

1. Use the pumping lemma to prove that $L = \{ww : w \in \{0, 1\}^*\}$ is nonregular.

If L is regular, then $\exists m$ such that $\forall x \in L, |x| \geq m, \exists u, v, t, |v| > 0, |uv| \leq m, x = uvt, \forall k \in \mathbb{N}, uv^k t \in L$.

Let's choose $x = 0^m 10^m 1 \in L$. In this string, $v = 0^i$ for some $i > 0$.

We see that when $k = 2$, $uv^2 t = 0^{m+i} 10^m 1 \notin L$ since pumping results in a string that is not ww . Therefore, by contradiction, $L = \{ww : w \in \{0, 1\}^*\}$ is nonregular.

2. Define a context-free grammar for the language $L = \{a^n b^m c^{n-m} : n \geq m\}$.

$G = (\{S, B\}, \{a, b, c\}, S, \{S \rightarrow aSc \mid B\epsilon, B \rightarrow aBb \mid \epsilon\})$

3. Define a context-free grammar for the language $L = \{a^n b^m : n \leq 3m\}$.

$G = (\{S\}, \{a, b\}, S, \{S \rightarrow Sb \mid aSb \mid aaSb \mid aaaSb \mid \epsilon\})$