

1. Are the following languages Turing decidable, Turing acceptable but not Turing-decidable, or not even Turing acceptable?
 - $L = \{\rho(M)\rho(w) : M \text{ uses a finite number of tape cells when running on input } w\}$.
 - $L = \{\rho(M)\rho(w)01^n0 : M \text{ uses at most } n \text{ tape cells when running on input } w\}$.

Here, “using n cells” means that the head of the (deterministic) TM M reaches the n -th cell from the left during its computation. Justify your answers clearly: both exercises require careful thinking. Note that this exercise is in a sense relevant to “real computing”, since one could argue that the computers we use in practice have a large but finite memory.

L where M uses a finite number of tape cells when running on input w is **Turing acceptable, but not Turing-decidable**. First off, it’s not a guarantee that M will halt on all w , so L cannot be Turing-decidable. However, if it does halt on an input and uses a finite number of tape cells, L can halt. This means that L is Turing acceptable.

L where M uses at most n tape cells when running on input w is **not even Turing acceptable**. Like the other L from above, M is not guaranteed to halt on all w , so L cannot be Turing-decidable. It also isn’t Turing acceptable because M can use at most n tape cells, but still not halt. Therefore, L may not accept everything that in fact uses at most n tape cells.