

Section 2.1: 10, 11, 13, 14, 20, 23, 25

Section 2.2: 1, 2, 10, 12, 17

2.1.10a)

If $a < b$ and $c \leq d$, then $a+c < b+d$.

Let's suppose that $c=d$. This implies that $a+c < b+d$.

So if $c < d$ we can construct $a < b$ where $a+c < b+d$ and $c \leq d$ to be constructed $b+c \leq b+d$.

$\therefore a < b$ and $c \leq d$ then $a+c < b+d$

2.1.10b)

We know that $a > b$ we multiply the multiplicative inverse of b which is b^{-1} to both sides.

$$a > b \Rightarrow b^{-1}a - b \cdot b^{-1} \Rightarrow \frac{a}{b} > 1$$

Same goes for $c < d$

$$c < d \Rightarrow d^{-1}c - d \cdot d^{-1} \Rightarrow \frac{c}{d} < 1$$

then multiplying $\frac{a}{b}$ and $\frac{c}{d}$ results in $\frac{ac}{bd} \leq 1$ by multiplying bd on both sides we get $ac \leq bd$.

If $c \leq d$ holds then $0 \leq ac \leq bd$ holds.

2.1.11a)

Proof by contradiction:

Assume $y_a > 0$. If $y_a = 0$, then $1 = a \cdot (y_a) = a \cdot 0 = 0$, which contradicts multiplicative inverse.

If $y_a < 0$, then this implies $1 = a(y_a) < 0$ contradicts the Trichotomy properties. Thus $y_a > 0$, and to show that $y(y_a) = a$.

We know that $a \cdot \frac{1}{a} = 1$ by the multiplicative inverse property.

$$a \cdot \frac{1}{a} = 1 \quad \begin{matrix} \text{multiplicative} \\ \text{inverse} \end{matrix}$$

$$a \cdot \left(\frac{1}{a}\right)^{-1} \left(\frac{1}{a}\right) = 1 \cdot \left(\frac{1}{a}\right)^{-1} \quad \begin{matrix} \text{multiplicative} \\ \text{inverse/identity} \end{matrix}$$

$$\therefore a = \frac{1}{\left(\frac{1}{a}\right)}$$

2.1.11 b)

If $a < b$,