

Lecture 13

Wednesday, October 18, 2023

10:05 AM

Spent a lot of time reviewing the exam

Subsequences of convergent sequences converge to the same limit

Theorem If a sequence $X = (x_n)$ of real numbers converges to x ($x \in \mathbb{R}$), then any subsequence of X also converges to x .

Proof Let $\epsilon > 0$ be given and let $N(\epsilon) \in \mathbb{N}$ be such that if $n \geq N(\epsilon)$, then $|x_n - x| < \epsilon$. Since $n_1 < n_2 < \dots < n_k < \dots$ is an increasing sequence of natural numbers, we can prove by Induction that $n_k \geq k$. So, if $k \geq N(\epsilon)$, we also have $n_k \geq k \geq N(\epsilon)$, so $|x_{n_k} - x| < \epsilon$. Thus, the subsequence (x_{n_k}) also converges to x . ■

Monotone Subsequence Theorem

If $X = (x_n)$ is a sequence of real numbers, then there is a subsequence of (x_n) that is monotone.