ComS 331 Spring 2024 Name:_____

HW 3 Due: 9 feb 2024

1. Consider the n-bit binary representation of a natural number x:

$$\text{the binary representation of } x \text{ is } (x_{n-1}x_{n-2}\cdots x_1x_0)_2 \iff x = \sum_{i=0}^{n-1} x_i \cdot 2^i$$

where each bit x_i is a binary digit, either zero or one. For example, $(00000101)_2$ is the 8-bit binary representation of the number 5, since $0\cdot2^7+0\cdot2^6+0\cdot2^5+0\cdot2^4+0\cdot2^3+1\cdot2^2+0\cdot2^1+1\cdot2^0 = 4+1=5$. This is the format normally employed by digital computers to store nonnegative integers.

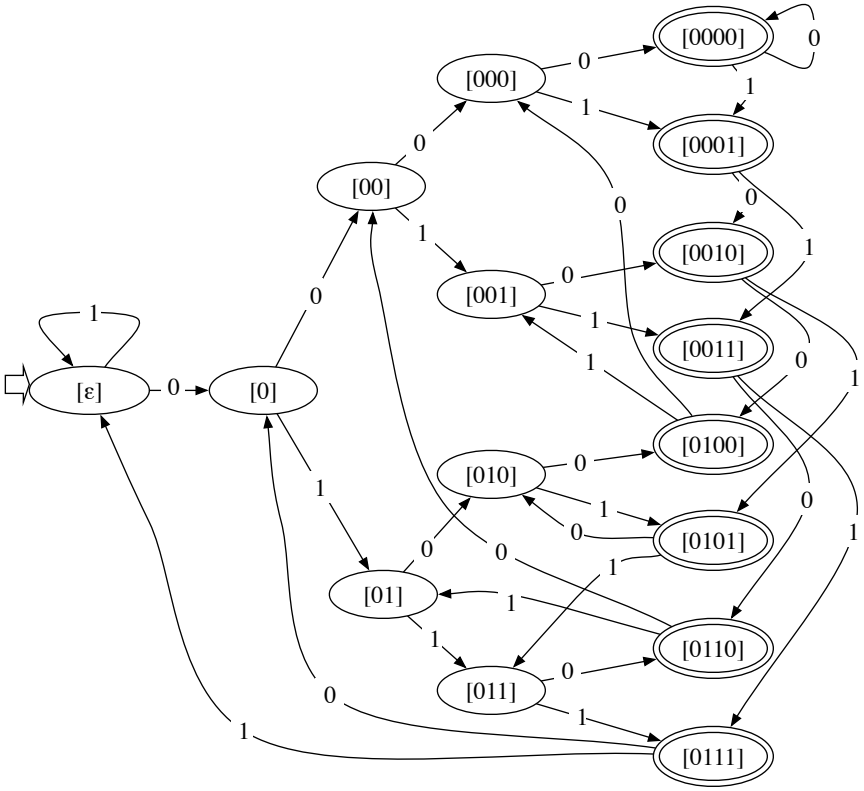
Consider the language

$$L = \{a_0b_0\cdots a_{n-1}b_{n-1} \ : \ n \in \mathbb{N} \wedge \forall i, 0 \le i < n, a_i \in \{0,1\} \wedge b_i \in \{0,1\} \wedge (a_{n-1}\cdots a_0)_2 > (b_{n-1}\cdots b_0)_2\}$$

For example, since $5 = (000101)_2$, $3 = (000011)_2$, and $5 > 3$, $110110000000 \in L$.

Draw a DFA, minimized to the best of your abilities, that accepts L. 100

2. Describe in a short English sentence the language accepted by the following DFA, and give a regular expression for it (hint: the "names" of the states reflect their meaning). Then, draw a 5-state NFA that accepts the same language.



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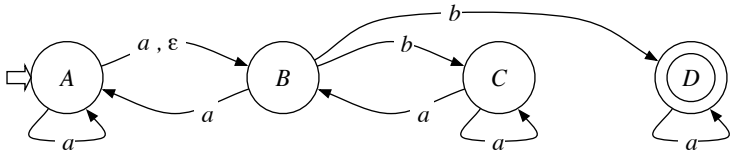
3. Prove or disprove: if a language $L \subseteq \Sigma^*$ is recognized by a FA, then there is a NFA $M = (K, \Sigma, \delta, s_0, F)$ with $|F| = 1$ such that $L = L(M)$. 50

4. Consider the following NFA M:



- Using the algorithm described in class, derive an equivalent (non-minimized) DFA M'.
- Using the algorithm described in class, minimize M' and obtain the minimized DFA M''.
- Describe in English, as succintly as you can, the essential characteristics of the language accepted by these automata.

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