Exam Preparation for
Introduction to Adaptive Control 2015/16

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Abstract

The following problem statements (together with your knowledge from the Project Homework) will be the main guide through the oral exam. Questions will be asked according to your work but also about the whole course. You will present the solutions and answer questions regarding them. The exam is open book, open laptop and open notes. There will be a projector at the exam.

If you do not have a computer to present your simulations, you may print the main results or simply bring a USB-drive.

1 Output Feedback Adaptive Control with Plant Relative Degree $n^* = 1$

In the following, you will design and implement a model-reference adaptive output-feedback controller. Prepare a simulation of your controller with the system in Simulink and a sketch of the stability proof. Here "sketch" means that you should know how it works and can explain it.

1.1 MRAC Design

Design an output feedback MRAC for the following. The plant G is

$$G(s) = \frac{s+1}{s^2 - 5s + 6}$$

where no coefficients are known. The reference model M is

$$M(s) = \frac{1}{s+1}$$

Chose the controller structure and the adaptive law at your discretion.

Compute the exact controller parameters (i. e., the parameters of the control law which would result in perfect tracking of the reference model without adaptation). Compare the estimated parameters, to the exact parameters. Show the comparison in a Presentation and/or a printout. Have Simulink ready.

1.2 Parameter Estimation

This does not build on the previous result - it does not require the controller. The plant G stays the same. Use a Persistetly Exciting input signal into the plant.

Design a Least-Squares Parameter Estimation Law (pure Least Squares). Show your results.

In the exam, be prepared to change the plant parameters to different values and answer questions regarding the resulting behavior.

2 Robustness Modifications

Rebuild the Rohr's Instability Examples as given in the course. Implement the robustness measures that were given. If you could pick any combination of two robustness measures, which ones would you pick and why?

3 Advanced Understanding

This section requires very good understanding of the course contents. You have all the tools necessary to figure it out, but don't start here if you have not finished everything above.

3.1 Alternative Adaptive Law for Relative Degree 1

From the standard adaptive control design, it should be clear that $e(t) \in \mathcal{L}_2$. Explain why.

Now, with the standard error equation for first order systems $\dot{e}(t) = -a_{\mathrm{m}} e(t) + k_{\mathrm{p}} \tilde{\theta}(t) \phi(t)$ consider the following Lyapunov-like function

$$V(e, \tilde{\theta}) = \frac{|e(t)|^{1+\alpha}}{1+\alpha} + \frac{1}{2} |k_{\mathrm{p}}| \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}$$

with $\Gamma = \Gamma^T > 0$ and a design parameter $0 < \alpha < \infty$.

With this as a starting point, design an adaptive law so that you end up with

$$\dot{V}(e, \tilde{\theta}) = -a_{\mathrm{m}} |e(t)|^{1+\alpha}.$$

What signal space does $e(t)$ belong to, now?

Compare your new adaptive law with the standard adaptive law with CRM. Prepare a simple example and plots.

3.2 Adaptive Dynamic Inversion

A slightly different representation of parameters of a plant may lead to a different adaptive controller. Investigate the following:

The plant is the first order system

$$G: \quad \dot{x} = ax + bu$$

where a and b are unknown parameters. As in MRAC, we introduce a stable reference model

$$M: \quad \dot{x}_{\mathrm{m}} = a_{\mathrm{m}} x_{\mathrm{m}} + b_{\mathrm{m}} r$$

and the goal of tracking the reference model (i. e., $\lim_{t\to\infty} (x(t) - x_{\mathrm{m}}(t)) = 0$) remains as in MRAC.

Unlike MRAC we now rewrite the plant dynamics

$$G: \quad \dot{x} = \hat{a}x + \hat{b}u - \underbrace{(\hat{a} - a)}_{\tilde{a}} x - \underbrace{\left(\hat{b} - b\right)}_{\tilde{b}} u$$

where '$\hat{\cdot}$' indicates a parameter to be estimated online and '$\tilde{\cdot}$' indicates a parameter error.

Pick the control input u as

$$u = \frac{1}{\hat{b}} \left[(a_{\mathrm{m}} - \hat{a})\, x + b_{\mathrm{m}} r \right].$$

Now continue from here:

1. Compute the closed-loop system
2. Compute the error dynamics $e = x - x_{\mathrm{m}}$
3. Select a Lyapunov-like function candidate $V(e, \tilde{a}, \tilde{b})$ (Hint: You are not to copy the Lyapunov-like function from the course. Start simple.)
4. Compute the time derivative $\dot{V}$ along the error dynamics and make it negative semi-definite
5. What are the adaptive laws?

Does the error e go to zero? What is different from MRAC? What are the advantages and the disadvantages?