

## Lecture 1

Thursday, August 24, 2023 6:35 PM

Welcome to Math301!

### Review of Sets/Functions (1.1)

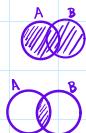
Ways to represent sets:

- Listing (good for finite sets)
- Use a property that determines the elements of the set

Ex.  $\mathbb{Q} = \left\{ \frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \right\}$

$\mathbb{N}$  : set of natural numbers including 0 (nonnegative integers)

Union  $A \cup B = \{x : x \in A \text{ or } x \in B\}$

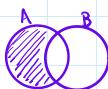


Intersection  $A \cap B = \{x : x \in A \text{ and } x \in B\}$



Complement of a set

Complement of B relative to A:



$$A \setminus B = \{x : x \in A \text{ and } x \notin B\}$$

A and B are disjoint if  $A \cap B = \emptyset$

\*  $A \cup B = (A \cap B) \cup (B \setminus A) \cup (A \setminus B)$

DeMorgan's Law

- $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$
- $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

$$A_1 \cup A_2 \cup \dots \cup A_n = \{x : x \text{ belongs to at least one of } A_1, \dots, A_n\}$$

$$\bigcup_{n=1}^{\infty} A_n = \{x : x \in A_n \text{ for some } n \in \mathbb{N}\}$$

$$\bigcap_{n=1}^{\infty} A_n = \{x : x \in A_n \text{ for all } n \in \mathbb{N}\}$$

Ex  $A_n = [n, \infty)$

so  $\bigcap_{n=1}^{\infty} A_n = \emptyset$  If  $x \in \bigcap_{n=1}^{\infty} A_n$ , then  $x \in A_n \text{ for all } n \in \mathbb{N}$

Then  $x \geq n \quad \forall n \in \mathbb{N}$  which cannot exist

Proof by Contradiction

Suppose the statement is false. Then get to a contradiction

which implies that the statement must be true.

You're about to see  
that this is Hong's favorite  
technique

### Code

- Black : Regular text
- Blue : definitions, theorems #1704C8
- Turquoise/Teal : proofs/notes for proofs #11918E
- Fun little notes to myself
- Review of past stuff
- Diagrams? Idk it's a fun color #6709E5

$\mathbb{Q}$      $\mathbb{R}$   
 $\mathbb{N}$      $\mathbb{Z}$

## Functions $f : A \rightarrow B$

A function from  $A$  to  $B$  is a set  $f$  of ordered pairs in  $A \times B$  such that for each  $a \in A$ , there exists a unique  $b \in B$  with  $(a, b) \in f$ .

3 main properties:

- domain : set of first elements
- codomain : set of second elements
- rule of correspondence

Ex  $f(x) = x^2 + 4$

Domain =  $\mathbb{R}$

$f : \mathbb{R} \rightarrow \mathbb{R}$  mapping from  $\mathbb{R}$  to  $\mathbb{R}$

Codomain (or range) =  $[0, \infty)$

\* personally I think it would be  $[4, \infty)$  but this is what they wrote

Direct Image : subset of codomain

$$f(E) = \{f(x) : x \in E\}$$

Inverse Image : subset of domain

$$f^{-1}(H) = \{x \text{ is in domain of } f(x) \in H\}$$

T/F?  $f^{-1}(f(E)) = E$

No.  $f^{-1}(f(E)) = E \Leftrightarrow E \subseteq f^{-1}(f(E))$  this is true  
 $f^{-1}(f(E)) \subseteq E$  this is false

First, prove  $E \subseteq f^{-1}(f(E))$

Let  $x \in E$ . Then  $f(x) \in f(E)$ , which implies  $x \in f^{-1}(f(E))$ .

So,  $E \subseteq f^{-1}(f(E))$

Show  $f^{-1}(f(E)) \not\subseteq E$  using a counterexample.

$$E = [12, \infty), f(E) = [0, \infty)$$

$$f^{-1}(f(E)) = f^{-1}([0, \infty))$$

So  $f^{-1}(f(E)) \subseteq E$  is not generally true.

Unless I say so

T/F  $f(f^{-1}(H)) = H$

No  $f(f^{-1}(H)) \subseteq H$  True  
 $H \subseteq f(f^{-1}(H))$  False

First prove  $f(f^{-1}(H)) \subseteq H$

Let  $y \in f(f^{-1}(H))$  and  $y = f(x)$ . Then  $x \in f^{-1}(H)$ .

Since  $y = f(x)$ ,  $f(x) \in H$ , so  $y \in H$ .

Attempt to prove  $H \subseteq f(f^{-1}(H))$  (and fail)

Let  $y \in H$ . Then  $y = f(x)$  and  $x \in f^{-1}(H)$ . Then,  $f(x) \in f(f^{-1}(H))$ , so  $y \in f(f^{-1}(H))$ .

But this is only true if  $H$  is in the range of  $f$ .

More definitions:

Injective (one-to-one) : if  $f(x_1) = f(x_2)$  implies  $x_1 = x_2$

Surjective (onto) : if the direct image of the domain is the codomain

Bijective : function is both injective and surjective