3.1.14

Let $b := \frac{1}{1+a}$, where $a > 0$.

Since $(1+a)^n > \frac{1}{2}n(n-1)a^2$, we have $0 < nb^n \leq \frac{n}{\frac{1}{2}n(n-1)a^2} \leq \frac{2}{(n-1)a^2}$

$\lim(nb^n) \leq \lim\limits_{n\to\infty} \frac{2}{(n-1)a^2} = 0$

$\therefore \lim(nb^n) = 0$

+7

3.2.2

a) Let $X := (n)$, $Y := (-n)$

$\lim(x_n + y_n) = \lim(n + (-n)) = \lim(0) = 0$

b) Let $X = Y := ((-1)^n)$

$\lim(x_n y_n) = \lim((-1)^n(-1)^n) = \lim((-1)^{2n}) = \lim(1) = 1$

+10

3.2.4

Since X converges to $x \neq 0$ there exists K such that $\forall n > K$: $x_n \neq 0$. Let $\lim(x_n y_n) = z$. X converges x for all n greater than a certain number and XY converges to z for all n greater than some number. Since $y_n = \frac{x_n y_n}{x_n}$ if $x_n \neq 0$, by properties of limits, Y converges $\frac{z}{x}$. Hence Y converges.

+10

3.2.6b

Squeeze Thm:

$\lim\left(\frac{(-1)^n}{n+2}\right) \Rightarrow -\frac{1}{n} \leq \left|\frac{(-1)^n}{n+2}\right| \leq \frac{1}{n}$

$0 \leq \lim\limits_{n\to\infty} \frac{(-1)^n}{n+2} \leq 0$

$\lim\left(\frac{(-1)^n}{n+2}\right) = 0$

+10

3.2.6c

$\lim\left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right) \Rightarrow * \ \frac{\sqrt{n}-1}{\sqrt{n}+1} = \frac{\cancel{\frac{\sqrt{n}}{\sqrt{n}}}^{\,1} - \frac{1}{\sqrt{n}}}{\cancel{\frac{\sqrt{n}}{\sqrt{n}}}_{\,1} + \frac{1}{\sqrt{n}}} = \frac{1 - \frac{1}{\sqrt{n}}}{1 + \frac{1}{\sqrt{n}}}$

$\lim\left(\frac{1 - \frac{1}{\sqrt{n}}}{1 + \frac{1}{\sqrt{n}}}\right) = \frac{1-0}{1+0} = \frac{1}{1} = 1$

$\lim\left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right) = 1$

+6

3.2.9

$y_n = \sqrt{n+1} - \sqrt{n}$

$$y_n = \sqrt{n+1} - \sqrt{n} \cdot \left(\frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}\right) = \frac{(\sqrt{n+1})^2 - (\sqrt{n})^2}{\sqrt{n+1} + \sqrt{n}} = \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

$$y_n = \frac{1}{\sqrt{n+1} + \sqrt{n}} \Rightarrow \lim(y_n) = 0$$

We have $\sqrt{n}\, y_n = \dfrac{\sqrt{n}}{\sqrt{n+1} + \sqrt{n}} = \dfrac{\frac{\sqrt{n}}{\sqrt{n}}}{\frac{\sqrt{n+1}}{\sqrt{n}} + \frac{\sqrt{n}}{\sqrt{n}}} = \dfrac{1}{\sqrt{1+\frac{1}{n}} + 1}$, so that $\lim(\sqrt{n}\, y_n) = \frac{1}{2}$