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1. Use reduction to prove that the language $L = \{\rho(M_1)\rho(M_2) : L(M_1) \cup L(M_2) = \Sigma^*\}$ is not decidable (you may assume that M_1 and M_2 have the same input alphabet Σ).

   We can reduce the Halting Problem to L by doing the following. Construct M_1 such that $L(M_1)$ accepts the input if M halts on w while M_2 is constructed such that $L(M_2)$ accepts all inputs.

   If M halts on w, then $\rho(M_1)\rho(M_2) \in L$ since $L(M_1) \cup L(M_2) = \Sigma^*$. If M doesn't halt, then $\rho(M_1)\rho(M_2) \notin L$ since $L(M_1) \cup L(M_2) \neq \Sigma^*$.

   This shows a potential way for L to be decided. However, this is a contradiction since we can reduce the Halting Problem to L, which means L is not decidable.

2. Define an unrestricted grammar for the language $\{a^{n^2} : n \in \mathbb{N}\}$.

   $G = (\{[,], S, A, L, R, X\}, \{a\}, S, P)$

   $P = \{S \rightarrow [AL],$
   $aL \rightarrow La,$
   $AL \rightarrow LA,$
   $[L \rightarrow [R,$
   $L] \rightarrow X,$
   $Ra \rightarrow aR,$
   $RA \rightarrow aAR,$
   $R] \rightarrow AAL],$
   $aX \rightarrow Xa,$
   $AX \rightarrow Xa,$
   $[X \rightarrow \epsilon\}$