Lecture 20 - Exam 2 Review (Class)

Wednesday, November 15, 2023 10:32 AM

* came late

Bolzano-Weierstrass is hard to test on so might not show up

lim sup is always ≥ lim inf

If $\overline{\lim} \neq \underline{\lim}$, the sequence diverges

Cauchy $|x_n - x_m| < \varepsilon$
where $n, m \geq N(\varepsilon)$

Cauchy implies convergent

How to verify a sequence is Cauchy

- using the def
- contractive

Properly Divergent Sequences

- Ratio Test
- Comparison Test $\lim_{n\to\infty} \frac{x_n}{y_n} = L > 0$

do not need to prove anything

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but need to know how to apply these results

☆ If a sequence is not bounded from above, it tends to ∞

☆ There will be limits of sequences

Ex Show $x_n = \frac{n}{n+1}$ is Cauchy.

$$|x_n - x_m| = \left|\frac{n}{n+1} - \frac{m}{m+1}\right|$$

$$= \left|\frac{1}{n} - \frac{1}{m}\right| \le \left|\frac{1}{n}\right| + \left|\frac{1}{m}\right| < \varepsilon$$

Triangle Inequality

For all $n, m \ge N(\varepsilon)$: $n > \frac{2}{\varepsilon}$, $m > \frac{2}{\varepsilon}$

So $\frac{1}{n} < \frac{\varepsilon}{2}$, $\frac{1}{m} < \frac{\varepsilon}{2}$

Ex $\overline{\lim} \frac{1}{x_n} = \frac{1}{\underline{\lim}\, x_n}$ (assuming $\underline{\lim}\, x_n > 0$)

$\overline{\lim} \frac{1}{x_n} = \inf V$ where $V = \{v : v < \frac{1}{x_n}$ for a finite number of n's$\}$.

$\underline{\lim}\, x_n = \sup W$ where $W = \{w : w > x_n$ for a finite number of n's$\}$.

$\forall v \in V$, $\frac{1}{v} \in \mathbb{N}$ and $\forall w \in W$, $\frac{1}{w} \in V$

$$\frac{1}{v} \leq \underline{\lim}\, x_n$$

$$\Longrightarrow \frac{1}{\underline{\lim}\, x_n} \leq v \Rightarrow \frac{1}{\underline{\lim}\, x_n} \leq \overline{\lim}\left(\frac{1}{x_n}\right)$$

$$\overline{\lim}\left(\frac{1}{x_n}\right) \leq \frac{1}{w} \Longrightarrow w \leq \frac{1}{\overline{\lim}\, x_n}$$

$$\underline{\lim}\, x_n \leq \frac{1}{\overline{\lim}\, \frac{1}{x_n}} \Longrightarrow \overline{\lim}\, \frac{1}{x_n} \leq \frac{1}{\underline{\lim}\, x_n}$$

Thus $\overline{\lim}\, \frac{1}{x_n} = \frac{1}{\underline{\lim}\, x_n}$