COMPSCI 230 MIDTERM 1 REFERENCE SHEET

proposition: statement that is either true or false

- propositions can be combined using logical operators (and = ^)

truth table: possibilities for compound propositions

implication: $P \rightarrow Q \leftrightarrow \neg P \lor Q$

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	F

De Morgan's laws

- $\neg(P \land Q) = \neg P \lor \neg Q \rightarrow$ not(P or Q) = neither
- $\neg(P \lor Q) = \neg P \land \neg Q \rightarrow$ not(P and Q) = either

distributive law

- $P \land (Q \lor R) = (P \land Q) \lor (P \land R)$
- $P \lor (Q \land R) = (P \lor Q) \land (P \lor R)$

valid: always true ($P \lor \neg P$)

satisfiable: not always false ($P \neg Q$, but not $P \land \neg Q$)

predicate: proposition quantified by a variable

domain: set from which variables are drawn

quantifier negation

- $\neg(\forall x\, P(x)) \equiv \exists x\, \neg(P(x))$
- $\neg(\exists x\, P(x)) \equiv \forall x\, \neg(P(x))$

inference

- start with a set of valid propositions/predicates, write conclusions (other statements that are also valid/always true)

rules

- modus ponens: $P,\ P \rightarrow Q,\ \therefore Q$
  - $P \land (P \rightarrow Q) \Rightarrow Q$
- modus tollens: $\neg Q,\ P \rightarrow Q,\ \therefore \neg P$
  - if P = T, then since $P \rightarrow Q$, Q = T but Q = F, $\therefore$ P must be F
- $P \rightarrow Q,\ Q \rightarrow R,\ \therefore P \rightarrow R$
- $P \land Q,\ \therefore P$
- $P \lor Q,\ \neg P,\ \therefore Q$
- $P,\ Q,\ P \land Q$

model: assignment of meaning to predicates

- inference is valid if it's T regardless of model
- inference is invalid if there is a model where preconditions are T but conclusion is F

axiom: a set of propositions we believe hold true

theorem: another true statement deduced from axioms with rules of inference

proof strategies

- direct proofs
- cases
- contrapositive (prove the contrapositive for $P \rightarrow Q$)
  - contrapositive: $\neg Q \rightarrow \neg P$
- contradiction (suppose the statement is false, go until contradicts)
- well ordering
  - WOP: any set of positive integers has a smallest element
  - assume min counterexample, show smaller counterexample
- note: iff $\leftrightarrow$
  - to show $P \leftrightarrow Q$, show $P \rightarrow Q$, $Q \rightarrow P$
  - to show $P \leftrightarrow Q \leftrightarrow R$, show $P \rightarrow Q \rightarrow R \rightarrow P$

induction

- base case: show P(1) is true
- inductive hypothesis: assume statement true
- inductive step: show $P(n) \rightarrow P(n+1)$ for all $n \geq 1$

strong induction: use truth of all P(1), P(2)...P(n) to show truth of P(n+1)

Let P be a predicate on nonnegative integers. If

- $P(0)$ is true, and
- for all $n \in \mathbb{N}$, $P(0), P(1), \ldots, P(n)$ *together* imply $P(n+1)$,

then $P(m)$ is true for all $m \in \mathbb{N}$.