

# MATH 300: Homework 1

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## Question 1

In the usual number system.  $P : 1 + 1 = 0$  and  $Q : 3 < 5$  Write the following truth values for expr:

1.  $P \vee Q$
2.  $P \wedge Q$
3.  $\sim P$
4.  $P \implies Q$
5.  $\sim Q \implies \sim P$

### 0.1 Truth Table

$P$	$Q$	$P \vee Q$	$P \wedge Q$	$\sim P$	$P \implies Q$	$\sim Q \implies \sim P$
T	T	T	T	F	T	T
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	F	T	T	T

## Question 2

The Pythagorean theorem from (high school) plane geometry says: If a and b are the legs of a right-angle triangle with c as the hypotenuse, then  $c^2 = a^2 + b^2$ . Write this statement as  $H \implies C$ . What are the converse and contra-positive statements?

1.  $H =$  "If a and b are the legs of a right-angle triangle with c as the hypotenuse"
2.  $C = "c^2 = a^2 + b^2"$

### 0.2 Writing Statments

1. Direct:  $P : H \implies C$

"If a and b are the legs of a right-angle triangle with c as the hypotenuse" then " $c^2 = a^2 + b^2$ "

2. Contra-positive:  $\sim C \implies \sim H : "c^2 = a^2 + b^2"$

If " $c^2 \neq a^2 + b^2$ " then "a and b are NOT the legs of a right-angle triangle with c as the hypotenuse"

3. Converse:  $C \implies H$

If " $c^2 = a^2 + b^2$ " then "a and b are the legs of a right-angle triangle with c as the hypotenuse"

## Question 3

Show using truth tables that  $\sim (P \wedge Q) \equiv \sim P \vee \sim Q$

### 0.3 Truth Table

$P$	$Q$	$P \wedge Q$	$\sim (P \wedge Q)$	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Observe columns corresponding to  $\sim (P \wedge Q)$  and  $\sim P \vee \sim Q$  are identical; therefore are logically equivalent.

### Question 4

Show that the statement  $(P \vee \sim Q) \vee (Q \vee \sim P)$  is a tautology

### 0.4 Truth Table

$P$	$Q$	$\sim Q$	$\sim P$	$P \vee \sim Q$	$Q \vee \sim P$	$(P \vee \sim Q) \vee (Q \vee \sim P)$
T	T	F	F	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

Observe that the column corresponding to  $(P \vee \sim Q) \vee (Q \vee \sim P)$  is always true; therefore is a tautology.

### Question 5

Show the addition table for  $\mathbb{Z}_5$ .

### 0.5 $\mathbb{Z}_5$ Addition Table

x	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3