The fractal market hypothesis (FMH)20 is based on the idea that price movements show certain

patterns, i.e. self-similarity instead of a pure random walk. A sign of self-similarity is the clustering

and the bursting of time series in raw returns or volatility. Long-memory is characterised by

hyperbolically decaying autocorrelations: the large (small) volatility are more likely to be followed

by large (small) volatility than small (large) volatility.  
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For self-similar processes, the local properties are reflected in the global ones and vice versa.

Hence, self-similar processes are invariant stochastic processes in distribution under suitable scaling

of time.9 To characterise such invariant stochastic process in a global sense, we use the Hurst

exponent (H ) as a parameter of self-similar process, whereas fractal dimension is used as a measure

of roughness.10 We often refer to the Hurst effect of time series when long-range dependence in

the time series is associated with power correlations.

In the previous chapter, the fractal analysis is limited to study a single scaling law for financial

time series, see Peters (1994). The long-term behaviour of financial time series is too complex to

be described just by means of a single fractal dimension. Thus, we will turn our point of view from

fractal to multi-fractal behaviour. The fractal formalism corresponds to the invariant probability

distribution, while the multi-fractal formalism studies a non-linear relation between the invariant

probability densities on a generic support [Chhabra and Jensen (1989)]. In the framework of multi-

fractals, the moments of distributions can be characterised by a non-linear exponent function