MATH 300: Homework 3

Aren Vista

October 13, 2025

1 14.3 a,c

For each of the following relations defined on the set 1,2,3,4,5 determine whether the relation is reflexive, irreflexive, symmetric, antisymetric, and or transitive.

Recall the following properties:

- R is called a reflexive iff $\forall\ a \in A,\ aRa$
- R is called a irrreflexive iff $\forall\ a \in A, (a,a) \notin R$
- R is symmetric iff $aRb \implies bRa$ when $a, b \in A$
- R is antisymetric iff $aRb \wedge bRa \implies a = b$
- R is transitive iff $aRb \wedge bRc \implies aRc$

$$R = \{(1,1),(2,2),(3,3),(4,4),(5,5)\}$$

- $aRa\ \forall\ a \in A$ therefore is reflexive
- $\forall (a,b) \in A, \exists (b,a) \in A \therefore aRb \implies bRa$ therefore is symmetric
- $a = 1, (1,1) \in R \implies (a,a) \in R \therefore$ not irreflexive
- R is antisymetric iff $aRb \wedge bRa \implies a = b$ Observe $aRb \wedge bRa$ is false; therefore is vaccously true;
- R is transitive iff $aRb \wedge bRc \implies aRc$. Observe $aRb \wedge bRc$ is false; therefore is vaccously true;

$$R = \{(1,1),(1,2),(1,3),(1,4),(1,5)\}$$

- $\exists a = 2 \ni (2,2) \notin R \therefore$ R is not reflexive
- $a = 1, (1,1) \in R \implies (a,a) \in R \therefore$ not irreflexive
- Obs $\exists\ a = 1, b = 2 \ni aRb \not\Longrightarrow bRa$ therefore is not symmetric
- R is antisymetric iff $aRb \wedge bRa \implies a = b$ Observe $aRb \wedge bRa$ is false; therefore is vaccously true;
- R is transitive iff $aRb \wedge bRc \implies aRc$. Observe $aRb \wedge bRc$ is false; therefore is vaccously true;

2 14.15

Prove: A relation on R on a set A is antisymetric iff

$$R \cap R^{-1} \subset \{(a,a) : a \in A\}$$

$$R \text{ is antisymetric iff } aRb \wedge bRa \implies a = b \therefore A = \{(a,b) | aRb \wedge bRa \implies a = b\}$$

Suppose R is antisymmetric.

- If $(a,b) \in R \cap R^{-1}$, then $(a,b) \in R$ and $(b,a) \in R^{-1}$
- By antisymmetry, $a = b$
- Hence, (a,b) is of the form (a,a)
- implying $R \cap R^{-1} \subset \{(a,a) : a \in A\}$

Suppose R is antisymmetric.

- If $(b,a) \in R \cap R^{-1}$, then $(b,a) \in R$ and $(a,b) \in R^{-1}$
- By antisymmetry, $a = b$
- Hence, (b,a) is of the form (b,b)
- implying $R \cap R^{-1} \subset \{(b,b) : a \in A\}$

Therefore R is antisymmetric iff $R \cap R^{-1} \subset \{(a,a) : a \in A\}$.

3 14.16

Give an example of a relation on a set that is both symmetric and transitive but not reflexive.

$$R = (a,b),(b,a),(b,c),(c,b),(a,c),(c,a)$$

Explain what is wrong with the "proof":

- "Suppose R is symmetric and transitive"
- Symmetric means that $xRy \implies yRx$
- Applying transitivity to $xRy \wedge yRx$ to give xRx therefore is reflexive

Answer:

- "Suppose R is symmetric and transitive"
- Symmetric means that $\{(x,y)|xRy \implies yRx\}$
- Transitive $\{(x,y),(y,z),(x,z)|xRy \wedge yRz \implies xRz\}$
- Observe $(x,x) \notin R \therefore$ not reflexive

4 15.3a

Which of the following are equivalence relations?

$$R = \{(1,1),(1,2),(2,1),(2,2),(3,3)\},\ A = \{1,2,3\}$$

$$\forall a \in R,\ aRa \therefore \text{ reflexive } \implies \forall a \in R,\ \exists(a,a) \in R$$

$$\forall a,b \in R,\ aRb \implies bRa \therefore \text{ symmetric } \implies \forall a,b \in R,\ \exists(a,b),(b,a) \in R$$

$$\forall a,b,c \in R,\ aRb \wedge bRc \implies aRc \therefore \text{ transitive by vaccuous truth}$$

Since R is reflexive, transitive, and symmetric, R is an equivalence relation

5 15.7f

For each equivalence relation below, find the requested equivalence class.

- R is has the same size as A, on A=$2^{\{1,2,3,4,5\}}$. Find $[\{1,3\}]$

$$[\{1,3\}] = \left\{ \begin{array}{lllll} \{1,1\}, & \{1,2\}, & \{1,3\}, & \{1,4\}, & \{1,5\}, \\ \{2,2\}, & \{2,3\}, & \{2,4\}, & \{2,5\}, & \\ \{3,3\}, & \{3,4\}, & \{3,5\}, & & \\ \{4,4\}, & \{4,5\}, & & & \\ \{5,5\} & & & & \end{array} \right\}$$