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HW1 — Maths Prerequisites

This homework covers the prerequisites of COM S 474/574 given for Spring 2024 at Iowa State University. It is due before the Thursday class on Feb 2nd 2024.

1 Calculus

1.1 Numbers

Give True/False for the following claims:

1. $\frac{5}{3}$ is a rational. **Answer:** True
2. $\sqrt{3}$ is a rational. **Answer:** False
3. -4 is an integer. **Answer:** True
4. -4 is a natural number. **Answer:** False
5. -4 is a real number. **Answer:** True

1.2 Limits

1. $\lim_{x\to+\infty} \frac{1}{x} = 0$
2. $\lim_{x\to 8} \frac{2x^2-17x+8}{8-x} = \lim_{x\to 8} \frac{(2x-1)(x-8)}{8-x} = \lim_{x\to 8} -2x+1 = -2(8)+1 = -15$
3. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \frac{\sqrt{4}-2}{4-4} = \frac{0}{0} = \lim_{x\to 4} \frac{d}{dx}(\frac{\sqrt{x}-2}{x-4}) = \lim_{x\to 4} \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

1.3 Integrals

Find the following integrals

1. $\int(2x^2+6x^9)dx = \frac{2}{3}x^3 + \frac{6}{10}x^{10} + C$

1.4 Derivatives

Calculate $\frac{df(x)}{dx}$ for the following functions ($f(x)$'s).

1. $f(x) = 2x^2+6x^9 \to \frac{df(x)}{dx} = 4x+54x^8$
2. $f(x) = \sqrt{x^4+1} \to \frac{df(x)}{dx} = \frac{1}{2\sqrt{x^4+1}}(4x^3) = \frac{2x^3}{\sqrt{x^4+1}}$
3. $f(x) = x\sin(5x) \to \frac{df(x)}{dx} = 5xcos(5x) + sin(5x)$
4. $f(x) = \frac{e^x}{x^2} \to \frac{df(x)}{dx} = \frac{x^2e^x - e^x(2x)}{(x^2)^2} = \frac{xe^x(x-2)}{x^4} = \frac{e^x(x-2)}{x^3}$

2 Linear Algebra

Please include your calculation steps whenever applicable.

2.1 Notations

1. $\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Rewrite the equation in the linear algebra form:

$$\begin{cases} 3x + 5y = 1 \\ 2x - y = 0 \end{cases} \tag{1-1}$$

   Answer: $5x + 4y = 1$

3. Let $A = \mathbf{I}_9$, what is a_{11}? What is a_{51}?

   Answer: $a_{11} = 1, a_{51} = 0$

2.2 Multiplications

Compute the following multiplications (note: not every question is valid):

1. $\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac + bd \end{bmatrix}$
2. $\begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} c & d \end{bmatrix} = \begin{bmatrix} ac & ad \\ bc & bd \end{bmatrix}$
3. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$
4. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \end{bmatrix} =$ **NOT VALID (columns of first matrix doesn't equal rows of second matrix)**
5. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ae + bf \\ ce + df \end{bmatrix}$

2.3 Other Operations

Unless specified otherwise, let $A, B \in \mathbb{R}^{n \times n}$.

1. $\mathbf{I}_3^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. $(AB)^T = B^T A^T$
3. $(a^T B)^T = B^T a$
4. $AA^{-1} = \mathbf{I}$
5. Let $A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$, what is A^{-1}?

   $A^{-1} = \frac{1}{0(3)-2(1)} \begin{bmatrix} 3 & -2 \\ -1 & 0 \end{bmatrix}$ **Answer:** $\begin{bmatrix} -3/2 & 1 \\ 1/2 & 0 \end{bmatrix}$

6. Let $x = \begin{bmatrix} 0 & 2 & 1 & 3 \end{bmatrix}^T$, what is $\|x\|_2$? What is $\|x\|_\infty$? What is $\|x\|_1$? What is $\|x^T\|_2$?
   $\|x\|_2 = \sqrt{0^2 + 2^2 + 1^2 + 3^2} = \sqrt{14}$
   $\|x\|_\infty = max(0, 2, 1, 3) = 3$
   $\|x\|_1 = max(6) = 6$
   $\|x^T\|_2 = \sqrt{0^2 + 2^2 + 1^2 + 3^2} = \sqrt{14}$
7. $|\mathbf{I}_2|(\det(\mathbf{I}_2)) = 1(1) - 0(0) = 1$
8. $|\mathbf{I}_{1024}| = 1$

3 Probability

1. Consider the data set $x = \{1, 2, 3, 4, 5, 6, 77, 88, 999\}$. What is the mean of x? What is the median of x? What is the standard deviation of x?
   Mean: $mean = \frac{1185}{9} \approx 132$
   Media: $median = 5$
   Standard Deviation: $SD = (\frac{1}{9}((1-132)^2 + (2-132)^2 + (3-132)^2 + (4-132)^2 + (5-132)^2 + (6-132)^2 + (77-132)^2 + (88-132)^2 + (999-132)^2))^{1/2} \approx 308$
2. If you were to roll the dice one time,
   - what is the probability it will land on a 1? **Answer:** $\frac{1}{6}$
   - what is the probability it will not land on a 1? **Answer:** $\frac{5}{6}$
3. (**The Monty Hall Problem**) You are presented with three doors, A, B, and C. Behind one door is a valuable prize (like a car), while behind the other two doors, there are less desirable items (like goats). The location of the prize is unknown to you. Suppose you choose one of the three doors (let's say A). Your selection is not opened immediately. Monty, who knows what's behind each door, opens one of the other two doors (let's say C), that reveals a goat (never the car). After Monty reveals a goat behind C, you are given a choice: stick with your original selection (A) or switch to the other unopened door (B). Which decision has a higher chance to win (the car)? Why?
   Answer: It is better to switch to the other unopened door. The reason is that there is a 1/3 chance that your original choice selected the door with the car behind it. This results in a 1/3 chance to win the car. However, you have a 2/3 chance of selecting a door with a goat behind it. When the other door with the goat gets revealed, that means switching to the other door yields a win (which I established earlier happens 2/3 times). Therefore, you have twice as likely of a chance to win the car if you switch to the other unopened door.

4 A "Bonus" Question (1 pt)

From a scale of 1 to 5, how difficult is HW1? 1 is "I can do it in my sleep". 5 is "Bowen is ridiculous". 0 is "I refuse to answer this question". This is for my own reference to improve the quality of future assignments. Thanks!

Answer: I would give this assignment a 2. It wasn't super diffult, I just needed to quite a bit of review since it has been 1.5+ years since I did most of this. Also, I had never heard of The Monty Hall Problem before this class! Very intriguing problem to understand...

Submitted by Aren Ashlock on January 26, 2024.