MATH 300: Dem Images and Stuff

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Forward and Inverse Images

Properties of Forward Images

Let $f : X \to Y$ be a function
Given $A \subset X$ define:
Forward image of A under f

$$f(A) = \{f(a)|a \in A\}$$

Consider $f(x) : x^2 : \mathbb{R} \to \mathbb{R}$

$$A = \{1\} \implies f(A) = \{f(1)\} = \{1\}$$

$$B = \{0, 1, -1\} \implies f(B) = \{f(0), f(-1), f(1)\} = \{0, 1\}$$

Let $A, B \subset X$

1. $A \subset B \implies f(A) \subset f(B)$
2. $f(A \cup B) = f(A) \cup f(B)$
3. $f(A \cap B) \subset f(A) \cap f(B)$

Proof of 2

Let $y \in f(A \cup B)$
$\leftrightarrow y = f(x), x \in (A \cup B)$
$\leftrightarrow$ Where $x \in A \vee x \in B$
$\leftrightarrow$ Thus $y = f(x), x \in A \vee y = f(x), x \in B$
$\leftrightarrow$ Thus $y \in f(A) \vee y \in f(B)$

Proof of 3

Let $f(x) = x^2 : \mathbb{R} \to \mathbb{R}$
$A = \{-1\} \implies f(A) = \{-1\}$ $B = \{-1\} \implies f(B) = \{1\}$
$A \cap B = \emptyset \implies f(A \cap B) = \emptyset \neq f(A) \cap f(B) = \{1\}$

Properties of Inverse Images

Given $C \subset Y$, the inverse image of c under f is given by

$$f^{-1}(C) = \{x \in X|f(x) \in C\}$$

Example: $f : \mathbb{R} \implies \mathbb{R}, f(x) = x^2$
Take $C \subset \mathbb{R}$ then $f^{-1}(C) = \{x \in \mathbb{R}|f(x) \in C\}$
Let $C = \{0\} \implies f^{-1}(C) = \{x \in \mathbb{R}|f(x) \in \{0\}\}$

1. $A \subset B \implies f^{-1}(A) \subset f^{-1}(B)$
2. $f^{-1}(A \cup B) = f(A) \cup f(B)$
3. $f^{-1}(A \cap B) = f(A) \cap f(B)$
4. $f^{-1}(A - B) = f(A) - f(B)$

Counting Principles

If A is a nonempty finite set, then: $|A|$ is the number of distinct objects.
Example: $A = \{1, a\}, B = \{1, a, 5\}$
$A \subset B \implies |A| \leq |B|$
If $A \subset B$ and $|A| = |B| \implies A = B$

0.1 Thm:

Suppose A and B are two finite empty sets.

1. If $A \subset B$ then $|A| \leq |B|$
2. If $A \subset B$ and $|A| = |B|$ then $A = B$