MATH 300: Homework 2

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1 Q1. 4.1

Each of the following statements can be recase in the if-then form. Rewrite the following sentences in the form "If A then B"

1. The product of an odd integer and an even integer is even
   - H: $x = 2k + 1, y = 2j, k \in \mathbb{Z}, j \in \mathbb{Z}$
   - C: $x * y = 2l, l \in \mathbb{Z}$
2. The square of an odd integer is odd
   - H: If we square an odd integer
   - C: Then result is an odd integer
3. The square of a prime numeber is not a prime
   - H: If we square a prime number
   - C: Then the result is not a prime
4. The product of two negative integers is negative
   - If we mutiple two negative integers
   - Then the product will be negative
5. The diagonals of a rhombus are perpendicular
   - If we have two diagonals from a rhombus
   - Then they will be perpendicular
6. Congruent triangles have the same area
   - If we have two congruent triangles
   - Then the area of the trianges will be equal
7. The sum of three conseutive integers is divisble by three
   - If we have the sum of three conseutive integers
   - Then the sum will be divisible by three

2 Q2. 4.2.k

Below you will find pairs of statments A and B. For each pair, please indicate which of the following three senetences are true and which are false

Refer to x, and y as real numbers

$$A : x + y = 0, B : x = 0 \land y = 0$$

False by counter-example:

- Suppose $x = 1, y = -1$
- Observe $1 + (-1) = 0 \land x \neq 0, y \neq 0$
- $\therefore$ the statement is false by counter example $\square$

3 Q3. 5.6

Prove that the product of two odd integers is odd

- Let $x = 2k+1, y = 2j+1$ s.t. $k \in \mathbb{Z} \wedge j \in \mathbb{Z}$
- Observe $x * y = (2k+1)(2j+1) = 4kj + 2k + 2j + 1$
- Observe $(4kj + 2k + 2j + 1) = 2(2kj + k + j) + 1$ where $(2kj + k + j) = z, z \in \mathbb{Z}$
- Thus $x * y = 2z + 1$
- $\therefore$ given two odd integers, if multipled together; then the product is odd $\quad\square$.

4 Q4. 5.13

Let x be an integer. Prove if x is odd iff x+1 is even.

- Case 1: $x \in \mathbb{Z}$ and x is odd s.t. $x = 2k+1, k \in \mathbb{Z}$ by definition of odd
  - Observe $x + 1 = 2k + 2$
  - Thus, $x + 1 = 2(k+1), (k+1) \in \mathbb{Z}$ which must be even by definition of even
  - $\therefore$ if x is odd then $x+1$ is even $\quad\square$
- Case 2: $x \in \mathbb{Z}$ and x+1 is even s.t. $x + 1 = 2k, k \in \mathbb{Z}$ by definition of even
  - Observe $x = 2k - 1$
  - Let $c \in \mathbb{Z}, 2c = 2k - 2$ s.t. $2k = 2c + 2$
  - Subtitute and observe $x = 2k - 1 = 2c + 1$ which is odd by definition of odd
  - $\therefore$ if $x + 1$ is even then x is odd $\quad\square$
- $\therefore$ By Case 1 and Case 2 x is odd iff x+1 is even $\quad\square$

5 Q5. pg 31 p.9

Prove or disprove the following statements:

Let $a, b, c \in \mathbb{Z}$. If $a|c$ and $b|c$, then $(a+b)|c$

- False by counter-example
- Let $a = b = c = 1$
- Observe $a|c = 1|1 \wedge b|c = 1|1$
- Observe $(1+1) \nmid 1 \therefore (a+b) \nmid c$
- $\therefore$ False by counter-example. $\quad\square$

Let $a, b, c \in \mathbb{Z}$. If $a|c$, then $(ac)|(bc)$

- Observe $a|b = ax = b, x \in \mathbb{Z}$
- Multiply both sides by c s.t. $ac(x) = bc$
- This can be rearanged by the defn. of divisiblity $ac(x) = bc = (ac)|(bc)$
- $\therefore$ Given $a, b, c \in \mathbb{Z}$. If $a|c$, then $(ac)|(bc)$ $\quad\square$