

MATH 301: Homework 2

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Question 1

Prove Theorem 1.3.4 (b)

If a set A with $m \in \mathbb{N}$ elements and $C \subseteq A$ is a set with one element then $A - C$ is a set with $m - 1$ elements

Proof. Suppose a set A with $m \in \mathbb{N}$ elements

$$A = \{a_1, a_2, \dots, a_{m-1}, a_m\} \implies |A| = m$$

Suppose $C \subseteq A$ is a set with one element

$$C = \{a\}, a \in A$$

Observe a is an arbitrary element of A

WLOG let $a = a_m$ s.t.

$$C = \{a_m\}$$

Thus

$$A - C = \{a_1, a_2, \dots, a_{m-1}\} \implies |A - C| = m - 1$$

□

Question 2

Prove Theorem 1.3.4 (c)

If C is an infinite set and B is a finite set

Then $C - B$ is an infinite set

Recall an infinite set is a set which is not finite (defn.)

Proof. Suppose C is an infinite set

$$C = \{c_1, c_2, \dots\}$$

Suppose B is a finite set

$$B = \{b_1, b_2, \dots, b_n\}, n \in \mathbb{N}$$

Suppose for the sake of contradiction $C - B$ is finite s.t

$$C - B = \{x_1, x_2, \dots, x_m\} m \in \mathbb{N}$$

Observe at most $C \cap B$ must be finite (n number of elements) by definition of set intersection

$$C \cap B = \{b_1, b_2, \dots, b_n\}$$

By rearrangement of the principle of exclusion

$$C = C - B \cup (C \cap B)$$

If $C - B$ is finite and $C \cap B$ is finite then C must be finite. This is a logical inconsistency.

Thus, $C - B$ must be infinite.

□

Question 3

Question 4

Question 5

Question 6