

3.1.14

Let $b := \frac{1}{1+a}$, where $a > 0$.

Since $(1+a)^n > \frac{1}{2}n(n-1)a^2$, we have $0 < nb^n \leq \frac{n}{\frac{1}{2}n(n-1)a^2} \leq \frac{2}{(n-1)a^2}$

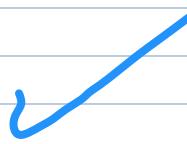
$$\lim(nb^n) \leq \lim \frac{2}{(n-1)a^2} = 0$$

$$\therefore \lim(nb^n) = 0 \quad +7$$

3.2.2

a) Let $x := (n)$, $y := (-n)$

$$\lim(x_n + y_n) = \lim(n + (-n)) = \lim(0) = 0$$



+10

b) Let $x = y := ((-1)^n)$

$$\lim(x_n y_n) = \lim((-1)^n (-1)^n) = \lim((-1)^{2n}) = \lim(1) = 1$$



+10

3.2.4

Since X converges to $x \neq 0$ there exists K such that $\forall n > K : x_n \neq 0$. Let $\lim(x_n y_n) = z$.

X converges x for all n greater than a certain number and XY converges to z for all n greater than some number. Since $y_n = \frac{x_n y_n}{x_n}$ if $x_n \neq 0$, by properties of limits, Y converges $\frac{z}{x}$. Hence Y converges.

+10

3.2.6b

Squeeze Thm:

$$\lim \left(\frac{(-1)^n}{n+2} \right) \Rightarrow -\frac{1}{n} \leq \left| \frac{(-1)^n}{n+2} \right| \leq \frac{1}{n}$$

$$0 \leq \lim_{n \rightarrow \infty} \frac{(-1)^n}{n+2} \leq 0$$

$$\lim \left(\frac{\sqrt{n}-1}{\sqrt{n}+1} \right) \Rightarrow \frac{\sqrt{n}-1}{\sqrt{n}+1} = \frac{\frac{\sqrt{n}}{\sqrt{n}} - \frac{1}{\sqrt{n}}}{\frac{\sqrt{n}}{\sqrt{n}} + \frac{1}{\sqrt{n}}} = \frac{1 - \frac{1}{\sqrt{n}}}{1 + \frac{1}{\sqrt{n}}}$$

1

$$\lim \left(\frac{(-1)^n}{n+2} \right) = 0 \quad +10$$

$$\lim \left(\frac{1 - \frac{1}{\sqrt{n}}}{1 + \frac{1}{\sqrt{n}}} \right) = \frac{1-0}{1+0} = \frac{1}{1} = 1$$

$$\lim \left(\frac{\sqrt{n}-1}{\sqrt{n}+1} \right) = 1 \quad +6$$

3.2.9

$$y_n = \sqrt{n+1} - \sqrt{n}$$

$$y_n = \sqrt{n+1} - \sqrt{n} \cdot \left(\frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}} \right) = \frac{(\sqrt{n+1})^2 - (\sqrt{n})^2}{\sqrt{n+1} + \sqrt{n}} = \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

$$y_n = \frac{1}{\sqrt{n+1} + \sqrt{n}} \rightarrow \lim(y_n) = 0$$

We have $\sqrt{n} y_n = \frac{\sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \frac{\frac{\sqrt{n}}{\sqrt{n}}}{\frac{\sqrt{n+1}}{\sqrt{n}} + \frac{\sqrt{n}}{\sqrt{n}}} = \frac{1}{\sqrt{1+\frac{1}{n}} + 1}$, so that $\lim(\sqrt{n} y_n) = \frac{1}{2}$