

Section 2.1: 10, 11, 13, 14, 20, 23, 25

Section 2.2: 1, 2, 10, 12, 17

2.1.10a)

If $a < b$ and $c \leq d$, then $a + c < b + d$.

Let's suppose that $c = d$. This implies that $a + c < b + d$.

So if $c < d$ we can construct $a < b$ where $a + c < b + c$ and $c \leq d$ to be constructed $b + c \leq b + d$.

$\therefore a < b$ and $c \leq d$ then $a + c < b + d$

2.1.10b)

We know that $a > b$ we multiply the multiplicative inverse of b which is b^{-1} to both sides.

$$a < b \Rightarrow b^{-1}a \quad b \cdot b^{-1} \Rightarrow \frac{a}{b} < 1$$

Same goes for $c \leq d$

$$c \leq d \Rightarrow d^{-1}c \quad d \cdot d^{-1} \Rightarrow \frac{c}{d} \leq 1$$

then multiplying $\frac{a}{b}$ and $\frac{c}{d}$ results in $\frac{ac}{bd} \leq 1$ by multiplying bd on both sides we get $ac \leq bd$.

If $c \leq d$ holds then $0 \leq ac \leq bd$ holds.

2.1.11a)

Proof by contradiction:

Assume $\frac{1}{a} \neq 0$. If $\frac{1}{a} = 0$, then $1 = a \cdot (\frac{1}{a}) = a \cdot 0 = 0$, which contradicts multiplicative inverse.

If $\frac{1}{a} < 0$, then this implies $1 = a(\frac{1}{a}) < 0$ contradicts the Trichotomy properties. Thus $\frac{1}{a} > 0$, and to show that $\frac{1}{(\frac{1}{a})} = a$.

We know that $a \cdot \frac{1}{a} = 1$ by the multiplicative inverse property.

$$a \cdot \frac{1}{a} = 1 \quad \text{multiplicative inverse}$$

$$a \cdot \left(\frac{1}{a}\right)^{-1} \left(\frac{1}{a}\right) = 1 \cdot \left(\frac{1}{a}\right)^{-1} \quad \text{multiplicative inverse/identity}$$

$$\therefore a = \frac{1}{(\frac{1}{a})}$$

2.1.11 b)

If $a < b$,