Automn School Series in Acoustics

Lecture 1: Fundamentals of Machine Learning

Eindhoven University of Technology
3-7 November 2025

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ASSA
AUTUMN SCHOOL SERIES
IN
ACOUSTICS

Outline

1. Introduction

   AI? Machine Learning? Neural Networks?

2. Machine Learning

   Definition, Characteristics, Types, Algorithms

3. Deep Neural Networks

   Definition, Motivation, How to Train Them

4. Limits of Machine Learning

   Overfitting, Data Biases

Then: Tutorial to put this in practice!

Artificial Intelligence

Machine Learning

Generative Models

Neural Networks

Deep Learning

ChatGPT, Grok, MidJourney, Stable Diffusion, …

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The study of **Algorithms** that build **Models** from **Data** to achieve **Tasks**.

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Model (in machine learning):

- A *"program created by a program"*
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Tasks:

- Sort / Visualize / Represent the **Data**
- **Infer** from **new input Data:**
  - Estimate explanatory quantities
  - Generate, complete, predict
  - Transform, convert, translate
  - Decide

Contrast it with *symbolic* A.I.

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def add_numbers(a, b):
    return a + b
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```python
def square(num):
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  - **Neuro-symbolic A.I.** aims at combining Symbolic A.I. and Neural Networks
  - Attractive approach: Hybridizing **physics-** and **data-**driven models

Types of ML based on available data

- Supervised Learning

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Ex 1: Sound Source Localization

- *A dataset of audio recordings, annotated by source position*
- *Given a new recording, where's the sound?*

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- *An annotated dataset of cat sounds and baby sounds*
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Input ↔ Output

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Inference / Evaluation Stage

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- Identify a gas leak, or a train rail defect from acoustic recordings (indutry monitoring)

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- Obtain a low bitrate representation of music tracks while preserving perceptual fidelity

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Unsupervised (representation) Learning

Supervised Learning

Labeled training data

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OUTPUT →

CONTINOUS

DISCRETE

Supervised Learning

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$\mathcal{T} = \{(\boldsymbol{x}_n, \boldsymbol{y}_n)\}_{n=1}^{N}$

Training

Test

$\tilde{\boldsymbol{x}}$

Learned Model f_θ

Estimate

$\tilde{\boldsymbol{y}}$

Unsupervised (representation) Learning

Unlabeled training data

$\mathcal{T} = \{\boldsymbol{x}_n\}_{n=1}^{N}$

Training

Learned Model f_θ

Representation

$[\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_N]$

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Representation $[\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_N]$

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Learned Model f_θ

Estimate $\tilde{\boldsymbol{y}}$

Unsupervised (representation) Learning

Unlabeled training data

$\mathcal{T} = \{\boldsymbol{x}_n\}_{n=1}^{N}$

Training

Learned Model f_θ

Representation $[\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_N]$

OUTPUT ↓

CONTINOUS

DISCRETE

Supervised Learning

Labeled training data

$\mathcal{T} = \{(\boldsymbol{x}_n, \boldsymbol{y}_n)\}_{n=1}^{N}$

Test $\tilde{\boldsymbol{x}}$ → **Training** → Learned Model f_θ → **Estimate** $\tilde{\boldsymbol{y}}$

Unsupervised (representation) Learning

Unlabeled training data

$\mathcal{T} = \{\boldsymbol{x}_n\}_{n=1}^{N}$

Training → Learned Model f_θ → **Representation** $[\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_N]$

OUTPUT →

CONTINOUS

$\tilde{\boldsymbol{x}}$

$\tilde{\boldsymbol{y}} = (\theta, \phi)$

$\{\boldsymbol{x}_n\}_{n=1}^{N}$ → $\{\boldsymbol{z}_n\}_{n=1}^{N}$

DISCRETE

$\tilde{\boldsymbol{x}}$

→ $\tilde{\boldsymbol{y}} \in \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

Supervised Learning

Labeled training data

$\mathcal{T} = \{(\boldsymbol{x}_n, \boldsymbol{y}_n)\}_{n=1}^{N}$

$[\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N]$ $[\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_N]$

Input — Labels/Targets

Training

Test $\tilde{\boldsymbol{x}}$ → Learned Model f_θ → **Estimate** $\tilde{\boldsymbol{y}}$

Unsupervised (representation) Learning

Unlabeled training data

$\mathcal{T} = \{\boldsymbol{x}_n\}_{n=1}^{N}$

$[\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_N]$

Input

Training

Learned Model f_θ → **Representation** $[\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_N]$

OUTPUT ↓

CONTINOUS

$\tilde{\boldsymbol{x}}$ → $\tilde{\boldsymbol{y}} = (\theta, \phi)$

$\{\boldsymbol{x}_n\}_{n=1}^{N}$ → $\{\boldsymbol{z}_n\}_{n=1}^{N}$

DISCRETE

$\tilde{\boldsymbol{x}}$ → $\tilde{\boldsymbol{y}} \in \left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}$

$\{\boldsymbol{x}_n\}_{n=1}^{N}$ → $\{\boldsymbol{z}_n\}_{n=1}^{N} \subset \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$

OUTPUT ↓

Supervised Learning

Labeled training data

$\mathcal{T} = \{(\boldsymbol{x}_n, \boldsymbol{y}_n)\}_{n=1}^{N}$

Training

Test $\tilde{\boldsymbol{x}}$

Learned Model f_θ

Estimate $\tilde{\boldsymbol{y}}$

Unsupervised (representation) Learning

Unlabeled training data

$\mathcal{T} = \{\boldsymbol{x}_n\}_{n=1}^{N}$

Training

Learned Model f_θ

Representation $[\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_N]$

CONTINOUS

Regression

$\tilde{\boldsymbol{x}}$

$\tilde{\boldsymbol{y}} = (\theta, \phi)$

Dimensionality Reduction

$\{\boldsymbol{x}_n\}_{n=1}^{N} \longrightarrow \{\boldsymbol{z}_n\}_{n=1}^{N}$

DISCRETE

Classification

$\tilde{\boldsymbol{x}} \longrightarrow \tilde{\boldsymbol{y}} \in \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

Clustering

$\{\boldsymbol{x}_n\}_{n=1}^{N} \longrightarrow \{\boldsymbol{z}_n\}_{n=1}^{N} \subset \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

A “Continuum” of Supervision

supervised — *Weakly- / Semi-supervised* — *unsupervised* → **Amount of supervision**

A “Continuum” of Supervision

Sound Source Localization

Image Classification

supervised — *Weakly- / Semi-supervised* — *unsupervised* → **Amount of supervision**

Audio / image Inpainting

bandwidth extension / Super-resolution

A “Continuum” of Supervision

Sound Source Localization

Image Classification

Speaker Diarization

Audio Compression

supervised — *Weakly- / Semi-supervised* — *unsupervised* → **Amount of supervision**

Audio / image Inpainting

bandwidth extension / Super-resolution

Text / image / audio / video generation from a class

A “Continuum” of Supervision

Sound Source Localization

Image Classification

Sound event recognition

Reinforcement learning

Speaker Diarization

Audio Compression

supervised — *Weakly- / Semi-supervised* — *unsupervised* → **Amount of supervision**

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Image or music generation from a prompt

ChatGPT

Text Translation

Text / image / audio / video generation from a class

A “Continuum” of Supervision

PREDICTIVE	Sound Source Localization Image Classification *supervised*	Sound event recognition *Reinforcement learning* *Weakly- / Semi-supervised*	Speaker Diarization Audio Compression *unsupervised*
GENERATIVE	Audio / image Inpainting bandwidth extension / Super-resolution	Image or music generation from a prompt ChatGPT Text Translation	Text / image / audio / video generation from a class

Amount of supervision →

A “Continuum” of Supervision

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PREDICTIVE	Sound Source Localization Image Classification	Sound event recognition *Reinforcement learning*	Speaker Diarization Audio Compression
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Amount of supervision →

Types of data used in machine learning

- Numerical Data / Continuous Data / Signals / Vectors in $\mathbb{R}^D$

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Types of data used in machine learning

- Categorical Data

ImageNet Labels

- airplane
- automobile
- bird
- cat
- deer
- dog
- frog
- horse
- ship
- truck
- …

```
Sex ∈ {male, female, other}
ZIP ∈ {67000, 75009, …}
Country ∈ {France, Spain, …}
```

Types of data used in machine learning

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- Text Data

Artificial intelligence

Article Talk

From Wikipedia, the free encyclopedia

Artificial intelligence (**AI**) is intelligence—perceiving, synthesizing, and inferring information—demonstrated by machines, as opposed to intelligence displayed by non-human animals and humans. Example tasks in which this is done include speech recognition, computer vision, translation between (natural) languages, as well as other mappings of inputs.

AI applications include advanced web search engines (e.g., Google Search), recommendation systems (used by YouTube, Amazon and Netflix), understanding human speech (such as Siri and Alexa), self-driving cars (e.g., Waymo), generative or creative tools (ChatGPT and AI art), automated decision-making and competing at the highest level in strategic game systems (such as chess and Go).[1]

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- Heterogenous / Tabular Data

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- Time series

$$\cdots \rightarrow \boxed{x_{t-1}} \rightarrow \boxed{x_t} \rightarrow \boxed{x_{t+1}} \rightarrow \cdots$$

- Heterogenous / Tabular Data

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- Time series

- Data on graphs

Machine learning algorithms

How to make a program that makes a program?

Machine learning algorithms

How to make a program that makes a program?

- Search in the set of all python functions?

```python
def add_numbers(a, b):
    return a + b
```

```python
def square(num):
    result = num * num
    return result
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Machine learning algorithms

How to make a program that makes a program?

- Search in the set of all python functions?
  - 🚫 a gigantic combinatorial set

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  Find the *best* model within a **parameterized family** $\mathcal{F} = \{m_\theta\}_{\theta \in \Theta}$

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Dataset

- $m_\theta =$ a jean
- $\theta =$ its (width, length)
- $\mathcal{F} =$ the shelves

Machine learning algorithms

How to make a program that makes a program?

- The model is found by minimizing a ***total loss*** / ***cost function*** L over the set of parameters, for a given training **dataset** $\mathcal{T}$:

$$\hat{m} = m_{\hat{\theta}} \quad \text{where} \quad \hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmin}}\, L(m_\theta, \mathcal{T})$$

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- Examples:
  - *mean localization error in degrees*
  - *mean classification error*
  - *mean signal-to-error ratio in dB*
  - *negative likelihood of observed data*

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- The total loss L is designed based on the **task**, the **data**, and the chosen family of **models**. It measures the **fit** of $m_{\hat{\theta}}$ for these data and task.
- Most modern machine learning **algorithms** can be interpreted as minimizing a loss, using **optimization**.

What is a model?

Example:

What is a model?

Example:

Tree diameter

Model

Tree Height

Data

What is a model?

Example:

Tree diameter

Tree Height

What is a model?

Example:

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Example:

What is a model?

Example:

Tree diameter

Tree Height

$$\underset{a,b}{\operatorname{argmin}}\left\{\operatorname{mean}_i (ax_i + b - y_i)^2\right\}$$

« The line is as close as possible to the points »

What is a model?

Training stage

What is a model?

Training stage

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Training stage

What is a model?

Training stage

What is a model?

Training stage

What is a model?

Training stage

Inference / evaluation stage

It's a cat !

Optimization Techniques

Optimization Techniques

1. Brute Force

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2. Population-Based

Optimization Techniques

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3. Zeroing gradient

Optimization Techniques

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$$
\begin{aligned}
\theta_1^{(i+1)} &= \underset{\theta_1}{\operatorname{argmin}}\ g(\theta_1, \theta_2^{(i)}, \ldots, \theta_P^{(i)}) \\
\theta_2^{(i+1)} &= \underset{\theta_2}{\operatorname{argmin}}\ g(\theta_1^{(i+1)}, \theta_2, \ldots, \theta_P^{(i)}) \\
&\vdots \\
\theta_P^{(i+1)} &= \underset{\theta_P}{\operatorname{argmin}}\ g(\theta_1^{(i+1)}, \theta_2^{(i+1)}, \ldots, \theta_P)
\end{aligned}
$$

4. Alternate Minimization

Optimization Techniques

1. Brute Force

2. Population-Based

3. Zeroing gradient

5. Gradient Descent

$$\begin{cases} \theta_1^{(i+1)} = \underset{\theta_1}{\operatorname{argmin}}\ g(\theta_1, \theta_2^{(i)}, \ldots, \theta_P^{(i)}) \\ \theta_2^{(i+1)} = \underset{\theta_2}{\operatorname{argmin}}\ g(\theta_1^{(i+1)}, \theta_2, \ldots, \theta_P^{(i)}) \\ \vdots \qquad\qquad \vdots \\ \theta_P^{(i+1)} = \underset{\theta_P}{\operatorname{argmin}}\ g(\theta_1^{(i+1)}, \theta_2^{(i+1)}, \ldots, \theta_P) \end{cases}$$

4. Alternate Minimization

Artificial Intelligence

"Applied" A.I.

Symbolic A.I.

Machine Learning

Neural Networks

Deep Learning

Artificial Intelligence

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Machine Learning

- **Algorithms** that (implicitly) build **models** from **data** to achieve **tasks**

Neural Networks

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- Different **flavors** depending on available data and task (sup./unsup.)

Neural Networks

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Artificial Neural Networks

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- $\boldsymbol{w}^\top \boldsymbol{x} = \langle \boldsymbol{w}, \boldsymbol{x} \rangle = \sum_{d=1}^{D} w_d x_d$, the ***dot product***

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Examples of activation functions:

Artificial Neural Networks

Artificial Neural Networks

- **Simple perceptron**: multiple neurons attending the same input

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- We have: $y_n = \sigma(\boldsymbol{w}_n^\top \boldsymbol{x} + b_n)$

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- Or in matrix form: $\boldsymbol{y} = \sigma(\mathbf{W}\boldsymbol{x} + \boldsymbol{b})$, where

$$\mathbf{W} = \begin{bmatrix} \boldsymbol{w}_1^\top \\ \boldsymbol{w}_2^\top \\ \vdots \\ \boldsymbol{w}_N^\top \end{bmatrix} = \begin{bmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,D} \\ w_{2,1} & w_{2,2} & \dots & w_{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N,1} & w_{N,2} & \dots & w_{N,D} \end{bmatrix}, \ \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix}, \ \theta = (\mathbf{W}, \boldsymbol{b})$$

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- Or in matrix form: $\boldsymbol{y} = \sigma(\mathbf{W}\boldsymbol{x} + \boldsymbol{b})$, where

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- Note: In this notation, σ is applied **elementwise** to a vector

Artificial Neural Networks

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… But the required $N^{(1)}$ may be **exponential** in the input dimension D .

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- **Multilayer Perceptron** (**MLP**) with 3 hidden layer (***Depth 4***)

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Artificial neural networks with 2 or more hidden layers are called **Deep Neural Networks** (DNNs)

Why Deep?

Why Deep?

1) Biological inspiration

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PowerPoint Lectures for Biology, Seventh Edition Neil Campbell and Jane Reece.

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Frank Rosenblatt. « The perceptron: A probabilistic model for information storage and **organization in the brain** ». ***Psychological Review*** (1958).

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3.7B year evolution + experiencing life	Designed by humans, trained on Internet

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- **Sufficiently large DNNs** (and in particular deep **convolutional** nets) tend to **learn features** automatically. ➔ The ***deeper*** the ***higher*** level.
- Can be successfully applied to **raw signals** (e.g. image pixels, audio samples)

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Theorem [G. Montufar et al., 2014]: *the max. number of linear regions modeled by a piecewise linear network (i.e., a network with ReLU neurons) with D inputs, L layers, and N units per layer is in the order of*

$$N^D \left(\frac{N}{D}\right)^{D(L-2)}$$

, i.e., the model capacity is ***exponential in the depth*** *L, i.e., in the* ***model size*** *(recall it is O(LN²))*

Why Deep?

3) The “Origami Effect”: Illustration with Sigmoids

How to train a DNN?

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1. Brute Force

2. Population-Based

3. Zeroing gradient

$$\begin{cases} \theta_1^{(i+1)} = \underset{\theta_1}{\operatorname{argmin}}\ g(\theta_1, \theta_2^{(i)}, \dots, \theta_P^{(i)}) \\ \theta_2^{(i+1)} = \underset{\theta_2}{\operatorname{argmin}}\ g(\theta_1^{(i+1)}, \theta_2, \dots, \theta_P^{(i)}) \\ \vdots \qquad\qquad \vdots \\ \theta_P^{(i+1)} = \underset{\theta_P}{\operatorname{argmin}}\ g(\theta_1^{(i+1)}, \theta_2^{(i+1)}, \dots, \theta_P) \end{cases}$$

4. Alternate Minimization

5. Gradient Descent

This one !

Gradient Descent

Intuition:

- Start from an initial **parameter vector** $\boldsymbol{\theta}^{(0)} \in \mathbb{R}^P$
- From here, follow the ***direction of steepest descent***
- Stop when things look ***flat***

Animation by Andrew Ng

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- Requires the function to be (almost everywhere) ***differentiable***

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$$\boldsymbol{\theta}^{(i+1)} \leftarrow \boldsymbol{\theta}^{(i)} - \epsilon \nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}^{(i)})$$

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- The ***gradient-step*** ϵ, also called ***learning rate***, is a critical ***hyper-parameter***

Small ϵ Large ϵ Optimal ϵ

Image by Gabriel Peyre

Problem: How to efficiently compute the gradient of a DNN with respect to its parameters?

Compute $\nabla_{\boldsymbol{\theta}} L(\boldsymbol{\theta}, \mathcal{T})$ *?!* 😱

$$\boldsymbol{y} = \mathrm{dnn}_{\boldsymbol{\theta}}(\boldsymbol{x})$$
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The Backpropagation Algorithm

Let's start by **cleaning** a bit the picture:

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- It's enough to calculate the gradient of the loss at **one sample** $(\boldsymbol{x}_t, \boldsymbol{y}_t)$:

$$\boldsymbol{G}_{\boldsymbol{\theta}} \stackrel{\text{def}}{=} \nabla_{\boldsymbol{\theta}} \ell(\mathrm{dnn}_{\boldsymbol{\theta}}(\boldsymbol{x}_t), \boldsymbol{y}_t)$$

and then sum.

The Backpropagation Algorithm

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For example, for the L2 loss $\ell(\boldsymbol{x}^4, \boldsymbol{y}_t) = \|\boldsymbol{x}^4 - \boldsymbol{y}_t\|_2^2$, we have $\boldsymbol{G}_{\boldsymbol{x}^4} = 2(\boldsymbol{x}_t^4 - \boldsymbol{y}_t)$, the **difference** between the network **prediction** and the **target**.

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$\mathbf{W}^4$

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- $$\boldsymbol{G}_{\boldsymbol{w}_d^4} = \left.\frac{\partial r}{\partial \boldsymbol{w}_d^4}\right|_{\boldsymbol{w}_d^4}^\top = \left(\left.\frac{\partial r}{\partial \boldsymbol{a}^4}\right|_{\boldsymbol{a}_t^4} \times \left.\frac{\partial \boldsymbol{a}^4}{\partial \boldsymbol{w}_d^4}\right|_{\boldsymbol{w}_d^4}\right)^\top$$

The Backpropagation Algorithm

2) Then:

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$x_{t,d}^3\mathbf{I}$

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The Backpropagation Algorithm

And so on…

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Putting everything together, we have for example:

$$G_{b^1} = \sigma'(a_t^1) \odot \mathbf{W}^{2\top} \sigma'(a_t^2) \odot \mathbf{W}^{3\top} \sigma'(a_t^3) \odot \mathbf{W}^{4\top} \sigma'(a_t^4) \odot \nabla_{x^4} \ell(x_t^4, y_t) .$$

Stochastic Gradient Descent (SGD) and Batching

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- We train DNNs using so called ***Empirical Risk Minimization***:

$$L(\boldsymbol{\theta}, \mathcal{T}) = \frac{1}{T} \sum_{t=1}^{T} \ell\left(\mathrm{dnn}_{\boldsymbol{\theta}}(\boldsymbol{x}_t), \boldsymbol{y}_t\right) \approx \mathbb{E}_{\boldsymbol{X},\boldsymbol{Y}}\{\ell(\mathrm{dnn}_{\boldsymbol{\theta}}(\boldsymbol{X}), \boldsymbol{Y})\}$$

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- We compute the gradient of the **total loss** by summing the gradients of the **loss** at individual data samples:

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- But doing so across the **entire dataset** (e.g.: 1 million images) for **every gradient step** would be very expansive.

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- At each iteration (i), compute the gradient over a **random subset** $\mathcal{T}^{(i)} \subseteq \mathcal{T}$ and perform one step of gradient descent:

$$\boldsymbol{\theta}^{(i+1)} \leftarrow \boldsymbol{\theta}^{(i)} - \epsilon \cdot \frac{1}{|\mathcal{T}^{(i)}|} \sum_{(\boldsymbol{x}_t, \boldsymbol{y}_t) \in \mathcal{T}^{(i)}} \nabla_{\boldsymbol{\theta}} \ell(\mathrm{dnn}_{\boldsymbol{\theta}}(\boldsymbol{x}_t), \boldsymbol{y}_t)$$

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+ compute gradients of 1 batch in parallel via GPU

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- Competing frameworks: **TensorFlow, Jax**

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2. Machine Learning ✅

   Definition, Characteristics, Types, Algorithms

3. Deep Neural Networks ✅

   Definition, Motivation, How to Train Them

4. Limits of Machine Learning

   Overfitting, Data Biases

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What's going on?

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- We sometimes have **millions** or **billions** of parameters in a model => it has the **capacity** to **store/encode** large amounts of data
- This may even happen for models of relatively small capacity, if the **amount of training data** is insufficient.

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- To achieve this, the split is done **at random**.

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- Looking at the test set if **forbidden** in any of those steps

Overfitting vs. Underfitting

Example of technique to reduce overfitting

➔ Data augmentation

Signal transformations / Noise

Simulate data

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Exercise: *Think of data augmentation strategies for your favorite acoustic application!*

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@getjonwithit

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Now let's code !