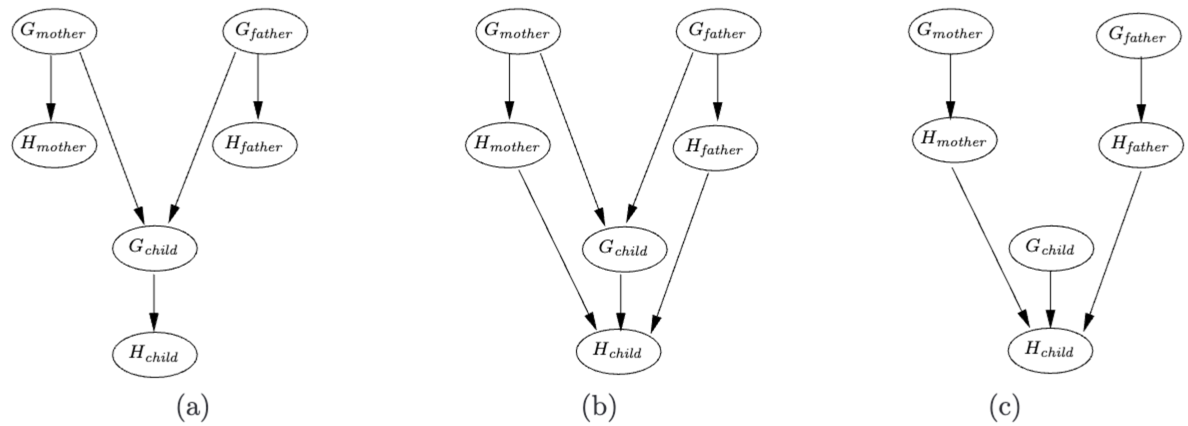
**ComS 472 - PS10 Due: Nov 24, 2024 Name: Aren Ashlock**

1. **(18 pts)** (Exercise 14.7) Let $H_x$ be a random variable denoting the handedness of an individual $x$, with possible values $l$ or $r$. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene $G_x$, also with values $l$ or $r$, and perhaps actual handedness turns out mostly the same (with some probability $s$) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability $m$ of a random mutation flipping the handedness.




(a) **(2 pts)** Which of the three networks in the above figure claim that

$$\mathbf{P}(G_{father}, G_{mother}, G_{child}) = \mathbf{P}(G_{father})\mathbf{P}(G_{mother})\mathbf{P}(G_{child})$$

**C**, all genes are independent of each other

(b) **(3 pts)** Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

**A and B**, the gene of the child is dependent on the parents, but the parents are independent, which both contain.

(c) **(3 pts)** Which of the three networks is the best description of the hypothesis?

**A**, it's either A or B since the hypothesis notes a dependence of the child's gene on the parents' gene. But, the hypothesis doesn't mention anything about the handedness of the child depending on the handedness of the parents.

(d) **(4 pts)** Write down the CPT for the $G_{child}$ node in network (a), in terms of $s$ and $m$.

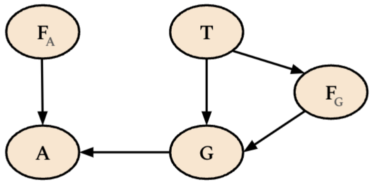
| $G_{father}$ | $G_{mother}$ | $P(G_{child} = l \mid G_{father}, G_{mother})$ |
|---|---|---|
| $l$ | $l$ | $1 - m$ |
| $l$ | $r$ | $0.5(1 - m) + 0.5(m) = 0.5$ |
| $r$ | $l$ | $0.5$ |
| $r$ | $r$ | $m$ |

(e) **(6 pts)** Suppose that $P(G_{father} = l) = P(G_{mother} = l) = q$. In network (a), derive an expression for $P(G_{child} = l)$ in terms of $m$ and $q$ only, by conditioning on its parent nodes.

$P(G_{child} = l) = \sum P(G_{child} = l \mid G_{father} \wedge G_{mother})P(G_{father})P(G_{mother})$
$P(G_{child} = l) = (1 - m)(q^2) + 2 \times (0.5)(q)(1 - q) + (m)(1 - q)^2$
$P(G_{child} = l) = q^2 - mq^2 + q - q^2 + m - 2mq + mq^2$
$P(G_{child} = l) = q + m - 2mq$

2. **(10 pts)** (Exercise 14.13) In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables $A$ (alarm sounds), $F_A$ (alarm is faulty), and $F_G$ (gauge is faulty) and the multivalued nodes $G$ (gauge reading) and $T$ (actual core temperature).

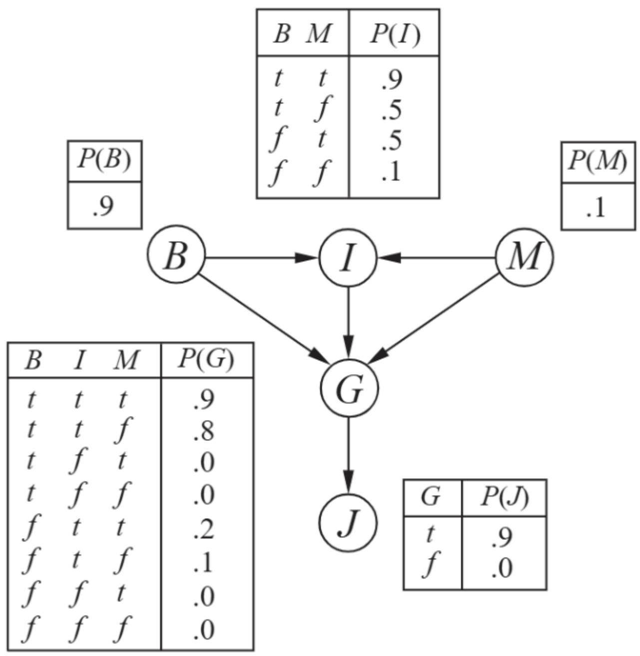
(a) **(6 pts)** Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.




(c) **(4 pts)** Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is $x$ when it is working, but $y$ when it is faulty. Give the conditional probability table associated with $G$.

| $T$ | $F_G$ | $P(G = normal \mid T, F_G)$ |
|---|---|---|
| $normal$ | $f$ | $x$ |
| $normal$ | $t$ | $y$ |
| $high$ | $f$ | $1 - x$ |
| $high$ | $t$ | $1 - y$ |

3. **(12 pts)** (Exercise 14.16) Consider the Bayes net shown below.




| $B$ | $M$ | $P(I)$ |
|---|---|---|
| $t$ | $t$ | .9 |
| $t$ | $f$ | .5 |
| $f$ | $t$ | .5 |
| $f$ | $f$ | .1 |

| $P(B)$ |
|---|
| .9 |

| $P(M)$ |
|---|
| .1 |

| $B$ | $I$ | $M$ | $P(G)$ |
|---|---|---|---|
| $t$ | $t$ | $t$ | .9 |
| $t$ | $t$ | $f$ | .8 |
| $t$ | $f$ | $t$ | .0 |
| $t$ | $f$ | $f$ | .0 |
| $f$ | $t$ | $t$ | .2 |
| $f$ | $t$ | $f$ | .1 |
| $f$ | $f$ | $t$ | .0 |
| $f$ | $f$ | $f$ | .0 |

| $G$ | $P(J)$ |
|---|---|
| $t$ | .9 |
| $f$ | .0 |

(a) **(3 pts)** Which of the following are asserted by the network structure?

$$\mathbf{P}(B, I, M) = \mathbf{P}(B)\mathbf{P}(I)\mathbf{P}(M) \tag{1}$$

$$\mathbf{P}(J|G) = \mathbf{P}(J|G, I) \tag{2}$$

$$\mathbf{P}(M|G, B, I) = \mathbf{P}(M|G, B, I, J) \tag{3}$$

**2** (conditional independence, $I$ is not in $MB(J)$ so $I$ has no impact)
**and 3** (conditional independence, $J$ is not in $MB(M)$ so $J$ has no impact)

(b) **(3 pts)** Calculate the probability $P(b, i, \neg m, g, j)$.
$P(b, i, \neg m, g, j) = P(b)P(\neg m)P(i|b \wedge \neg m)P(g|b \wedge \neg m \wedge i)P(j|g)$
$P(b, i, \neg m, g, j) = 0.9 * (1 - 0.1) * 0.5 * 0.8 * 0.9$
**Answer:** $P(b, i, \neg m, g, j) = 0.2916$

(c) **(6 pts)** Calculate the probability that someone goes to jail given that they broke the law, have been indicted, and face a politically motivated prosecutor.
$P(j|b \wedge i \wedge m) = \sum P(j|G)P(G|b \wedge i \wedge m)$
$P(j|b \wedge i \wedge m) = P(j|g)P(g|b \wedge i \wedge m) + P(j|\neg g)P(\neg g|b \wedge i \wedge m)$
$P(j|b \wedge i \wedge m) = 0.9(0.9) + 0.0(1 - 0.9)$
**Answer:** $P(j|b \wedge i \wedge m) = 0.81$

4. **(10 pts)** (Exercise 14.18(a)) Consider the variable elimination algorithm given below. For the shown burglary-alarm network, applies variable elimination to the query

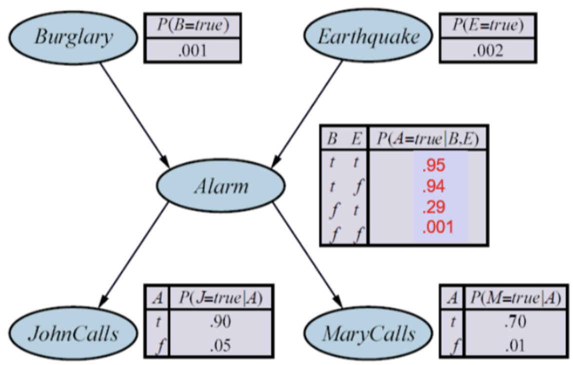
$$\mathbf{P}(Burglary|JohnCalls = True, MaryCalls = True).$$

Perform the calculations indicated and check that the answer is correct.

```
function ELIMINATION-ASK(X, e, bn) returns a distribution over X
  inputs: X, the query variable
          e, observed values for variables E
          bn, a Bayesian network specifying joint distribution P(X_1, ..., X_n)

  factors ← [ ]
  for each var in ORDER(bn.VARS) do
     factors ← [MAKE-FACTOR(var, e)|factors]
     if var is a hidden variable then factors ← SUM-OUT(var, factors)
  return NORMALIZE(POINTWISE-PRODUCT(factors))
```

**Figure 14.11** The variable elimination algorithm for inference in Bayesian networks.




$\alpha P(B)\sum_{e'} P(e')\sum_{a'} P(a'|B,e')P(j|a')P(m|a')$
$\alpha P(B)\sum_{e'} P(e')(\begin{pmatrix}0.95 & 0.94 & 0.29 & 0.001\end{pmatrix} * 0.90 * 0.70 + \begin{pmatrix}0.05 & 0.06 & 0.71 & 0.999\end{pmatrix} * 0.05 * 0.01)$
$\alpha P(B)\sum_{e'} P(e')(\begin{pmatrix}0.5985 & 0.5922 & 0.1827 & 0.00063\end{pmatrix} + \begin{pmatrix}0.000025 & 0.00003 & 0.000355 & 0.0004995\end{pmatrix})$
$\alpha P(B)\sum_{e'} P(e')\begin{pmatrix}0.598525 & 0.59223 & 0.183055 & 0.0011295\end{pmatrix}$
$\alpha P(B)(0.002 * \begin{pmatrix}0.598525 & 0.183055\end{pmatrix} + 0.998 * \begin{pmatrix}0.59223 & 0.0011295\end{pmatrix})$
$\alpha P(B)(\begin{pmatrix}0.00119705 & 0.00036611\end{pmatrix} + \begin{pmatrix}0.59104554 & 0.001127251\end{pmatrix})$
$\alpha P(B)\begin{pmatrix}0.59224259 & 0.001493351\end{pmatrix}$
$\alpha\begin{pmatrix}0.001 & 0.999\end{pmatrix}\begin{pmatrix}0.59224259 & 0.001493351\end{pmatrix}$
$\alpha\begin{pmatrix}0.00059224259 & 0.00149185765\end{pmatrix}$
$\approx \langle 0.284, 0.716\rangle$, **which is correct**