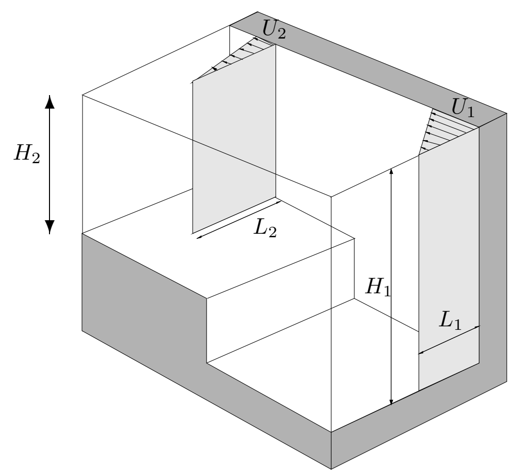
# GFD Homework 2

DUE 7 Feb in class (or by the end of the week)

1. Following the question at the end of class today, take the bottom Ekman layer solution we came up with, and integrate vertically over a reasonable definition of the height of the bottom Ekman layer. What is the depth-averaged magnitude and direction of the Ekman component of the flow in this layer? How about the total flow?

2. The attached classic paper by MacCready and Rhines explores some of the odd things that happen when you have a bottom Ekman layer not on a flat bottom, but over a slope, in particular a slope where isopycnals intersect the slope at some angle. If confused (and it's admittedly confusing), you could also read their 1991 paper, or others cited therein).
   (a) Describe in relatively plan language what they mean by a 'slippery' boundary layer. Why does this happen?
   (b) Discuss one of the conundrums they bring up in the conclusion - what it means, why it's potentially problematic, and what your personal guesses are.
   (c) How would you design an experiment (observational or numerical or laboratory) to evaluate any of these ideas?




3. As depicted in the figure above (from C-R Chapter 7), a vertically uniform but laterally sheared northern hemisphere coastal current must climb a bottom escarpment. Assuming that the jet velocity still vanishes offshore, use conservation of the *nonlinear shallow water PV* (e.g. DQ/Dt=0 following a water parcel) to determine the velocity prole and the width of the jet downstream of the escarpment. Use $H_1 = 200$ m, $H_2 = 160$ m, $U_1 = 0.5$ m/s, $L_1 = 10$ km and f $= 10^{-4}$ s$^{-1}$. What would happen if the downstream depth were only 100 m?

4. Two-layer geostrophic adjustment. Consider a two-layer fluid with resting depths $H_1, H_2$ and densities $\rho_1, \rho_2$. We will discuss the setup for this problem in class on Monday, or you can start by yourself. Let's consider the baroclinic version of the "dam break problem"- in particular assume that at time=0 both layers are at rest and the surface height ($\eta$) and the interface

height ($h$) are given by:

$$\begin{aligned} \eta(t=0) &= 0 && (1)\\ h(t=0) &= h_0 \quad x <= 0 && (2)\\ h(t=0) &= -h_0 \quad x > 0 && (3) \end{aligned}$$

(a) Making a rigid lid assumption ($\eta << h$), and assuming a small density difference ($\rho_1/\rho_2 \approx 1$) and assume that PV is conserved within each layer (we'll go over this in class Monday, but it's also straightforward to just extrapolate what you have in your notes from 1 to 2 layers), derive a single differential equation that governs the interface height once the system has reached a steady-state solution. It's easier to do this for $x > 0$ and $x < 0$ separately.

(b) Assuming the initial displacement is small $h_0 << H_1, H_2$, rewrite this equation in terms of the baroclinic Rossby radius: $a'^2 = g' H_{\text{eff}}/f^2$, $H_{\text{eff}} = (H_1 H_2)/(H_1 + H_2)$

(c) Solve for $h(x)$ in the entire domain and sketch the solution. You'll need to apply reasonable matching conditions at x=0.

(d) What are typical values of the barotropic and baroclinic rossby radii of deformation? (use $H_1 = 100m, H_2 = 1000m, f = 10^{-4}s^{-1}$)