

MEMORY, POINTERS, LINKEDLIST ([slides](#))

LinkedList API perspective

- implements List, has the same methods
- reference (pointer) to the first node in a list connected by a reference (pointer) to the next node
- no constant time access to nodes in the middle
 - `get()` → $O(n)$ for singly linked, $O(\min(k, \text{size}-k))$ for doubly (go backward or forward)
 - to get every element one at a time is $O(n^2)$
 - better to use iterator or while `hasNext()` → $O(n)$
- removing from the front is much more efficient than ArrayList
 - update node connections → $O(1)$

memory and references (generally)

- variables for reference types (non primitive) store the location of an object in memory
 - can have multiple references to the same object in memory
- references are copied, changes persist outside of a method
 - vs primitive values are copied, changes won't persist outside of a method
 - still can't "lose" a reference inside a method because the reference is copied
- null reference/pointer
 - null: default value for an uninitialized object
 - check `== null`

LINKED LIST IMPLEMENTATION & POINTER PROBLEMS ([slides](#), [slides](#))

linked list nodes

- each node has a reference (pointer, memory location) to another node
- calling `new Node(...)` always creates a Node in memory that didn't exist before
- `node.next = otherNode` makes `node` → `otherNode`
 - `next` returns the node, not the value
- if node is null (uninitialized), `node.next` or `node.info` gives an error
 - no error to get a null reference, yes error to call something on a null reference
- the variable for the "linked list itself" is a reference to the first ListNode

methods

- `add()` and `remove()` at front are $O(1)$
 - removing first → save reference to second element, set first node to null (break connection), set first node to second element
- `get()` → loops through each element, checks for null for `indexOutOfBounds`, $O(N)$
- `contains()` → loops through list until you find it, $O(N)$
- traverse with a while loop (`list != null`)

pointer problems

- append linked lists → $O(1)$, no copying values, just changing pointers
- get last node → go to next until `list.next == null`

ex: reverse a LinkedList

- `temp = list.next list.next = rev rev = list list = temp` (rev is reversed so far)

RECURSION ([slides](#))

recursion

- base case: easy answer with small input
- recursive calls: get answer on subset of input
- do something with the result of recursive calls and return
- method does not call itself → calls identical clone with its own state, methods/calls are stacked
- call stack (methods are in a queue)
 - each method call gets its own frame (local vars etc)
 - invoking method does not continue until invoked method returns

random note: invariant

- invariant: what's true for each loop, may become false partway through loop, re-established before guard check

ex: copying a linked list

- iterative → initialize first, call new and link, advance pointers (traverse front to back)
- recursive → create one node that is linked to copy of rest, base case is null

developing and verifying code

- verify base case → always null, sometimes one node
- trace through and ensure result of call is used with small size, generalize from n nodes

ex: recursive reverse

- base case is if list or list.next is null
- store recursive call on list.next
- set list.next.next to list, set list.next to null
- return stored recursive call
- runtime = $O(N)$ → will not have to be able to solve and simplify down
 - $T(N) = T(g(N) = N-1) + (f(N) = O(1))$

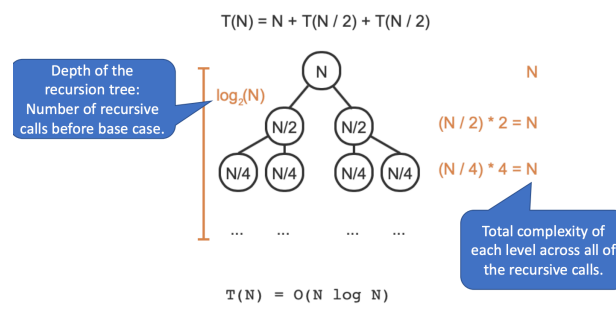
recursive runtime

- $T(N) = a * T(g(N)) + f(N)$
 - $T(N)$: runtime of method with input size N
 - a : number of recursive calls
 - $g(N)$: how much input size decreases on each recursion
 - $f(N)$: runtime of non-recursive code on N

Recurrence	Algorithm	Solution
$T(n) = T(n/2) + O(1)$	binary search	$O(\log n)$
$T(n) = T(n-1) + O(1)$	sequential search	$O(n)$
$T(n) = 2T(n/2) + O(1)$	tree traversal	$O(n)$
$T(n) = T(n/2) + O(n)$	qsort partition, find k^{th}	$O(n)$
$T(n) = 2T(n/2) + O(n)$	mergesort, quicksort	$O(n \log n)$
$T(n) = T(n-1) + O(n)$	selection or bubble sort	$O(n^2)$

mergesort

- insertion sort without recursion $\rightarrow O(N^2)$
 - loop through original list, maintain a new sorted list, insert one value at a time in order (looping through new sorted list)
- base case \rightarrow size 1, return list
- recursive case \rightarrow mergesort(first half), mergesort(second half), return merged sorted halves with helper merge(first, second)
 - merge() \rightarrow add to a new list, traverse listA and listB with 2 indices with looping, compare the value at each of 2 indices and add the smaller
- $O(N \log(N)) \rightarrow$ halves at each level so $O(\log(N))$ levels, merge at each level with $O(N)$
 - merge() $\rightarrow O(nA + nB) = \text{linear}$



- mergeSort() \rightarrow

SORTING, COMPARABLE, COMPARATOR ([slides](#))

Java API sort algorithms

- Collections.sort for List, Arrays.sort for an Array
 - will actually alter the object, even just called within a method
- both implement Timsort (variant of Mergesort)
 - $O(N \log(N))$, nearly linear
 - sorts in place, mutates input instead of making new List/Array
 - stable: does not reorder elements if not needed (ex: 2 elements are equal)

Comparable

- class objects that implement Comparable interface can be sorted → have a naturalOrder
- compareTo()
 - < 0 if this comes before parameter, 0 if equal, > 0 if this comes after parameter
 - ^ if it's in the right order, return negative
 - method is in class of which this object is an instance
- Strings
 - compareTo() for natural lexicographic ordering (dictionary order, starts at first char)
 - ex: "a".compareTo("b"); → -1
- can implement Comparable interface and compareTo()
 - defines a natural ordering, can sort (Collections.sort, Arrays.sort)

Comparator

- use Comparator when not changing the object itself to compare non natural order
 - comparing(Comparator.naturalOrder()) for natural order
- Comparator c, c.compare(a,b)
 - < 0 if a comes before b, 0 if equal, > 0 if a comes after b
 - method is part of Comparator

- ex: sort by length

```
copy = Arrays.copyOf(a, a.length);
Arrays.sort(copy, Comparator.comparing(String::length));
```

- ex: Collections.sort(schools, Comparator.comparing(University::getName));
 - ^no () because passing in the getName function name itself, not the return value
- to combine sequence of comparison → Comparator.comparing(_::_).thenComparing
- Comparator with lambdas (for when you compare something without a getter)
 - Comparator<Object type> c = (given a,b) -> (what c.compare(a,b) should return)
 - ex: sort by first elements

```
Comparator<int[]> comp =
    (a, b) -> (a[0] - b[0]);
```

- can use sort(_, createdComparator)

runtime complexity of sort() and Comparator → $O(N \log(N))$

- Arrays.sort() and Collections.sort() call either compareTo() (default) or compare() (if Comparator)

comparisons in mergesort

- one comparison per loop iteration / per element merged
- $O(CN \log(N))$, where C is complexity of compareTo() or compare()
- C is not constant

```
public class ListComp implements Comparator<List<Integer>> {
    @Override
    public int compare(List<Integer> list1, List<Integer> list2) {
        int minLength = Math.min(list1.size(), list2.size());
        for (int i=0; i<minLength; i++) {
            int diff = list1.get(i) - list2.get(i);
            if (diff != 0) {
                return diff;
            }
        }
        return 0;
    }
}
```

- worst case to sort here is $O(MN \log(N))$ (N ArrayLists, each with M elements)

- M is complexity of each call of `compare()`

binary search

- given a sorted list of N elements and a target, return index i of target or -1 if not in list
- $O(\log(N)) \rightarrow$ cut down search space by half at each step
- process
 - low (initially 0) and high (initially $N-1$) mark the limits of active search space
 - loop while (low \neq high) \rightarrow while there's anything left to search
 - loop invariant \rightarrow if target is in array/list, it is in [low, high]
 - compare mid $((\text{low} + \text{high}) / 2)$ to target
 - search in lower (high = mid-1) or upper half (low = mid+1)
 - not high = mid or low = mid to prevent infinite loop in edge cases \rightarrow you could get stuck at just a few elements where the range won't decrease
- does not guarantee the first or last index of a match if there are multiple