Lecture 25 - Final Lecture

(12/11)

Monday, January 8, 2024 9:21 PM

Review day

Random example $\lim_{x \to 5} \frac{x^2+4}{x-6} = \frac{29}{-1} = -29$

Prove with def: $\left| \frac{x^2+4}{x-6} + 29 \right| = \left| \frac{x^2+4+29x-(6\cdot 29)}{x-6} \right| = \left| \frac{x^2+29x-170}{x-6} \right|$ factor using quadratic formula

$\delta = \min\left\{ \frac{1}{2}, \frac{\varepsilon}{79} \right\}$

Or can use sequential criterion

$\forall (x_n) \in A$, $(x_n) \to 5$ as $n \to \infty$, where $x_n \neq 5$

$$\frac{x_n^2+4}{x_n-6} \to -29$$

$$\Rightarrow \frac{5^2+4}{5-6} = -29 \checkmark$$

For continuity, if c is isolated, then it is not a cluster point

$\forall (x_n) \to c$, $(f(x_n)) \to f(c)$

Convergence def

A sequence $(x_n) \in \mathbb{R}$ converges to $x \in \mathbb{R}$ if $\forall \varepsilon > 0$, $\exists N(\varepsilon) \in \mathbb{N}$ such that $\forall n \geq N(\varepsilon)$, $|x_n - x| < \varepsilon$.

Continuity def

Let $A \subseteq \mathbb{R}$, let $f: A \to \mathbb{R}$, and let $c \in A$. f is **continuous** at c if

$\forall \varepsilon > 0$, $\exists\ \delta > 0$ such that if x is any point $\in A$ such that

$|x - c| < \delta$, then $|f(x) - f(c)| < \varepsilon$.

Ex $f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$

Using the density properties, we can show that this does not converge

Lipschitz condition

$$|f(x) - f(y)| \leq K\,|x - y| \qquad K \in \mathbb{R}$$

Ex $\frac{1}{x}$ on $[a, \infty)$ where $a > 0$

$$\left| \frac{1}{x} - \frac{1}{y} \right| = \left| \frac{x-y}{xy} \right| \leq \frac{1}{|a^2|} |x - y|$$