

Lecture 1

Thursday, August 24, 2023 6:35 PM

Welcome to Math301!

Review of Sets/Functions (1.1)

Ways to represent sets:

- Listing (good for finite sets)
- Use a property that determines the elements of the set

Ex. $\mathbb{Q} = \left\{ \frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \right\}$

\mathbb{N} : set of natural numbers including 0 (nonnegative integers)

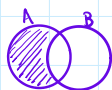
Union $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Intersection $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Complement of a set

Complement of B relative to A:

$$A \setminus B = \{x : x \in A \text{ and } x \notin B\}$$



A and B are disjoint if $A \cap B = \emptyset$

$$* A \cup B = (A \cap B) \cup (B \setminus A) \cup (A \setminus B)$$

DeMorgan's Law

- $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$
- $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

$$A_1 \cup A_2 \cup \dots \cup A_n = \{x : x \text{ belongs to at least one of } A_1, \dots, A_n\}$$

$$\bigcup_{n=1}^{\infty} A_n = \{x : x \in A_n \text{ for some } n \in \mathbb{N}\}$$

$$\bigcap_{n=1}^{\infty} A_n = \{x : x \in A_n \text{ for all } n \in \mathbb{N}\}$$

Ex $A_n = [n, \infty)$

so $\bigcap_{n=1}^{\infty} A_n = \emptyset$

If $x \in \bigcap_{n=1}^{\infty} A_n$, then $x \in A_n$ for all $n \in \mathbb{N}$

Then $x \geq n \quad \forall n \in \mathbb{N}$ which cannot exist

Proof by Contradiction

Suppose the statement is false. Then get to a contradiction

which implies that the statement must be true.

You're about to see
that this is King's favorite
technique

Code

- Black : Regular text
- Blue : definitions, theorems #1704C8
- Turquoise/Teal : proofs/notes for proofs #11918E
- Fun little notes to myself
- Review of past stuff
- Diagrams? Idk it's a fun color #6709E5

\mathbb{Q} \mathbb{R}
 \mathbb{N} \mathbb{Z}

Functions $f: A \rightarrow B$

A function from A to B is a set f of ordered pairs in $A \times B$ such that for each $a \in A$, there exists a unique $b \in B$ with $(a, b) \in f$.

3 main properties:

- domain: set of first elements
- codomain: set of second elements
- rule of correspondence

Ex $f(x) = x^2 + 4$

Domain = \mathbb{R}

$f: \mathbb{R} \rightarrow \mathbb{R}$ mapping from \mathbb{R} to \mathbb{R}

Codomain (or range) = $[0, \infty)$

* personally I think it would be $[4, \infty)$ but this is what they want

Direct Image: subset of codomain

$$f(E) = \{f(x) : x \in E\}$$

Inverse Image: subset of domain

$$f^{-1}(H) = \{x \text{ is in domain of } f(x) \in H\}$$

T/F? $f^{-1}(f(E)) = E$

No. $f^{-1}(f(E)) = E \iff E \subseteq f^{-1}(f(E))$ this is true
 $f^{-1}(f(E)) \subseteq E$ this is false

First, prove $E \subseteq f^{-1}(f(E))$

Let $x \in E$. Then $f(x) \in f(E)$, which implies $x \in f^{-1}(f(E))$.

So, $E \subseteq f^{-1}(f(E))$

Show $f^{-1}(f(E)) \not\subseteq E$ using a counterexample.

$$E = [12, \infty), f(E) = [0, \infty)$$

$$f^{-1}(f(E)) = f^{-1}([0, \infty))$$

So $f^{-1}(f(E)) \subseteq E$ is not generally true.

Unless I say so

T/F $f(f^{-1}(H)) = H$

No. $f(f^{-1}(H)) \subseteq H$ True
 $H \subseteq f(f^{-1}(H))$ False

First prove $f(f^{-1}(H)) \subseteq H$

Let $y \in f(f^{-1}(H))$ and $y = f(x)$. Then $x \in f^{-1}(H)$.

Since $y = f(x)$, $f(x) \in H$, so $y \in H$.

Attempt to prove $H \subseteq f(f^{-1}(H))$ (and fail)

Let $y \in H$. Then $y = f(x)$ and $x \in f^{-1}(H)$. Then, $f(x) \in f(f^{-1}(H))$,

so $y \in f(f^{-1}(H))$.

But this is only true if H is in the range of f .

More definitions:

Injective (one-to-one) : if $f(x_1) = f(x_2)$ implies $x_1 = x_2$

Surjective (onto) : if the direct image of the domain is the codomain

Bijective : function is both injective and surjective