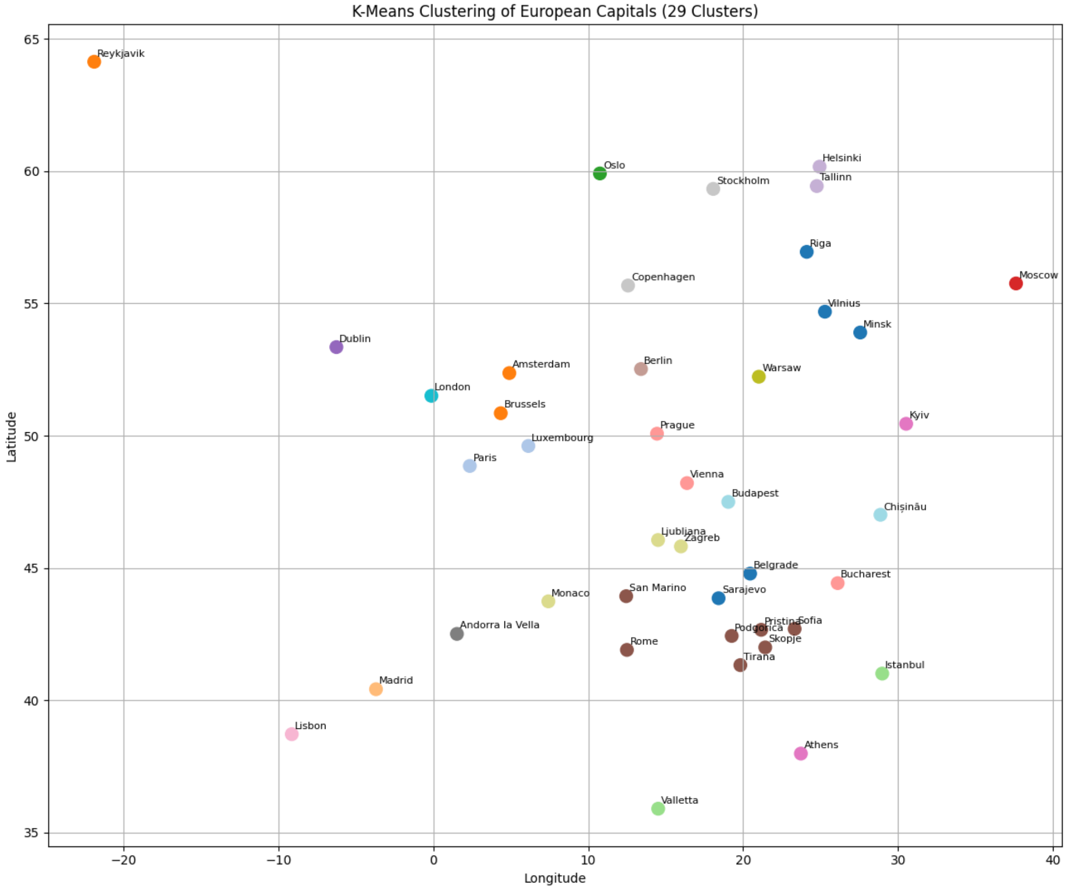


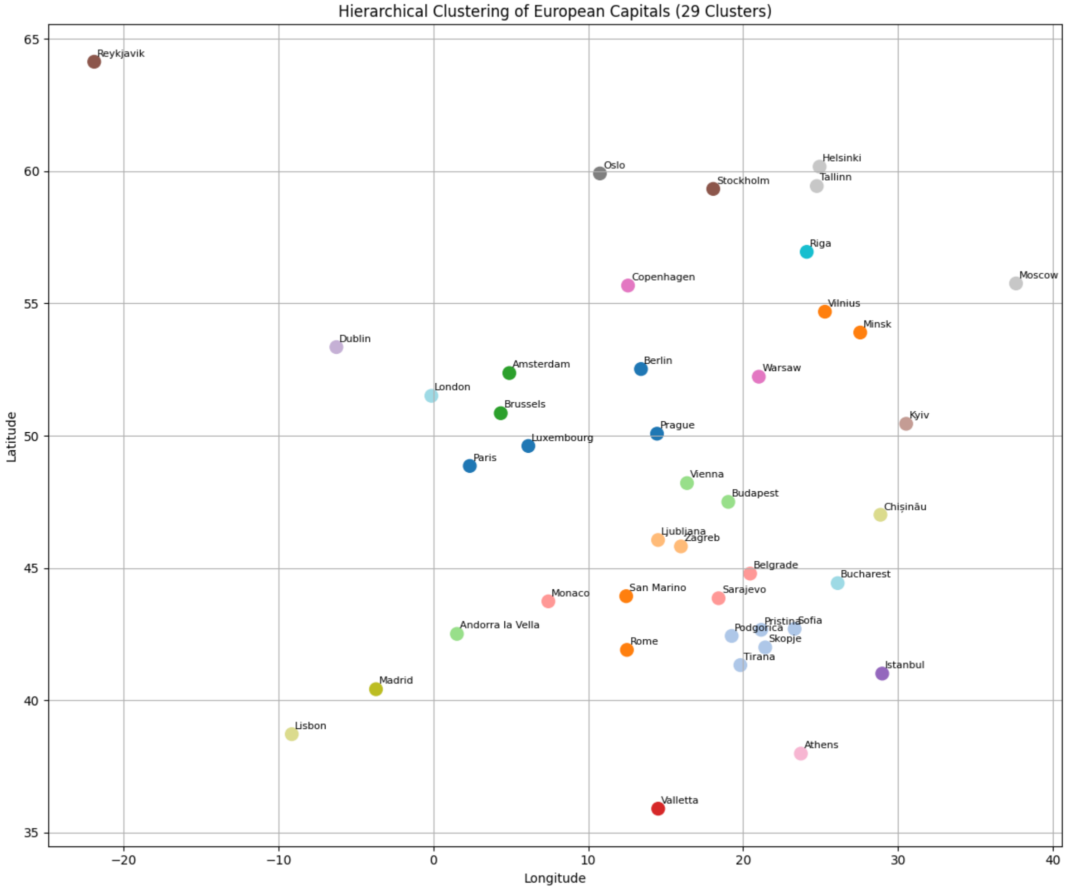
K-Means vs Hierarchical clustering (k=4)





K-mean vs Hierarchical clustering (k=29)





K-means clustering - properties

- Non-deterministic
  - Choice of initial centroids
    - Informed decision / corners / sufficiently distant / multiple realization
  - Choice of # of clusters
    - From application / SME
- Depends on the distance definition: Euclidean
- Similar to Ward’s method, which minimizes the increase in total within-cluster variance (similar to k-means). Weighting makes the difference: penalizes larger clusters with large variations.

Gaussian Mixture Model (GMM) & Expectation Maximization (EM)

GMM Description

- Probabilistic clustering minimizing log likelihood:

$$\log L = \sum_{i=1}^{n} \log\left(\sum_{k=1}^{K} \pi_k \mathcal{N}(x_i|\mu_k, \Sigma_k)\right)$$

$$\mathcal{N}(x_i|\mu_k, \Sigma_k) = \left((2\pi)^n \det\Sigma_k\right)^{-1/2} e^{(x_i-\mu_k)^T\Sigma_k^{-1}(x_i-\mu_k)/2}$$

- Soft assignments
- Handles elliptical clusters
- Uses Expectation-Maximization

EM Algorithmic steps

- E-Step (Expectation):
  Compute the **responsibility** γ_{ik} —the probability that point $x_i$belongs to cluster k

$$\gamma_{\mathrm{ik}} = \frac{\pi_k \mathcal{N}(x_i|\mu_k, \Sigma_k)}{\sum_l \pi_l \mathcal{N}(x_i|\mu_{\mathrm{l}}, \Sigma_{\mathrm{l}})}$$

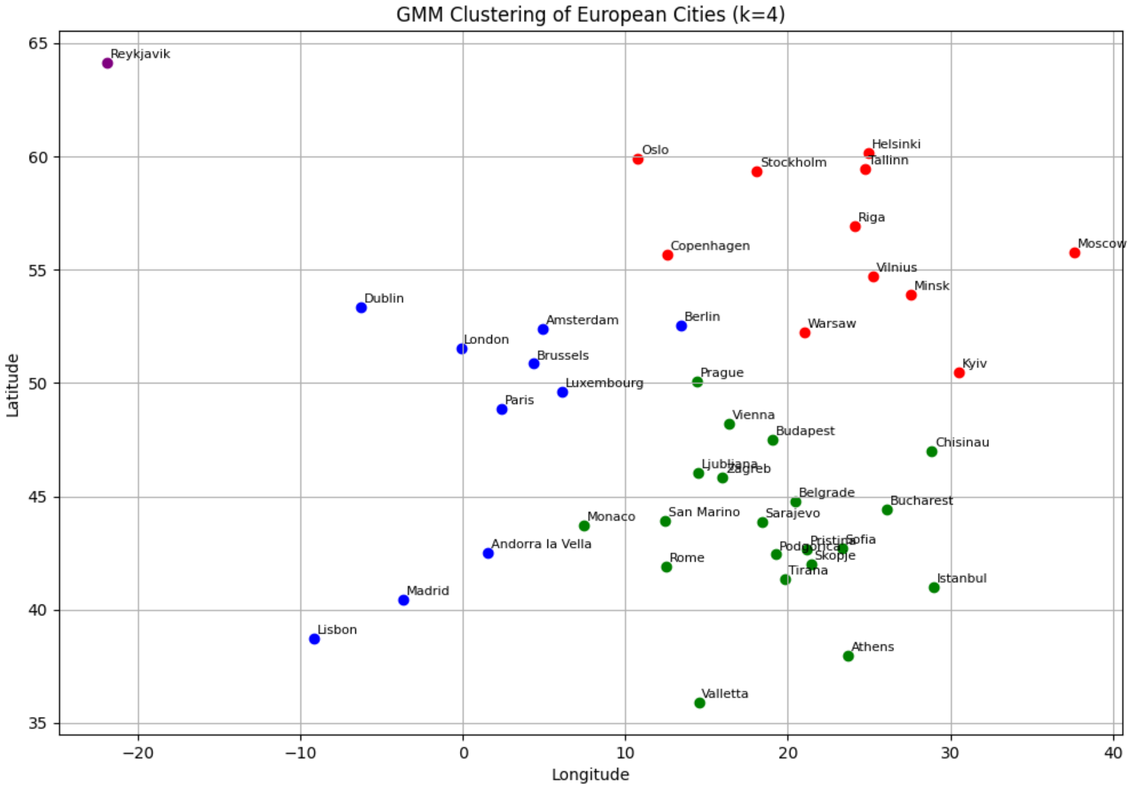
- M-Step (Maximization):
  1. Effective number of points assigned to cluster: $N_k = \sum_{i=1}^{n} \gamma_{ik}$
  2. Update means: $\mu_k = N_k^{-1} \sum_{i=1}^{n} \gamma_{ik} x_i$
  3. Update covariance:

$$\Sigma_k = N_k^{-1} \sum_{i=1}^{n} \gamma_{ik}(x_i - \mu_k)(x_i - \mu_k)^T$$

  4. Update mixing coefficient: $\pi_k = N_k/n$

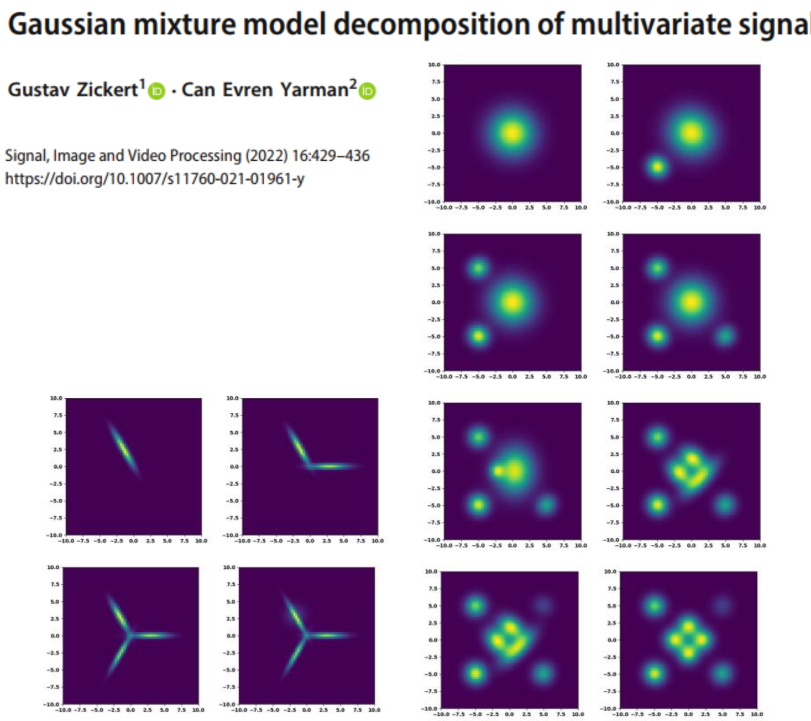
- Repeat EM steps until $\log L$ is below a threshold.

Example – European cities



GMM clustering + EM - properties

- EM is non-deterministic
  - Initial centroids & (inv) covariance matrices
    - Informed decision / corners / sufficiently distant / multiple realization
    - Modes, maxima, etc. of histogram
  - Choice of optimization scheme
    - Greedy + Stochastic descent
  - # of clusters
    - From application / SME
  - Assumptions – sufficiently distinct peaks
- Alternative
  - Histogram decomposition
  - Challenge handling multivariate histograms



Gaussian mixture model decomposition of multivariate signa

Gustav Zickert[1] · Can Evren Yarman[2]

Signal, Image and Video Processing (2022) 16:429–436
https://doi.org/10.1007/s11760-021-01961-y

DBSCAN

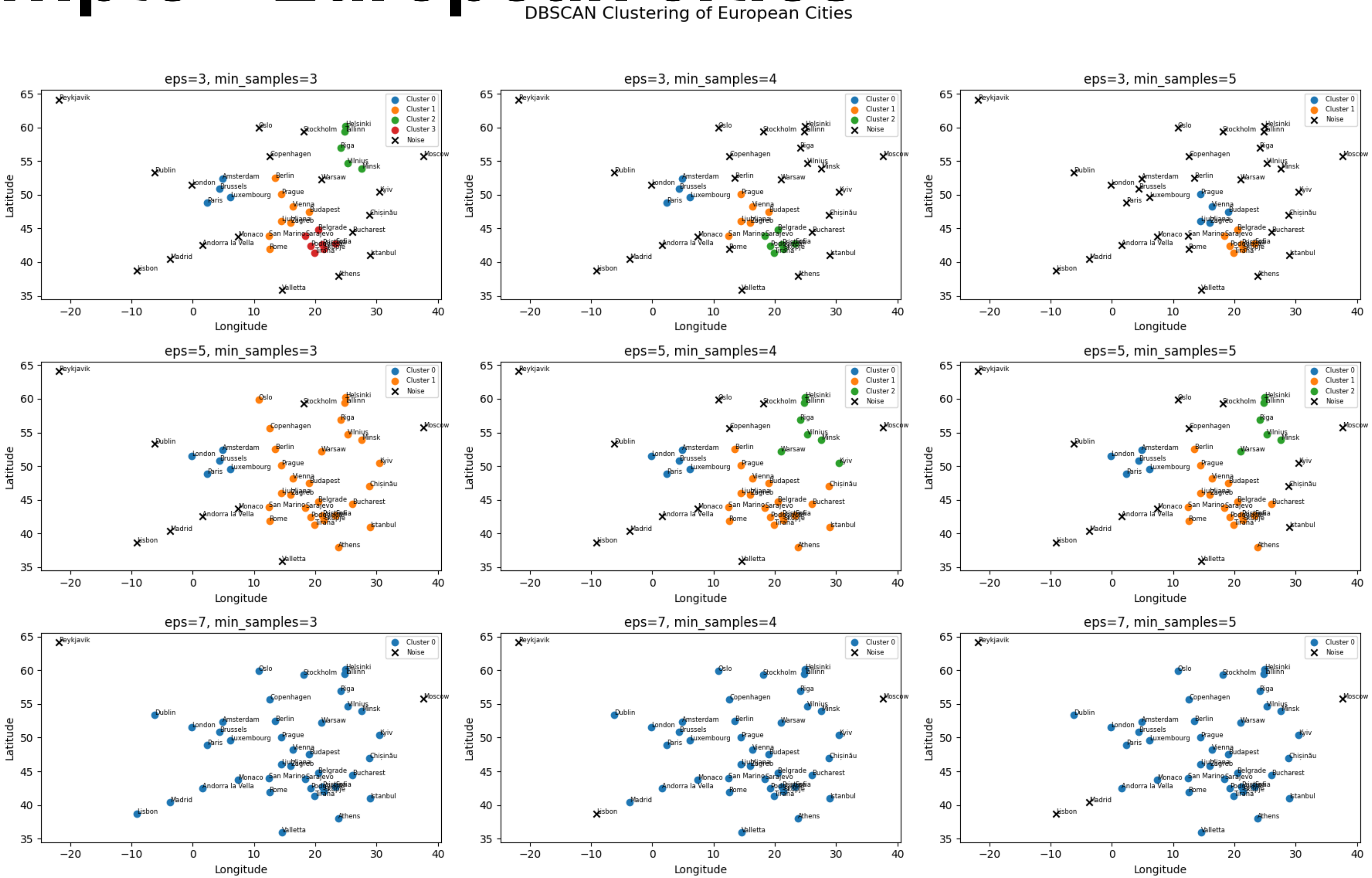
Description

- Groups together points that are closely packed and marks points that lie alone in low-density regions as outliers.
  ~ # of modes in histogram
- Does not require specifying the number of clusters in advance.
- Can find arbitrarily shaped clusters.
- Can identify noise/outliers.
- Sensitive to the choice of ε and minPts.
- Struggles with clusters of varying density.
- Example: Detecting anomalous acoustic events (e.g., microseismic outliers)

Algorithmic steps

1. For each point in the dataset:
   - Find all points within distance ε (its neighborhood).
   - If the number of neighbors ≥ minPts, mark it as a core point.
2. Form a cluster:
   - For each core point not yet assigned to a cluster:
     - Create a new cluster.
     - Add the core point and all its density-reachable points (i.e., points within ε of the core or other reachable core points).
3. Repeat until all points are visited.
4. Label remaining points that are not part of any cluster as noise.

Example – European cities



DBSCAN Clustering of European Cities

eps=3, min_samples=3

eps=3, min_samples=4

eps=3, min_samples=5

eps=5, min_samples=3

eps=5, min_samples=4

eps=5, min_samples=5

eps=7, min_samples=3

eps=7, min_samples=4

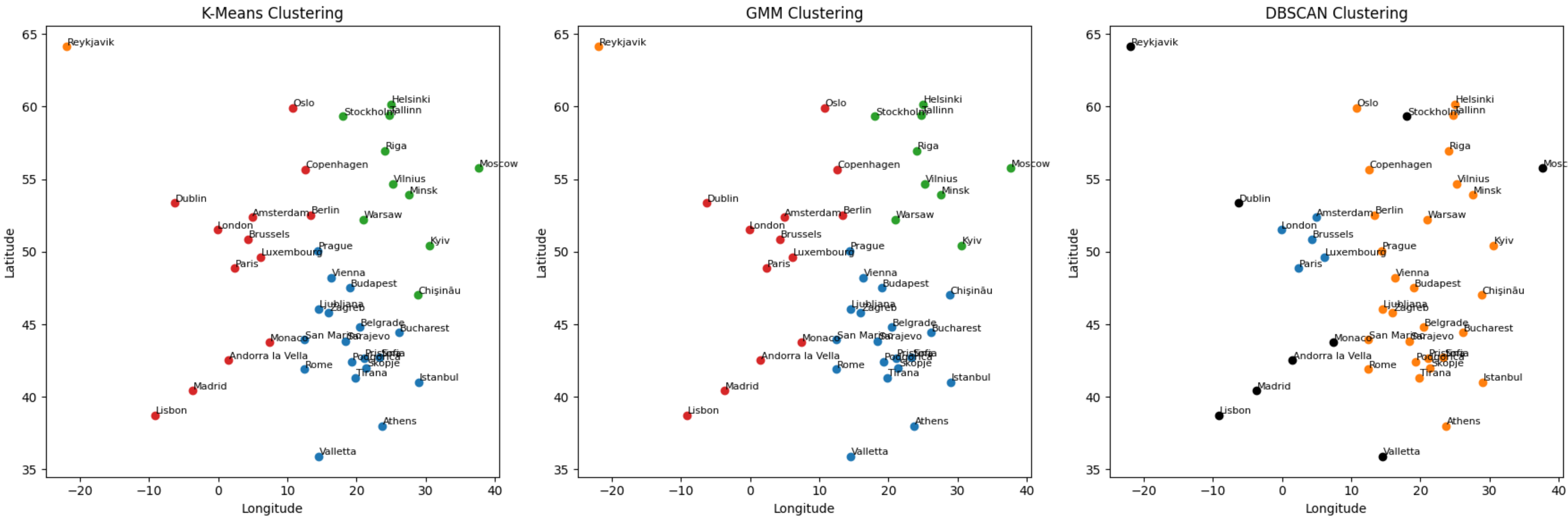
eps=7, min_samples=5

DBScan Choice of parameters

Convert degrees to kilometers:

- 1° latitude ≈ 111 km.
- 1° longitude ≈ 111 km × cos(latitude).
- So eps = 0.5° ≈ 55 km — too small for cities spread across Europe.
- For clustering cities into regional groups, you need eps in the range of 3° to 6° (≈ 300–600 km).

Comparison of clustering methods



Choosing the Right Clustering Algorithm

Feature	**K-Means**	**GMM (Gaussian Mixture Models)**	**DBSCAN**	**Hierarchical Clustering**
Cluster Shape Assumption	Spherical, equal size	Elliptical, variable size	Arbitrary shapes	Arbitrary shapes
Cluster Assignment	Hard (each point belongs to one)	Soft (probabilistic)	Hard	Hard
Number of Clusters Required	Yes	Yes	No	Optional (can cut dendrogram)
Noise Handling	Poor	Poor	Good (identifies noise points)	Poor
Scalability	High	Moderate	Moderate	Low (especially with large datasets)
Interpretability	Easy	Moderate	Moderate	Easy (via dendrograms)
Distance Metric	Euclidean	Mahalanobis (can vary)	Any (usually Euclidean)	Any (usually Euclidean)
Best Use Case in Acoustics	Speaker clustering, waveform grouping	Speaker modeling, seismic facies	Anomaly detection in acoustic signals	Grouping similar acoustic events
Strengths	Fast, simple	Flexible, probabilistic	Detects noise, no need for k	Reveals hierarchy, no need for k
Limitations	Sensitive to initial centroids	Computationally intensive	Sensitive to parameters (ε, minPts)	Computationally expensive
Computational complexity	$\mathcal{O}(nkdi)$	$\mathcal{O}(nkd^2i)$	$\mathcal{O}(n\log n)$ (1) to $\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$ (2) to $\mathcal{O}(n^3)$

n = number of points
k = number of clusters
d = dimensionality
i = number of iterations

(1) with spatial indexing like KD-tree or R-tree.

(2) with efficient data structures (e.g., using priority queues).

- **GMM** → Best for modeling complex distributions.
- **K-Means** → Acceptable for moderate dimensions.
- **Hierarchical** → Not ideal.
- **DBSCAN** → Worst in high dimensions.

Evaluation Metrics

Metric	Formula	Type	Best Value	Use Case
Silhouette Score	$s(i) = \frac{b(i) - a(i)}{\max\big(a(i), b(i)\big)}$	Internal	Close to 1	General clustering quality
Davies-Bouldin Index	$DBI = \frac{1}{k}\sum_{i=1}^{k} \max_{j \neq i}\left(\frac{\sigma_i - \sigma_j}{d\big(\mu_i, \mu_j\big)}\right)$	Internal	Close to 0	Compactness & separation
AIC / BIC	$\mathrm{AIC} = 2k - 2\log(L)$ $\mathrm{BIC} = k\log(n) - 2\log(L)$	Model-based	Lower is better	GMM model selection

$a(i)$ = average distance from point i to all other points in the same cluster.
$b(i)$ = minimum average distance from point i to points in other clusters.
$\sigma(i)$= average distance of all points in cluster i to its centroid μ_i.

- **Akaike Information Criterion**: $\mathrm{AIC} = 2k - 2\log(L)$
- **Bayesian Information Criterion**: $\mathrm{BIC} = k\log(n) - 2\log(L)$

k: # of model parameters, n: # of data points, L: likelihood function
Evaluates how well a probabilistic model (like GMM) fits the data, while penalizing model complexity.

Real-World Applications

- Seismic characterization
  - https://www.marine-geo.org/doi/10.60521/331931
  - https://wiki.seg.org/wiki/Open_data
- Speaker diarization
  - https://dihardchallenge.github.io/dihard3/
  - https://github.com/wq2012/awesome-diarization
- Environmental sound clustering
  - **The Marinexplore and Cornell University Whale Detection Challenge** https://www.kaggle.com/competitions/whale-detection-challenge/data

Challenges and Future Directions

- Lack of ground truth
- Interpretability
- Evaluation without labels
- Self-supervised learning
- High-dimensionality of audio features
  - Temporal & directional dependencies

Recent & Evolving Trends

- Self-supervised learning in audio (e.g., wav2vec)
- Integration with deep learning (autoencoders, contrastive learning)

Summary and Q&A

- Unsupervised learning reveals **hidden** (expected or unexpected) patterns

!!! Hidden ≠ Desired or Useful !!!

- Key techniques:
  - choice of representation (i.e. features)
  - dimension reductions (relevant features)
  - clustering (of features)
- Applications in seismic, speech, and audio
- Question the status quo

Tutorial – Suggestion (others welcomed)

- **Load audio signals** (whale sounds).
- **Listen the audio signal**
- **Computes time varying features (i.e. STFT, Wavelet transform)**
- Extracts **feature vectors** by taking the mean of the feature across time.
  - Other features (e.g., mean, variance, or log-mel spectrogram).
- **Standardizes features** for better clustering performance.
- **Reduces dimensionality** with PCA (2 components for visualization).
- **Clusters signals** using Kmeans, etc.
- **Plots clusters** in PCA space.
- **V&V**

  Verification: Are we building solution right

  Validation: Are we building the right solution