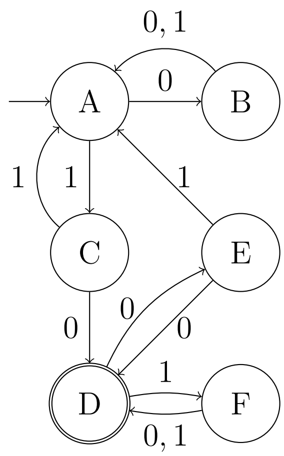
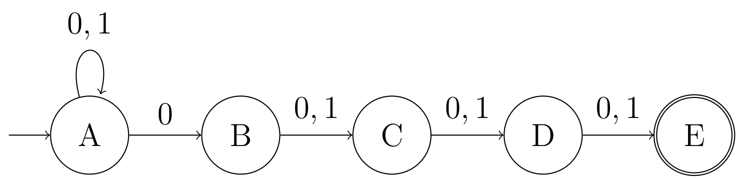
ComS 331 Spring 2024 Name: Aren Ashlock

1. $L = \{a_0b_0...a_{n-1}b_{n-1} | n \in \mathbb{N} \wedge \forall i, 0 \leq i < n, a_i \in \{0,1\} \wedge b_i \in \{0,1\} \wedge (a_{n-1}...a_0)_2 > (b_{n-1}...b_0)_2\}$



2. "The language which the final 4 bits are less than 8 (in decimal)."
$L = \{a_0a_1...a_{n-2}a_{n-1} | n \in \mathbb{N} \wedge n \geq 4 \wedge \forall i, 0 \leq i < n, a_i \in \{0,1\} \wedge (a_{n-4}a_{n-3}a_{n-2}a_{n-1})_2 < 8_{10}\}$



3. Prove or disprove: if a language $L \subseteq \Sigma^*$ is recognized by a FA, then there is a NFA $M = (K, \Sigma, \delta, s_0, F)$ with $|F| = 1$ such that $L = L(M)$.

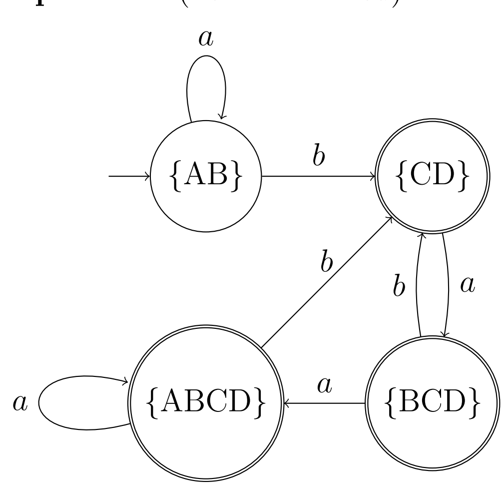
   Let's start by defining an FA which recognizes the language L. This is defined as $M' = (K', \Sigma, \delta', s_0', F')$.

   Then, we construct the NFA $M = (K, \Sigma, \delta, s_0, F)$ where $K = K' \cup \{k_f\}$, $s_0 = s_0'$, and $F = \{k_f\}$. This satisfies the condition that $|F| = 1$. For δ, we construct it as follows: $\forall k \in K', \forall x \in \Sigma, \delta(k, x) = \delta'(k, x)$ AND $\forall k \in F', \delta(k, \epsilon) = \{k_f\}$.

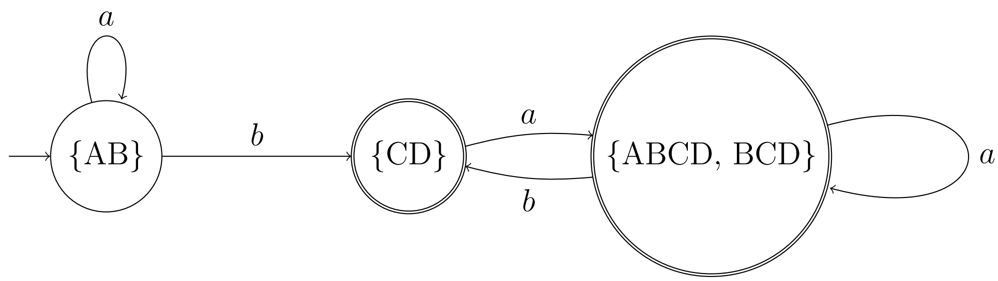
   As a result, we get an NFA with only 1 final state. This NFA is essentially the same in terms of states and transitions. However, we added in a new, singular final state that each of the final states from the FA transition to by ϵ. This ends up accepting the same language as the FA, but with only 1 final state.

   Therefore, there is a NFA $M = (K, \Sigma, \delta, s_0, F)$ with $|F| = 1$ such that $L = L(M)$.

4. **Equivalent** (non-minimized) DFA M':



Minimized DFA M'':



English: "The language which all strings contain at least 1 b and there are no consecutive b inputs."