

MATH 300: Homework 7

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Question 1: 24.1

Answer the following:

- Is it a function? Explain if not.
- What is the domain and range?
- Is it injective? Explain if not.
- What is the inverse function?

24.1d

$$f = \{(x, y) | x, y \in \mathbb{Z}; xy = 0\}$$

Observe $f(x)$ contains (x, y) s.t. $x, y \in \mathbb{Z} \wedge xy = 0$

Let $x = 0$. Observe $(0, 1), (0, 2) \in f(x) \implies f(x)$ is not a function.

24.1h

$$f = \{(x, y) | x, y \in \mathbb{Z}; x|y\}$$

Let $x = 1$

Observe 1 divides any value in \mathbb{Z} thus f is not a function.

Question 2: 24.4

Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write down all functions $A \rightarrow B$. Indicate which are injective and which are surjective

Bijjective:

$$f_1 = (1, 3), (2, 4)$$

$$f_2 = (2, 3), (1, 4)$$

Neither:

$$f_3 = (1, 3), (2, 3)$$

$$f_4 = (1, 4), (2, 4)$$

Question 3: 24.8

Let $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7\}$. Let f be a the relation:

$$f = \{(1, 5), (2, 5), (3, 6), (?, ?)\}$$

Presume elements of f are of member $A \times B$ Such that the following conditions are met (independently).

- a. The relation f is not a function
- b. The relation is a function from $A \rightarrow B$ but is not onto
- c. The relation is a function from $A \rightarrow B$ and is onto

a. The relation f is not a function

Presume elements of f are of member $A \times B \implies (?, ?) = (a, b) \ a \in A, b \in B$

For f to not be a function make $a \in (a, b), a \neq 4$

Any element of $a \in A, a \neq 4$ will satisfy the condition.

b. The relation is a function from $A \rightarrow B$ but is not onto

Presume elements of f are of member $A \times B \implies (?, ?) = (a, b) \ a \in A, b \in B$

For f to be a function make $a \in (a, b), a = 4$

For f to not be onto make $b \in (a, b), b \neq 7$

Any element of $b \neq 7, a = 4$ will satisfy the condition.

c. The relation is a function from $A \rightarrow B$ and is onto

Presume elements of f are of member $A \times B \implies (?, ?) = (a, b) \ a \in A, b \in B$

For f to be a function make $a \in (a, b), a = 4$

For f to be onto make $b \in (a, b), b = 7$

Any element of $b = 7, a = 4$ will satisfy the condition.

Question 4

Within \mathbb{R} , let $A = \{x : 0 < x < 1\}$ and $B = \{x : 0 < x < \inf\}$

Define $f : A \rightarrow B$ by $f(x) = \frac{x}{1-x}$

Show f is a bijection and find it's inverse function

f is injective

Let $a_1, a_2, \in A$

$$\frac{a_1}{1-a_1} = \frac{a_2}{1-a_2}$$

Cross multiply and simplify:

$$a_1 - a_1a_2 = a_2 - a_1a_2$$

Simplify:

$$a_1 = a_2$$

Thus f is injective

f is surjective

Let $a \in A$ and $b \in B$

$$b = f(a)$$

Substitute

$$b = \frac{a}{1-a}$$

Multiply by $1-a$

$$b(1-a) = a$$

Simplify

$$b - ab = a$$

Group a terms

$$b = a + ab$$

Factor a terms

$$b = a(1 + b)$$

Isolate a

$$\frac{b}{1 + b} = a$$

Thus f is surjective

The $f^{-1} : B \rightarrow A = f^{-1}(b) : \frac{b}{1+b} = a$

As f is injective and surjective it is bijective

Question 5

Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$ are functions. Show the following

- a. gof is an injection implies f is an injection
- b. gof is a surjection implies g is a surjection

Let $a \in A, b \in B, c \in C$.

a. gof is an injection implies f is an injection

If gof is an injection, then:

$$g(f(a_1)) = g(f(a_2)) \implies a_1 = a_2$$

Thus

$$f(a_1) = f(a_2) \implies a_1 = a_2$$

Therefore f is an injection

b. gof is a surjection implies g is a surjection

If gof is a surjection, then:

$$\forall c \in \text{range}(gof) \implies \exists a \ni g(f(a)) = c \equiv g(b) = c$$

Observe $g(b) = c \implies \exists b \in B \ni \forall c, c = g(b)$ Thus g is surjective.