**ComS 472 - PS5 Due: Oct 13, 2024 Name: Aren Ashlock**

1. **(6 pts)** (Exercise 6.1) How many solutions are there for the map-coloring problem in Figure 6.1 below? How many solutions if four colors are allowed? Two colors?

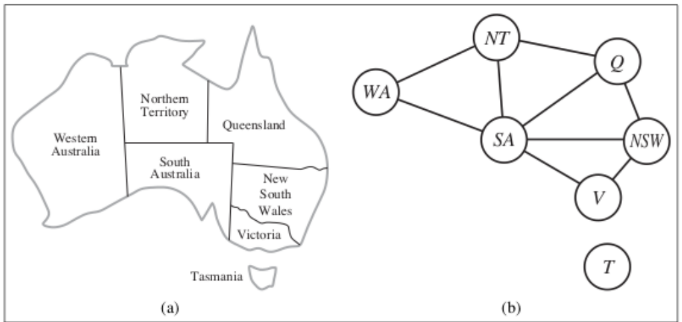


**Figure 6.1** (a) The principal states and territories of Australia. Coloring this map can be viewed as a constraint satisfaction problem (CSP). The goal is to assign colors to each region so that no neighboring regions have the same color. (b) The map-coloring problem represented as a constraint graph.

Solutions $= 3 * 2 * 1 * 1 * 1 * 1 * 3 = 18$
Solutions (for 4 colors) $= 4 * 3 * 2 * 2 * 2 * 2 * 4 = 768$
Solutions (for 2 colors) = None

2. **(10 pts)** (Exercise 6.5) Solve the cryptarithmetic problem in Figure 6.2 on the next page by hand, using the strategy of backtracking with forward checking and the MRV and least-constraining-value heuristics.

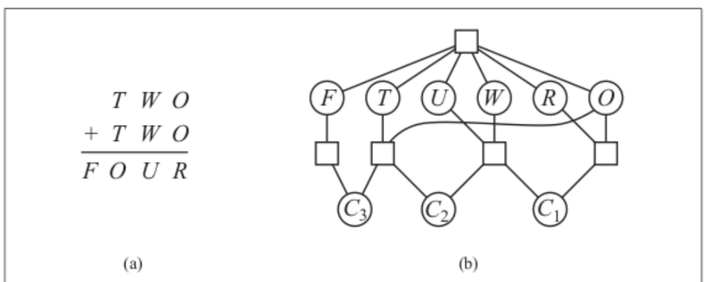


**Figure 6.2** (a) A cryptarithmetic problem. Each letter stands for a distinct digit; the aim is to find a substitution of digits for letters such that the resulting sum is arithmetically correct, with the added restriction that no leading zeroes are allowed. (b) The constraint hypergraph for the cryptarithmetic problem, showing the *Alldiff* constraint (square box at the top) as well as the column addition constraints (four square boxes in the middle). The variables $C_1$, $C_2$, and $C_3$ represent the carry digits for the three columns.

1. Choose $F$ since the domain is $\{1\}$ (MRV), so $F = 1$.
2. Then choose $T$ since the domain is $\{2 - 9\}$ (MRV), which we have $C_2 + 2T = O + 10 * C_3$, and we know $C_3 = 1$ since $C_3 = F$. This reduces to $C_2 + 2T - 10 = O$. Using least-constraining-value, we can chose $\{6 - 9\}$ since any the other values result in impossible values for $O$. So, try $T = 6$.
3. By choosing $T = 6$, that reduces the domain of $O$ to $\{2, 3\}$ (MRV), since $C_2 + 2 = O$. Checking for the least-constraining-value, we can see $2O = R + 10 * C_1$. $C_1 = 0$ since it being 1 would result in a negative number. Either value in the domain works in this case, so choose $O = 2$, which also means $C_2 = 0$.
4. Since we chose $O = 2$, the domain of $R$ is only $\{4\}$ (MRV), so we have to choose $R = 4$.
5. The domains of the last 2 letters are the same $\{0, 3, 5, 7, 8, 9\}$, so find a value for $U$. To try and find the least-constraining-value, this equation exists: $2W = U$. The only value that could work would be 8 since $U$ needs to be an even number due to the equation. So choose $U = 8$.
6. The final letter, $W$, has no valid options since the only value that satisfies $2W = U$ since $R = 4$. Therefore, we must backtrack, which we have to backtrack all the way back to $O$.
7. Now we have $F = 1$, $T = 6$, and now we choose $O = 3$ (since it is the only other value in the domain), which means $C_2 = 1$.
8. However, choosing $O = 3$ results in no options for $R$. This is because our equation is now $6 = R + 10 * C_1$, which means the only option for $R$ is 6. But, we already chose $T$ as 6, so we have to backtrack further to $T$.
9. The domain for $T$ was $\{6 - 9\}$, so we can now try $T = 7$.
10. Reanalyzing $O$ for the least-constraining-value, we have the equation $C_2 + 4 = O$. So the new domain is

$\{4, 5\}$. Try $O = 4$.
11. $R$ has only 1 value in the domain since $O = 4$, it must mean $R = 8$.
12. For the final letters, the domain is now $\{0, 2, 3, 5, 6, 9\}$. As learned earlier, the equation we must satisfy is $2W = U$, so we can only choose $U = 6$ to have a legal value for $W$.
13. Finally, we reduce the equation down to $2W = 6$, so we assign $W = 3$.
**Answer:** $F = 1$, $T = 7$, $O = 4$, $R = 8$, $U = 6$, $W = 3$.

3. **(9 pts)** (Exercise 6.11) Use the AC-3 algorithm to show that arc consistency can detect the inconsistency of the partial assignment $\{WA = green,\ V = red\}$ for the map-coloring problem shown in Figure 6.1 on the previous page.

   * I am skipping over the algorithm iterations which have no effect on the domains *
   Arc = (SA, WA), $D_i$ = {red, green, blue}, $D_j$ = {green}, remove green from $D_i$.
   Arc = (SA, V), $D_i$ = {red, blue}, $D_j$ = {red}, remove red from $D_i$.
   Arc = (NT, SA), $D_i$ = {red, green, blue}, $D_j$ = {blue}, remove blue from $D_i$.
   Arc = (NT, WA), $D_i$ = {red, green}, $D_j$ = {green}, remove green from $D_i$.
   Arc = (Q, NT), $D_i$ = {red, green, blue}, $D_j$ = {red}, remove red from $D_i$.
   Arc = (Q, SA), $D_i$ = {green, blue}, $D_j$ = {blue}, remove blue from $D_i$.
   Arc = (NSW, Q), $D_i$ = {red, green, blue}, $D_j$ = {green}, remove green from $D_i$.
   Arc = (NSW, SA), $D_i$ = {red, blue}, $D_j$ = {blue}, remove blue from $D_i$.
   Arc = (NSW, V), $D_i$ = {red}, $D_j$ = {red}, remove red from $D_i$. Since the size of $D_i$ is 0, this shows an arc inconsistency!