Home assignment N2 for the course ODE and mathematical modelling MMG511/TMV161 in year 2015

Logistic equation and Lotka Volterra two species competition model

The report for a modeling assignment must be written as of small scientific article that anybody without deep knowlege in the subjec must be able to read.

Each modeling modeling assignment consists of

i) a theoretical part, with theoretical conclusions

ii) an implementation part including writing a simple Matlab code solving an ODE, graphical output, and

iii) an analysis part including interpretation of numerical results and conclusions based on them.

You are welcome to contact the teacher in his office L2034 or by e-mail to ask questions. If a larger group of students needs help with Matlab programming an additional lecture can be given.

You are encouraged to work in small groups of 2-3 people, but each of you must write an own report (in TEX or Word) including:

1) Intorduction to and motivation of the subject 2) analytical work, 3) theoretical argumentation, and conclusions 4) numerical results with graphical output, their interpretation, and conclusions, 5) names of all group members (I will not accept reports without specifying group members!)

Send Matlab codes with clear comments by a separate e-mail.

Remember that copying of others programs is plagiarism and is prosecuted by Chalmers and GU administrations.

Grades for your reports on home assignments will contribute 30% to the final marks for the course.

Good to be ready before 25-th of may, dead line is 29-th of may. Grades for your reports on home assignments will contribute 30% to the final marks for the course.

Consider the logistic equation and the Lotka-Volterra two species competition model.

Let $x_i(t), i = 1, 2.$ be populations of two species. Each of the species grows with intrinsic growth rate r_i in case when infinite resources are available: $x_i' = r_i x_i$, $r_i > 0$.

Limited resources lead to competition within the population and a limited growth rate for large size of the population: $r_i(1 - \frac{x_i}{K_i})$. This model is called the logistic equation:

$$x_i' = r_i x_i \left(1 - \frac{x_i}{K_i}\right)$$

Here the constant $K_i > 0$ is called carrying capacity.

The competition between different species leads a decrease in each population with the decreasing rate proportional to the competitor population size: $-\alpha x_2$ for the population x_1 and $-\beta x_1$ for the population x_2 with competition coefficients $\alpha > 0$ and $\beta > 0$. The corresponding system is called Lotka-Volterra two species competition model:

$$\begin{aligned} x_1' &= r_1 x_1 \left(1 - \frac{x_1}{K_1}\right) - \alpha x_1 x_2 \\ x_2' &= r_2 x_2 \left(1 - \frac{x_2}{K_2}\right) - \beta x_2 x_1 \end{aligned}$$

The theory for the Lotka-Volterra two species competition model is described in the course book on pages 88-91.

Questions for the fome assignment N1

1. Consider the logistic equation and prove that $x_1(t) > 0$ and $x_2(t) > 0$ $x_1(0) > 0$ and $x_2(0) > 0$.

2. Clarify the meaning of the carrying capacity constant K_i both from theoretical analysis of the logistic equation. To do it check the sign of the right hand side in the equation depending on the relation between $x_i(t)$ and K_i. It will show in which direction up or down the solution $x_i(t)$ goes depending on that. Make conclusions about the behaviour of solutions for $t \to \infty$.

3. Study the description of the theory for the Lotka-Volterra two species competition model in the course book pages 88-91. Many detailes of calculations are missed there!

4. Consider the Lotka-Volterra two species competition model and show that $x_1(t) \geq 0$ and $x_2(t) \geq 0$ if $x_1(0) > 0$ and $x_2(0) > 0$ and that in this case

$$x_1' \leq r_1 x_1 \left(1 - \frac{x_1}{K_1}\right); \quad x_2' \leq r_2 x_2 \left(1 - \frac{x_2}{K_2}\right)$$

5. Write you own detailed version of the stability analysis of all stationary points an all possible qualitatively different scenarios on the phace portrait for the Lotka-Volterra two species competition model including ALL detailes of calculations.

6. Run your program for the logistic equation for large(!) time intervals, and illustrate the behaviour of solutions using the graphical output for different initial data $x_i(0) > K_i > 0$ and $0 < x_i(0) < K_i$ and for 3-4 different values of (positive!) constants in the model. Make conclusions about the behaviour of solutions for $t \to \infty$. In this case the equations are independent and it is enough to consider only one of them. Save figures as png or jpg files to use them in the report.

7. Illustrate your theoretical analysis of the Lotka-Volterra two species competition model by running your numerical program for this system for large time intervals, with graphical output for the phase portrait consisting of many trajectories with different initial data $x_i(0) > 0$ for the values of parameters in the model corresponding to ALL qualitatively different variants of the phase portrait. Save figures as png or jpg files to use them in the report.

8. Do you observe a qualitative difference between the behaviour solutions to the logistic equation and to the Lotka-Volterra two species competition model when $t \to \infty$ for some particular combinations of parameters ? How this difference depends on the facts that α and β are small or large?

9. Estimate using Gronwall inequality difference between solutions to the Lotka-Volterra two species competition model and a system of two uncoupled logistic equations with the same K_1 and K_2 parameters as in the Lotka-Volterra system .

Necessary programming.

1. Write two Matlab codes: one that can solve a scalar ODE and one that can solve a system of two ODEs. Let both programs choosing several initial data points and draw corresponding integral curves both as functions of time. In the case of a system of two equations the code must draw paths of $(x_1(t), x_2(t))$ in the phase plane of (x_1, x_2) in a separate figure. You can use function *plot* for graphics. In the case of the system of two equations the function *ginput* in Matlab can be used (not obligatory) to choose a point of initial data from the plane of (x_1, x_2). Define coordinates limits in the figure at least twise larger than K_i before using *ginput* to make the set of possible initial data large enough for choosing them by *ginput*

2. Write a Matlab function describing the right hand side of the logistic equation.

3. Write a Matlab function describing the right hand side of the Lotka-Volterra two species competition model.